

Computer algebra independent integration tests

4-Trig-functions/4.2-Cosine/4.2.4.1-a+b-cos^m-A+B-cos+C-cos²-

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3.172	$\int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	602
3.173	$\int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	605
3.174	$\int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	608
3.175	$\int \frac{(A+C \cos^2(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{4/3}} dx$	611
3.176	$\int \cos^m(c+dx)(b \cos(c+dx))^{4/3} (A+C \cos^2(c+dx)) dx$	614
3.177	$\int \cos^m(c+dx)(b \cos(c+dx))^{2/3} (A+C \cos^2(c+dx)) dx$	617
3.178	$\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A+C \cos^2(c+dx)) dx$	620
3.179	$\int \frac{\cos^m(c+dx)(A+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$	623
3.180	$\int \frac{\cos^m(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$	626
3.181	$\int \frac{\cos^m(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$	629
3.182	$\int (a \cos(c+dx))^m (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	632
3.183	$\int \cos^2(c+dx) (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	635
3.184	$\int \cos(c+dx) (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	638
3.185	$\int (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	641
3.186	$\int (b \cos(c+dx))^n (A+C \cos^2(c+dx)) \sec(c+dx) dx$	644
3.187	$\int (b \cos(c+dx))^n (A+C \cos^2(c+dx)) \sec^2(c+dx) dx$	647
3.188	$\int (b \cos(c+dx))^n (A+C \cos^2(c+dx)) \sec^3(c+dx) dx$	650
3.189	$\int (b \cos(c+dx))^n (A+C \cos^2(c+dx)) \sec^4(c+dx) dx$	653
3.190	$\int \cos^{\frac{5}{2}}(c+dx) (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	656
3.191	$\int \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	659
3.192	$\int \sqrt{\cos(c+dx)} (b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx$	662
3.193	$\int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$	665
3.194	$\int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\sqrt[3]{\cos^2(c+dx)}} dx$	668
3.195	$\int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\sqrt[5]{\cos^2(c+dx)}} dx$	671
3.196	$\int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\sqrt[7]{\cos^2(c+dx)}} dx$	674
3.197	$\int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\sqrt[9]{\cos^2(c+dx)}} dx$	677
3.198	$\int (a + a \cos(e+fx))^m (A+C \cos^2(e+fx)) dx$	680
3.199	$\int (a + a \cos(c+dx))^{2/3} (A+C \cos^2(c+dx)) dx$	683

3.200	$\int \sqrt[3]{a + a \cos(c + dx)} (A + C \cos^2(c + dx)) dx$	686
3.201	$\int \frac{A+C \cos^2(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx$	689
3.202	$\int \frac{A+C \cos^2(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx$	692
3.203	$\int (a + b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx$	695
3.204	$\int \sqrt[3]{a + b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$	699
3.205	$\int \frac{A+C \cos^2(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$	703
3.206	$\int \frac{A+C \cos^2(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$	707
3.207	$\int (a + b \cos(e + fx))^m (A - A \cos^2(e + fx)) dx$	711
3.208	$\int (a + b \cos(e + fx))^m (A + C \cos^2(e + fx)) dx$	714
3.209	$\int (a \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx$	717
3.210	$\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx$	720
3.211	$\int \cos^m(c + dx) (b \cos(c + dx))^{2/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx$	723
3.212	$\int \cos^m(c + dx) (b \cos(c + dx))^{4/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx$	726
3.213	$\int \frac{\cos^m(c+dx)(B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$	729
3.214	$\int \frac{\cos^m(c+dx)(B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$	732
3.215	$\int \frac{\cos^m(c+dx)(B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$	735
3.216	$\int (a \cos(c + dx))^m (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	738
3.217	$\int \cos^2(c + dx) (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	741
3.218	$\int \cos(c + dx) (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	744
3.219	$\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	747
3.220	$\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$	750
3.221	$\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$	753
3.222	$\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$	756
3.223	$\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx$	759
3.224	$\int \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	762
3.225	$\int \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	765
3.226	$\int \sqrt{\cos(c + dx)} (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx$	768
3.227	$\int \frac{(b \cos(c+dx))^n (B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$	771
3.228	$\int \frac{(b \cos(c+dx))^n (B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	774
3.229	$\int \frac{(b \cos(c+dx))^n (B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	777
3.230	$\int \frac{(b \cos(c+dx))^n (B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	780
3.231	$\int \frac{(b \cos(c+dx))^n (B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	783
3.232	$\int (a + a \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx$	786
3.233	$\int (a + b \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx$	789
3.234	$\int (a + b \cos(c + dx))^{2/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx$	793
3.235	$\int \sqrt[3]{a + b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx$	797
3.236	$\int \frac{B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$	801
3.237	$\int \frac{B \cos(c+dx)+C \cos^2(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$	805
3.238	$\int (a \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx)) dx$	809
3.239	$\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$	812
3.240	$\int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$	816
3.241	$\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$	820

3.242	$\int \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx) dx$	824
3.243	$\int \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx) dx$	828
3.244	$\int \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx) dx$	832
3.245	$\int \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^4(c+dx) dx$	836
3.246	$\int \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^5(c+dx) dx$	840
3.247	$\int \cos(c+dx)(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	844
3.248	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	848
3.249	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx) dx$	852
3.250	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx) dx$	856
3.251	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx) dx$	860
3.252	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^4(c+dx) dx$	864
3.253	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^5(c+dx) dx$	868
3.254	$\int (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^6(c+dx) dx$	872
3.255	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	876
3.256	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx) dx$	880
3.257	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx) dx$	884
3.258	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx) dx$	888
3.259	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^4(c+dx) dx$	892
3.260	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^5(c+dx) dx$	896
3.261	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^6(c+dx) dx$	900
3.262	$\int (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^7(c+dx) dx$	904
3.263	$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	908
3.264	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	912
3.265	$\int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$	916
3.266	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	920
3.267	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	923
3.268	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	927
3.269	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	931
3.270	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^4(c+dx)}{\sqrt{b \cos(c+dx)}} dx$	935
3.271	$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	939
3.272	$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	943
3.273	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	947
3.274	$\int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$	951
3.275	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	955
3.276	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	959
3.277	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	963
3.278	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{3/2}} dx$	967
3.279	$\int \frac{\cos^5(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	971
3.280	$\int \frac{\cos^4(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	975
3.281	$\int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	979

3.282	$\int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	983
3.283	$\int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$	987
3.284	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$	991
3.285	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{5/2}} dx$	995
3.286	$\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$	999
3.287	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{7/2}} dx$	1003
3.288	$\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	1007
3.289	$\int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	1011
3.290	$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	1015
3.291	$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$	1018
3.292	$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	1021
3.293	$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	1024
3.294	$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	1027
3.295	$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	1031
3.296	$\int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$	1035
3.297	$\int \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	1040
3.298	$\int \sqrt{\cos(c+dx)} (b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	1044
3.299	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$	1048
3.300	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	1051
3.301	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	1054
3.302	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	1057
3.303	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	1060
3.304	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$	1064
3.305	$\int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx$	1068
3.306	$\int \sqrt{\cos(c+dx)} (b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx$	1073
3.307	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$	1077
3.308	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$	1081
3.309	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$	1084
3.310	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$	1087
3.311	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$	1090
3.312	$\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$	1093

- 3.313 $\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx \dots\dots\dots 1097$
- 3.314 $\int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)} dx \dots\dots\dots 1101$
- 3.315 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1106$
- 3.316 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1110$
- 3.317 $\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1113$
- 3.318 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1116$
- 3.319 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1119$
- 3.320 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1122$
- 3.321 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1126$
- 3.322 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{9}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx \dots\dots\dots 1130$
- 3.323 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1135$
- 3.324 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1139$
- 3.325 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1142$
- 3.326 $\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1145$
- 3.327 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1148$
- 3.328 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1151$
- 3.329 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1155$
- 3.330 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx \dots\dots\dots 1159$
- 3.331 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1164$
- 3.332 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1168$
- 3.333 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1171$
- 3.334 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1174$
- 3.335 $\int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1177$
- 3.336 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1180$
- 3.337 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1184$
- 3.338 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx \dots\dots\dots 1188$
- 3.339 $\int \cos(c+dx)(b \cos(c+dx))^{2/3} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx \dots\dots\dots 1193$
- 3.340 $\int (b \cos(c+dx))^{2/3} (A+B \cos(c+dx)+C \cos^2(c+dx)) dx \dots\dots\dots 1196$
- 3.341 $\int (b \cos(c+dx))^{2/3} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx) dx \dots\dots\dots 1199$
- 3.342 $\int (b \cos(c+dx))^{2/3} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx) dx \dots\dots\dots 1202$
- 3.343 $\int (b \cos(c+dx))^{2/3} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx) dx \dots\dots\dots 1205$
- 3.344 $\int (b \cos(c+dx))^{2/3} (A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^4(c+dx) dx \dots\dots\dots 1208$

- 3.345 $\int \cos(c + dx)(b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1211$
- 3.346 $\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1214$
- 3.347 $\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \dots\dots\dots 1217$
- 3.348 $\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx \dots\dots\dots 1220$
- 3.349 $\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx \dots\dots\dots 1223$
- 3.350 $\int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx \dots\dots\dots 1226$
- 3.351 $\int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1229$
- 3.352 $\int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1232$
- 3.353 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1235$
- 3.354 $\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1238$
- 3.355 $\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1241$
- 3.356 $\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1245$
- 3.357 $\int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1248$
- 3.358 $\int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1251$
- 3.359 $\int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1254$
- 3.360 $\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1257$
- 3.361 $\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1260$
- 3.362 $\int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1264$
- 3.363 $\int \cos^m(c + dx)(b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1267$
- 3.364 $\int \cos^m(c + dx)(b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1270$
- 3.365 $\int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1273$
- 3.366 $\int \frac{\cos^m(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx \dots\dots\dots 1276$
- 3.367 $\int \frac{\cos^m(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx \dots\dots\dots 1280$
- 3.368 $\int \frac{\cos^m(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx \dots\dots\dots 1283$
- 3.369 $\int (a \cos(c + dx))^m (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1286$
- 3.370 $\int \cos^2(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1289$
- 3.371 $\int \cos(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1292$
- 3.372 $\int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1295$
- 3.373 $\int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \dots\dots\dots 1298$
- 3.374 $\int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx \dots\dots\dots 1301$
- 3.375 $\int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx \dots\dots\dots 1304$
- 3.376 $\int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx \dots\dots\dots 1307$
- 3.377 $\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1310$
- 3.378 $\int \sqrt{\cos(c + dx)}(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots\dots\dots 1313$
- 3.379 $\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx \dots\dots\dots 1316$
- 3.380 $\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx \dots\dots\dots 1319$
- 3.381 $\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx \dots\dots\dots 1322$
- 3.382 $\int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx \dots\dots\dots 1325$
- 3.383 $\int (a + a \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx)) dx \dots\dots\dots 1328$

3.384	$\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots \dots \dots$	1331
3.385	$\int \sqrt[3]{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots \dots \dots$	1334
3.386	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx \dots \dots \dots$	1337
3.387	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx \dots \dots \dots$	1340
3.388	$\int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots \dots \dots$	1343
3.389	$\int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \dots \dots \dots$	1347
3.390	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx \dots \dots \dots$	1351
3.391	$\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx \dots \dots \dots$	1355
3.392	$\int (a + b \cos(e + fx))^m (A + (A + C) \cos(e + fx) + C \cos^2(e + fx)) dx \dots \dots \dots$	1359
3.393	$\int (a + b \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx)) dx \dots \dots \dots$	1363

4 Listing of Grading functions

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Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from <https://rulebasedintegration.org>

The number of integrals in this report is [393]. This is test number [93].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or Hypergeometric2F1 functions. RootSum and RootOf are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (393)	% 0. (0)
Mathematica	% 98.98 (389)	% 1.02 (4)
Maple	% 60.05 (236)	% 39.95 (157)
Maxima	% 30.28 (119)	% 69.72 (274)
Fricas	% 30.79 (121)	% 69.21 (272)
Sympy	% 1.78 (7)	% 98.22 (386)
Giac	% 3.82 (15)	% 96.18 (378)

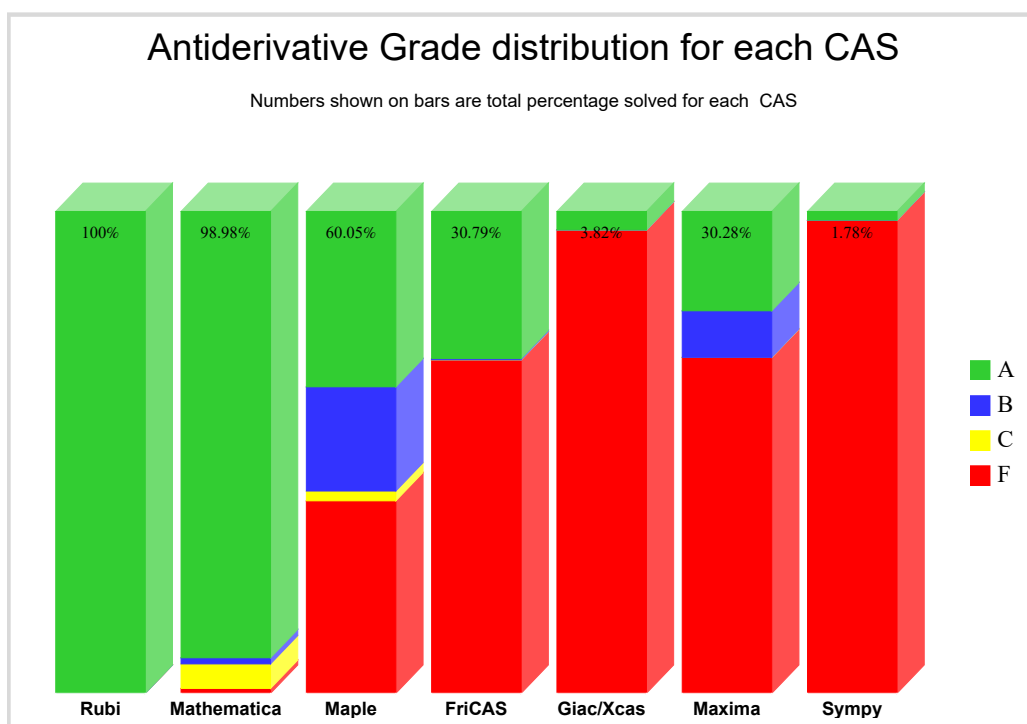
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

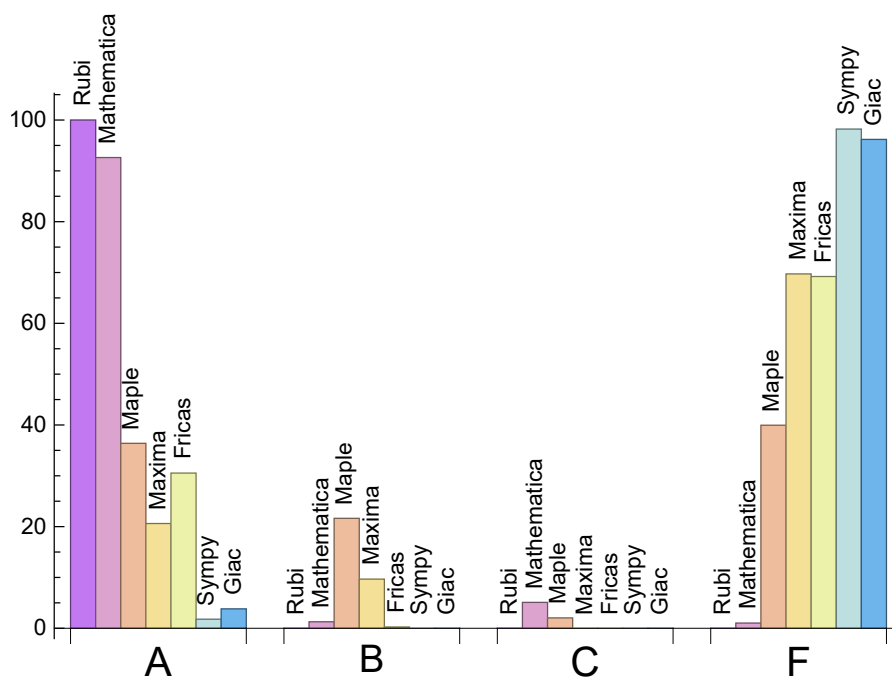
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	92.62	1.27	5.09	1.02
Maple	36.39	21.63	2.04	39.95
Maxima	20.61	9.67	0.	69.72
Fricas	30.53	0.25	0.	69.21
Sympy	1.78	0.	0.	98.22
Giac	3.82	0.	0.	96.18

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.12	132.62	1.	120.	1.
Mathematica	0.82	219.95	1.27	92.	0.74
Maple	2.73	246.95	2.1	216.	1.77
Maxima	2.22	638.75	5.05	144.	1.58
Fricas	1.81	528.97	4.91	597.	4.65
Sympy	10.97	183.	2.39	158.	2.16
Giac	1.17	87.07	1.46	81.	1.43

1.4 list of integrals that has no closed form antiderivative

{

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {

Mathematica {

Maple {

Maxima {

Fricas {

Sympy {

Giac {

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {

Mathematica {67, 76, 198, 203, 204, 205, 206, 207, 208, 233, 234, 235, 236, 237, 267, 268, 276, 354, 355, 383, 388, 389, 390, 391, 393}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using `sagemath` (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```

from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')

```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

1.9.2 Important note about FriCAS and Giac/X-CAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error Exception raised: NotImplementedError

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the builtin system function LeafSize is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

#see <https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in->

```

def tree(expr):
    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()+map(tree, expr.operands())

```

```

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1

```

For Sympy, called directly from Python, the following code is used

```

try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1

```

When these cas systems have a builtin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
June 22, 2018

Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 69, 70, 71, 72, 73, 74, 75, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 162, 164, 165, 166, 168, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 199, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293,

294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391 }

B grade: { 208, 233, 355, 361, 393 }

C grade: { 35, 36, 66, 67, 68, 76, 161, 163, 167, 169, 198, 200, 232, 267, 268, 276, 283, 383, 384, 386 }

F grade: { 202, 385, 387, 392 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 239, 240, 241, 242, 243, 247, 248, 249, 250, 251, 255, 256, 257, 258, 259, 263, 264, 265, 266, 267, 271, 272, 273, 274, 275, 279, 280, 281, 282, 283, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 95, 97, 121, 123, 244, 245, 246, 252, 253, 254, 260, 261, 262, 268, 269, 270, 276, 277, 278, 284, 285, 286, 287 }

C grade: { 26, 27, 28, 29, 30, 31, 32, 33 }

F grade: { 34, 35, 36, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 89, 90, 91, 92, 93, 94, 98, 99, 100, 101, 102, 103, 107, 108, 109, 110, 111, 112, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 132, 133, 134, 135, 136, 288, 289, 290, 291, 292, 293, 297, 298, 299, 300, 301, 302, 306, 307, 308, 309, 310, 311, 315, 316, 317, 318, 319, 323, 324, 325, 326, 327, 331, 332, 333, 334, 335 }

B grade: { 35, 36, 95, 96, 97, 104, 105, 106, 113, 114, 115, 121, 122, 123, 129, 130, 131, 137, 138, 139, 294, 295, 296, 303, 304, 305, 312, 313, 314, 320, 321, 322, 328, 329, 330, 336, 337, 338 }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, }

365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 24, 25, 35, 36, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338 }

B grade: { 12 }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 26, 27, 28, 29, 30, 31, 32, 33, 34, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.6 SymPy

A grade: { 1, 2, 3, 4, 9, 10, 11 }

B grade: { }

C grade: { }

F grade: { 5, 6, 7, 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15 }

B grade: { }

C grade: { }

F grade: { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table, the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	133	94	101	207	199	126
normalized size	1	1.	1.45	1.02	1.1	2.25	2.16	1.37
time (sec)	N/A	0.071	0.049	0.044	1.069	1.648	34.283	1.15

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	101	74	81	162	151	103
normalized size	1	1.	1.4	1.03	1.12	2.25	2.1	1.43
time (sec)	N/A	0.066	0.027	0.037	1.048	1.616	11.133	1.177

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	71	54	58	113	105	77
normalized size	1	1.	1.42	1.08	1.16	2.26	2.1	1.54
time (sec)	N/A	0.052	0.018	0.04	1.043	1.626	3.173	1.131

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	50	33	46	69	56	46
normalized size	1	1.	1.67	1.1	1.53	2.3	1.87	1.53
time (sec)	N/A	0.023	0.015	0.039	1.011	1.651	0.717	1.16

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	32	51	107	0	54
normalized size	1	1.	1.46	1.33	2.12	4.46	0.	2.25
time (sec)	N/A	0.031	0.016	0.059	1.031	1.692	0.	1.203

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	48	59	78	192	0	81
normalized size	1	1.	1.2	1.48	1.95	4.8	0.	2.02
time (sec)	N/A	0.037	0.028	0.064	1.071	1.705	0.	1.189

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	54	98	131	243	0	132
normalized size	1	1.	0.77	1.4	1.87	3.47	0.	1.89
time (sec)	N/A	0.047	0.115	0.079	1.035	1.667	0.	1.239

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	75	138	170	293	0	163
normalized size	1	1.	0.77	1.41	1.73	2.99	0.	1.66
time (sec)	N/A	0.06	0.3	0.067	1.067	1.73	0.	1.203

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	93	106	176	216	354	117
normalized size	1	1.	0.79	0.91	1.5	1.85	3.03	1.
time (sec)	N/A	0.067	0.156	0.037	1.565	1.697	19.178	1.151

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	86	139	167	258	92
normalized size	1	1.	0.76	0.97	1.56	1.88	2.9	1.03
time (sec)	N/A	0.053	0.096	0.052	1.586	1.657	6.598	1.161

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	99	119	158	58
normalized size	1	1.	0.74	1.07	1.62	1.95	2.59	0.95
time (sec)	N/A	0.041	0.086	0.037	1.683	1.608	1.72	1.122

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	21	27	76	0	27
normalized size	1	1.	1.	1.4	1.8	5.07	0.	1.8
time (sec)	N/A	0.024	0.009	0.064	1.495	1.593	0.	1.163

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	35	36	95	0	46
normalized size	1	1.	0.84	0.81	0.84	2.21	0.	1.07
time (sec)	N/A	0.038	0.087	0.067	1.091	1.545	0.	1.154

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	61	58	58	140	0	77
normalized size	1	1.	0.94	0.89	0.89	2.15	0.	1.18
time (sec)	N/A	0.044	0.201	0.07	1.019	1.689	0.	1.156

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	81	78	81	188	0	107
normalized size	1	1.	0.93	0.9	0.93	2.16	0.	1.23
time (sec)	N/A	0.05	0.302	0.069	1.013	1.609	0.	1.193

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	324	0	0	0	0
normalized size	1	1.	0.78	2.87	0.	0.	0.	0.
time (sec)	N/A	0.082	0.404	3.893	0.	0.	0.	0.

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	296	0	0	0	0
normalized size	1	1.	0.76	2.62	0.	0.	0.	0.
time (sec)	N/A	0.083	0.367	3.369	0.	0.	0.	0.

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	0	0	0
normalized size	1	1.	0.91	3.39	0.	0.	0.	0.
time (sec)	N/A	0.06	0.131	3.912	0.	0.	0.	0.

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	236	0	0	0	0
normalized size	1	1.	0.77	3.15	0.	0.	0.	0.
time (sec)	N/A	0.057	0.149	3.829	0.	0.	0.	0.

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	0	0	0
normalized size	1	1.	0.77	2.92	0.	0.	0.	0.
time (sec)	N/A	0.062	0.148	4.411	0.	0.	0.	0.

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	0	0	0
normalized size	1	1.	0.74	3.77	0.	0.	0.	0.
time (sec)	N/A	0.064	0.212	3.996	0.	0.	0.	0.

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	601	0	0	0	0
normalized size	1	1.	0.7	5.23	0.	0.	0.	0.
time (sec)	N/A	0.087	0.264	9.671	0.	0.	0.	0.

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	77	413	0	0	0	0
normalized size	1	1.	0.67	3.59	0.	0.	0.	0.
time (sec)	N/A	0.09	0.375	7.382	0.	0.	0.	0.

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	23	99	0	51	0	0
normalized size	1	1.	1.1	4.71	0.	2.43	0.	0.
time (sec)	N/A	0.023	0.056	1.698	0.	1.582	0.	0.

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	99	0	51	0	0
normalized size	1	1.	1.	4.71	0.	2.43	0.	0.
time (sec)	N/A	0.023	0.057	2.026	0.	1.623	0.	0.

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	78	249	0	0	0	0
normalized size	1	1.	0.68	2.17	0.	0.	0.	0.
time (sec)	N/A	0.125	0.808	0.6	0.	0.	0.	0.

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	79	668	0	0	0	0
normalized size	1	1.	0.69	5.81	0.	0.	0.	0.
time (sec)	N/A	0.134	0.467	0.648	0.	0.	0.	0.

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	199	0	0	0	0
normalized size	1	1.	0.74	2.55	0.	0.	0.	0.
time (sec)	N/A	0.096	0.221	0.587	0.	0.	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	55	590	0	0	0	0
normalized size	1	1.	0.74	7.97	0.	0.	0.	0.
time (sec)	N/A	0.096	0.141	0.513	0.	0.	0.	0.

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	190	0	0	0	0
normalized size	1	1.	0.77	2.53	0.	0.	0.	0.
time (sec)	N/A	0.1	0.128	0.652	0.	0.	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	61	608	0	0	0	0
normalized size	1	1.	0.79	7.9	0.	0.	0.	0.
time (sec)	N/A	0.099	0.232	0.592	0.	0.	0.	0.

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	79	241	0	0	0	0
normalized size	1	1.	0.69	2.1	0.	0.	0.	0.
time (sec)	N/A	0.133	0.525	0.533	0.	0.	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	636	0	0	0	0
normalized size	1	1.	0.7	5.53	0.	0.	0.	0.
time (sec)	N/A	0.128	0.623	0.547	0.	0.	0.	0.

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	114	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.173	1.754	0.	0.	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	113	0	236	81	0	0
normalized size	1	1.	3.65	0.	7.61	2.61	0.	0.
time (sec)	N/A	0.042	0.193	1.934	2.218	1.395	0.	0.

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	B	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	119	0	236	80	0	0
normalized size	1	1.	3.72	0.	7.38	2.5	0.	0.
time (sec)	N/A	0.05	0.198	1.523	2.159	1.328	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	88	322	0	0	0	0
normalized size	1	1.	0.79	2.88	0.	0.	0.	0.
time (sec)	N/A	0.099	0.304	3.27	0.	0.	0.	0.

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	89	294	0	0	0	0
normalized size	1	1.	0.81	2.67	0.	0.	0.	0.
time (sec)	N/A	0.092	0.332	3.734	0.	0.	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	70	261	0	0	0	0
normalized size	1	1.	0.91	3.39	0.	0.	0.	0.
time (sec)	N/A	0.055	0.08	3.621	0.	0.	0.	0.

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	59	237	0	0	0	0
normalized size	1	1.	0.81	3.25	0.	0.	0.	0.
time (sec)	N/A	0.073	0.133	2.95	0.	0.	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	55	214	0	0	0	0
normalized size	1	1.	0.8	3.1	0.	0.	0.	0.
time (sec)	N/A	0.088	0.201	3.233	0.	0.	0.	0.

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	56	292	0	0	0	0
normalized size	1	1.	0.74	3.84	0.	0.	0.	0.
time (sec)	N/A	0.089	0.174	4.084	0.	0.	0.	0.

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	84	598	0	0	0	0
normalized size	1	1.	0.76	5.44	0.	0.	0.	0.
time (sec)	N/A	0.114	0.307	8.442	0.	0.	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	83	411	0	0	0	0
normalized size	1	1.	0.73	3.64	0.	0.	0.	0.
time (sec)	N/A	0.114	0.48	7.637	0.	0.	0.	0.

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	91	324	0	0	0	0
normalized size	1	1.	0.83	2.95	0.	0.	0.	0.
time (sec)	N/A	0.093	0.242	3.417	0.	0.	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	86	296	0	0	0	0
normalized size	1	1.	0.76	2.62	0.	0.	0.	0.
time (sec)	N/A	0.081	0.067	3.625	0.	0.	0.	0.

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	71	263	0	0	0	0
normalized size	1	1.	0.95	3.51	0.	0.	0.	0.
time (sec)	N/A	0.08	0.066	3.659	0.	0.	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	61	239	0	0	0	0
normalized size	1	1.	0.8	3.14	0.	0.	0.	0.
time (sec)	N/A	0.105	0.082	3.189	0.	0.	0.	0.

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	57	216	0	0	0	0
normalized size	1	1.	0.79	3.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.137	4.014	0.	0.	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	0	0	0
normalized size	1	1.	0.74	3.77	0.	0.	0.	0.
time (sec)	N/A	0.099	0.17	3.707	0.	0.	0.	0.

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	84	599	0	0	0	0
normalized size	1	1.	0.74	5.3	0.	0.	0.	0.
time (sec)	N/A	0.127	0.229	8.875	0.	0.	0.	0.

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	413	0	0	0	0
normalized size	1	1.	0.72	3.59	0.	0.	0.	0.
time (sec)	N/A	0.123	0.304	7.	0.	0.	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	88	324	0	0	0	0
normalized size	1	1.	0.78	2.87	0.	0.	0.	0.
time (sec)	N/A	0.08	0.063	3.396	0.	0.	0.	0.

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	296	0	0	0	0
normalized size	1	1.	0.78	2.64	0.	0.	0.	0.
time (sec)	N/A	0.109	0.075	4.09	0.	0.	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	73	263	0	0	0	0
normalized size	1	1.	0.94	3.37	0.	0.	0.	0.
time (sec)	N/A	0.096	0.053	3.398	0.	0.	0.	0.

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	65	239	0	0	0	0
normalized size	1	1.	0.83	3.06	0.	0.	0.	0.
time (sec)	N/A	0.092	0.154	3.631	0.	0.	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	0	0	0
normalized size	1	1.	0.77	2.92	0.	0.	0.	0.
time (sec)	N/A	0.097	0.134	3.762	0.	0.	0.	0.

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	0	0	0
normalized size	1	1.	0.74	3.77	0.	0.	0.	0.
time (sec)	N/A	0.097	0.171	3.911	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	601	0	0	0	0
normalized size	1	1.	0.7	5.23	0.	0.	0.	0.
time (sec)	N/A	0.121	0.226	9.168	0.	0.	0.	0.

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	413	0	0	0	0
normalized size	1	1.	0.72	3.59	0.	0.	0.	0.
time (sec)	N/A	0.127	0.392	7.737	0.	0.	0.	0.

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	94	349	0	0	0	0
normalized size	1	1.	0.64	2.37	0.	0.	0.	0.
time (sec)	N/A	0.126	0.375	3.71	0.	0.	0.	0.

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	83	321	0	0	0	0
normalized size	1	1.	0.72	2.79	0.	0.	0.	0.
time (sec)	N/A	0.093	0.395	3.267	0.	0.	0.	0.

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	293	0	0	0	0
normalized size	1	1.	0.69	2.62	0.	0.	0.	0.
time (sec)	N/A	0.089	0.234	3.243	0.	0.	0.	0.

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	73	260	0	0	0	0
normalized size	1	1.	0.91	3.25	0.	0.	0.	0.
time (sec)	N/A	0.063	0.066	3.135	0.	0.	0.	0.

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	58	236	0	0	0	0
normalized size	1	1.	0.77	3.15	0.	0.	0.	0.
time (sec)	N/A	0.054	0.119	3.214	0.	0.	0.	0.

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	200	213	0	0	0	0
normalized size	1	1.	2.82	3.	0.	0.	0.	0.
time (sec)	N/A	0.076	1.236	3.432	0.	0.	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	73	73	141	291	0	0	0	0
normalized size	1	1.	1.93	3.99	0.	0.	0.	0.
time (sec)	N/A	0.089	1.425	3.503	0.	0.	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	522	601	0	0	0	0
normalized size	1	1.	4.66	5.37	0.	0.	0.	0.
time (sec)	N/A	0.112	6.281	9.168	0.	0.	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	74	412	0	0	0	0
normalized size	1	1.	0.67	3.75	0.	0.	0.	0.
time (sec)	N/A	0.123	0.365	7.625	0.	0.	0.	0.

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	97	729	0	0	0	0
normalized size	1	1.	0.66	4.96	0.	0.	0.	0.
time (sec)	N/A	0.151	0.796	10.628	0.	0.	0.	0.

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	0	0	0
normalized size	1	1.	0.75	2.82	0.	0.	0.	0.
time (sec)	N/A	0.091	0.4	3.818	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	0	0	0
normalized size	1	1.	0.7	2.57	0.	0.	0.	0.
time (sec)	N/A	0.088	0.252	3.627	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	263	0	0	0	0
normalized size	1	1.	0.86	3.29	0.	0.	0.	0.
time (sec)	N/A	0.071	0.164	3.815	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	239	0	0	0	0
normalized size	1	1.	0.78	3.06	0.	0.	0.	0.
time (sec)	N/A	0.068	0.138	3.462	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	0	0	0
normalized size	1	1.	0.77	2.92	0.	0.	0.	0.
time (sec)	N/A	0.088	0.113	3.459	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	75	75	140	294	0	0	0	0
normalized size	1	1.	1.87	3.92	0.	0.	0.	0.
time (sec)	N/A	0.084	1.416	3.602	0.	0.	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	81	601	0	0	0	0
normalized size	1	1.	0.72	5.32	0.	0.	0.	0.
time (sec)	N/A	0.132	0.272	8.447	0.	0.	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	77	413	0	0	0	0
normalized size	1	1.	0.69	3.69	0.	0.	0.	0.
time (sec)	N/A	0.148	0.36	7.003	0.	0.	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	86	324	0	0	0	0
normalized size	1	1.	0.75	2.82	0.	0.	0.	0.
time (sec)	N/A	0.112	0.394	3.361	0.	0.	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	80	296	0	0	0	0
normalized size	1	1.	0.7	2.57	0.	0.	0.	0.
time (sec)	N/A	0.103	0.239	3.497	0.	0.	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	69	263	0	0	0	0
normalized size	1	1.	0.86	3.29	0.	0.	0.	0.
time (sec)	N/A	0.07	0.157	3.615	0.	0.	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	61	239	0	0	0	0
normalized size	1	1.	0.78	3.06	0.	0.	0.	0.
time (sec)	N/A	0.066	0.126	3.711	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	57	216	0	0	0	0
normalized size	1	1.	0.77	2.92	0.	0.	0.	0.
time (sec)	N/A	0.071	0.115	3.707	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	58	294	0	0	0	0
normalized size	1	1.	0.74	3.77	0.	0.	0.	0.
time (sec)	N/A	0.066	0.186	3.767	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	81	601	0	0	0	0
normalized size	1	1.	0.72	5.37	0.	0.	0.	0.
time (sec)	N/A	0.107	0.092	8.561	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	77	413	0	0	0	0
normalized size	1	1.	0.68	3.65	0.	0.	0.	0.
time (sec)	N/A	0.122	0.295	7.829	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	81	601	0	0	0	0
normalized size	1	1.	0.7	5.23	0.	0.	0.	0.
time (sec)	N/A	0.092	0.085	9.02	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	77	413	0	0	0	0
normalized size	1	1.	0.67	3.59	0.	0.	0.	0.
time (sec)	N/A	0.116	0.092	7.634	0.	0.	0.	0.

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	70	70	150	170	0	0
normalized size	1	1.	0.6	0.6	1.29	1.47	0.	0.
time (sec)	N/A	0.059	0.223	0.461	2.107	1.485	0.	0.

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	67	88	101	549	0	0
normalized size	1	1.	0.59	0.78	0.89	4.86	0.	0.
time (sec)	N/A	0.073	0.169	0.489	2.051	1.747	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	77	126	0	0
normalized size	1	1.	0.7	0.64	1.04	1.7	0.	0.
time (sec)	N/A	0.038	0.086	0.379	2.037	1.392	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	52	54	70	459	0	0
normalized size	1	1.	0.58	0.6	0.78	5.1	0.	0.
time (sec)	N/A	0.023	0.073	0.428	1.897	1.719	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	44	55	108	554	0	0
normalized size	1	1.	0.65	0.81	1.59	8.15	0.	0.
time (sec)	N/A	0.039	0.046	0.415	2.067	1.749	0.	0.

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	108	518	0	0
normalized size	1	1.	0.76	0.76	1.83	8.78	0.	0.
time (sec)	N/A	0.031	0.066	0.377	1.89	1.688	0.	0.

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	59	134	983	591	0	0
normalized size	1	1.	0.76	1.72	12.6	7.58	0.	0.
time (sec)	N/A	0.043	0.098	0.411	2.143	1.722	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	51	54	473	128	0	0
normalized size	1	1.	0.65	0.68	5.99	1.62	0.	0.
time (sec)	N/A	0.044	0.178	0.397	2.124	1.436	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	214	3129	687	0	0
normalized size	1	1.	0.66	1.75	25.65	5.63	0.	0.
time (sec)	N/A	0.065	0.262	0.334	2.507	1.805	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	70	70	158	182	0	0
normalized size	1	1.	0.59	0.59	1.33	1.53	0.	0.
time (sec)	N/A	0.058	0.244	0.303	2.112	1.52	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	67	88	111	568	0	0
normalized size	1	1.	0.58	0.76	0.96	4.9	0.	0.
time (sec)	N/A	0.057	0.184	0.515	2.065	1.679	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	53	47	81	134	0	0
normalized size	1	1.	0.7	0.62	1.07	1.76	0.	0.
time (sec)	N/A	0.033	0.031	0.356	2.07	1.473	0.	0.

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	52	54	74	467	0	0
normalized size	1	1.	0.56	0.58	0.8	5.02	0.	0.
time (sec)	N/A	0.03	0.095	0.25	1.863	1.779	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	44	55	112	562	0	0
normalized size	1	1.	0.63	0.79	1.6	8.03	0.	0.
time (sec)	N/A	0.033	0.056	0.226	2.017	1.76	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	45	45	108	527	0	0
normalized size	1	1.	0.74	0.74	1.77	8.64	0.	0.
time (sec)	N/A	0.032	0.06	0.228	1.829	1.615	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	59	134	1027	599	0	0
normalized size	1	1.	0.74	1.68	12.84	7.49	0.	0.
time (sec)	N/A	0.04	0.111	0.27	2.122	1.669	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	52	54	479	134	0	0
normalized size	1	1.	0.64	0.67	5.91	1.65	0.	0.
time (sec)	N/A	0.048	0.143	0.262	2.087	1.352	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	81	214	3286	701	0	0
normalized size	1	1.	0.65	1.71	26.29	5.61	0.	0.
time (sec)	N/A	0.062	0.205	0.266	2.558	1.641	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	70	70	171	190	0	0
normalized size	1	1.	0.56	0.56	1.37	1.52	0.	0.
time (sec)	N/A	0.062	0.277	0.408	2.117	1.423	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	67	88	124	582	0	0
normalized size	1	1.	0.55	0.72	1.02	4.77	0.	0.
time (sec)	N/A	0.054	0.18	0.431	2.135	1.722	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	52	47	86	139	0	0
normalized size	1	1.	0.65	0.59	1.08	1.74	0.	0.
time (sec)	N/A	0.034	0.133	0.261	2.142	1.351	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	52	54	80	475	0	0
normalized size	1	1.	0.53	0.55	0.81	4.8	0.	0.
time (sec)	N/A	0.026	0.108	0.234	1.974	1.67	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	55	117	570	0	0
normalized size	1	1.	0.59	0.74	1.58	7.7	0.	0.
time (sec)	N/A	0.037	0.08	0.221	2.082	1.752	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	45	108	535	0	0
normalized size	1	1.	0.69	0.69	1.66	8.23	0.	0.
time (sec)	N/A	0.033	0.076	0.21	1.829	1.621	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	134	1108	608	0	0
normalized size	1	1.	0.7	1.6	13.19	7.24	0.	0.
time (sec)	N/A	0.045	0.101	0.27	2.277	1.668	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	54	495	139	0	0
normalized size	1	1.	0.6	0.64	5.82	1.64	0.	0.
time (sec)	N/A	0.052	0.218	0.42	2.124	1.441	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	214	3594	714	0	0
normalized size	1	1.	0.61	1.63	27.44	5.45	0.	0.
time (sec)	N/A	0.064	0.216	0.231	2.691	1.734	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	67	88	101	560	0	0
normalized size	1	1.	0.59	0.78	0.89	4.96	0.	0.
time (sec)	N/A	0.059	0.133	0.506	2.122	1.806	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	52	47	77	128	0	0
normalized size	1	1.	0.7	0.64	1.04	1.73	0.	0.
time (sec)	N/A	0.032	0.086	0.394	2.71	1.387	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	52	54	70	470	0	0
normalized size	1	1.	0.58	0.6	0.78	5.22	0.	0.
time (sec)	N/A	0.025	0.063	0.421	2.48	1.631	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	44	55	108	559	0	0
normalized size	1	1.	0.65	0.81	1.59	8.22	0.	0.
time (sec)	N/A	0.035	0.045	0.47	2.954	1.767	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	45	45	115	525	0	0
normalized size	1	1.	0.76	0.76	1.95	8.9	0.	0.
time (sec)	N/A	0.035	0.047	0.452	2.382	1.669	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	59	135	983	597	0	0
normalized size	1	1.	0.76	1.73	12.6	7.65	0.	0.
time (sec)	N/A	0.047	0.08	0.534	2.994	1.761	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	51	54	479	131	0	0
normalized size	1	1.	0.65	0.68	6.06	1.66	0.	0.
time (sec)	N/A	0.061	0.11	0.396	2.634	1.488	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	80	214	3129	693	0	0
normalized size	1	1.	0.66	1.75	25.65	5.68	0.	0.
time (sec)	N/A	0.07	0.153	0.422	3.008	1.76	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	67	88	101	566	0	0
normalized size	1	1.	0.55	0.72	0.83	4.64	0.	0.
time (sec)	N/A	0.059	0.129	0.362	3.202	1.765	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	52	47	77	131	0	0
normalized size	1	1.	0.65	0.59	0.96	1.64	0.	0.
time (sec)	N/A	0.034	0.094	0.245	2.879	1.412	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	52	54	70	475	0	0
normalized size	1	1.	0.53	0.55	0.71	4.8	0.	0.
time (sec)	N/A	0.028	0.077	0.273	2.567	1.661	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	44	55	108	564	0	0
normalized size	1	1.	0.59	0.74	1.46	7.62	0.	0.
time (sec)	N/A	0.039	0.05	0.374	2.792	1.716	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	45	126	531	0	0
normalized size	1	1.	0.69	0.69	1.94	8.17	0.	0.
time (sec)	N/A	0.033	0.055	0.414	2.917	1.678	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	134	994	602	0	0
normalized size	1	1.	0.7	1.6	11.83	7.17	0.	0.
time (sec)	N/A	0.045	0.072	0.345	2.792	1.632	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	54	513	134	0	0
normalized size	1	1.	0.6	0.64	6.04	1.58	0.	0.
time (sec)	N/A	0.054	0.129	0.276	3.03	1.397	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	214	3173	698	0	0
normalized size	1	1.	0.61	1.63	24.22	5.33	0.	0.
time (sec)	N/A	0.067	0.156	0.283	3.055	2.019	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	70	88	101	566	0	0
normalized size	1	1.	0.57	0.72	0.83	4.64	0.	0.
time (sec)	N/A	0.064	0.101	0.353	3.014	2.057	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	55	47	77	131	0	0
normalized size	1	1.	0.69	0.59	0.96	1.64	0.	0.
time (sec)	N/A	0.033	0.062	0.256	2.694	1.645	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	55	54	70	475	0	0
normalized size	1	1.	0.56	0.55	0.71	4.8	0.	0.
time (sec)	N/A	0.027	0.058	0.279	2.459	1.986	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	47	55	108	564	0	0
normalized size	1	1.	0.64	0.74	1.46	7.62	0.	0.
time (sec)	N/A	0.035	0.045	0.345	2.84	1.774	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	45	45	126	531	0	0
normalized size	1	1.	0.69	0.69	1.94	8.17	0.	0.
time (sec)	N/A	0.033	0.055	0.417	2.542	1.641	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	59	135	1018	602	0	0
normalized size	1	1.	0.7	1.61	12.12	7.17	0.	0.
time (sec)	N/A	0.042	0.074	0.392	2.924	1.663	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	51	54	556	134	0	0
normalized size	1	1.	0.6	0.64	6.54	1.58	0.	0.
time (sec)	N/A	0.047	0.112	0.256	2.931	1.341	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	80	214	3264	698	0	0
normalized size	1	1.	0.61	1.63	24.92	5.33	0.	0.
time (sec)	N/A	0.062	0.121	0.246	2.647	1.736	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	0.112	0.408	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.167	0.362	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.056	0.112	0.27	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	88	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	0.125	0.346	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.093	0.163	0.346	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	96	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.117	0.454	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.11	0.354	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.166	0.306	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.115	0.269	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	88	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.121	0.339	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	88	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.159	0.398	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	96	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.125	0.409	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.262	0.349	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	0.164	0.309	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	88	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.059	0.147	0.277	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.075	0.068	0.374	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	90	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.095	0.103	0.361	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	90	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.098	0.163	0.393	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.111	0.299	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.107	0.382	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	87	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.088	0.237	0.	0.	0.	0.

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	283	0	0	0	0	0
normalized size	1	1.	3.14	0.	0.	0.	0.	0.
time (sec)	N/A	0.077	3.57	0.302	0.	0.	0.	0.

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	101	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.092	0.764	0.335	0.	0.	0.	0.

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	404	0	0	0	0	0
normalized size	1	1.	4.39	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	5.986	0.379	0.	0.	0.	0.

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	0.12	0.319	0.	0.	0.	0.

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	91	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.064	0.112	0.343	0.	0.	0.	0.

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.061	0.11	0.252	0.	0.	0.	0.

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	277	0	0	0	0	0
normalized size	1	1.	3.08	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	3.649	0.329	0.	0.	0.	0.

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	103	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	0.675	0.352	0.	0.	0.	0.

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	402	0	0	0	0	0
normalized size	1	1.	4.37	0.	0.	0.	0.	0.
time (sec)	N/A	0.101	6.13	0.398	0.	0.	0.	0.

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.067	0.193	0.348	0.	0.	0.	0.

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	90	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.066	0.092	0.326	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	87	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.146	0.234	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	104	0	0	0	0	0
normalized size	1	1.	1.16	0.	0.	0.	0.	0.
time (sec)	N/A	0.081	0.654	0.331	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	90	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.208	0.333	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	91	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.209	0.393	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	138	142	0	0	0	0	0
normalized size	1	0.93	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.279	0.309	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0
normalized size	1	0.93	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.117	0.211	0.306	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0
normalized size	1	0.93	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.3	0.302	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	136	142	0	0	0	0	0
normalized size	1	0.93	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.271	0.302	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	142	0	0	0	0	0
normalized size	1	1.	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.241	0.283	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	139	142	0	0	0	0	0
normalized size	1	0.93	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.122	0.258	0.283	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.239	2.474	0.	0.	0.	0.

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	122	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	0.2	1.911	0.	0.	0.	0.

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	120	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.103	0.176	1.463	0.	0.	0.	0.

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	114	0	0	0	0	0
normalized size	1	1.	0.97	0.	0.	0.	0.	0.
time (sec)	N/A	0.072	0.163	1.42	0.	0.	0.	0.

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	111	0	0	0	0	0
normalized size	1	1.	1.11	0.	0.	0.	0.	0.
time (sec)	N/A	0.094	0.227	1.338	0.	0.	0.	0.

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	117	0	0	0	0	0
normalized size	1	1.	1.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.195	1.25	0.	0.	0.	0.

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	114	0	0	0	0	0
normalized size	1	1.	0.91	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.155	1.506	0.	0.	0.	0.

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	122	0	0	0	0	0
normalized size	1	1.	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	0.158	1.289	0.	0.	0.	0.

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0
normalized size	1	0.93	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.112	0.214	0.605	0.	0.	0.	0.

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0
normalized size	1	0.93	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	0.191	0.582	0.	0.	0.	0.

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0
normalized size	1	0.93	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.106	0.17	0.606	0.	0.	0.	0.

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	140	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.17	0.74	0.	0.	0.	0.

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	140	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.1	0.164	0.702	0.	0.	0.	0.

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	132	140	0	0	0	0	0
normalized size	1	0.94	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.166	0.612	0.	0.	0.	0.

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0
normalized size	1	0.93	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.115	0.173	0.631	0.	0.	0.	0.

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	132	140	0	0	0	0	0
normalized size	1	0.93	0.99	0.	0.	0.	0.	0.
time (sec)	N/A	0.113	0.171	0.644	0.	0.	0.	0.

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	170	170	242	0	0	0	0	0
normalized size	1	1.	1.42	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	1.413	1.687	0.	0.	0.	0.

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	175	0	0	0	0	0
normalized size	1	1.	1.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.793	0.371	0.	0.	0.	0.

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	240	0	0	0	0	0
normalized size	1	1.	1.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.838	0.302	0.	0.	0.	0.

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	144	0	0	0	0	0
normalized size	1	1.	1.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.441	0.28	0.	0.	0.	0.

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	138	138	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.177	0.134	0.286	0.	0.	0.	0.

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	279	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.356	2.594	0.283	0.	0.	0.	0.

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	277	277	276	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.311	2.514	0.27	0.	0.	0.	0.

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	274	274	256	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.31	1.691	0.286	0.	0.	0.	0.

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	272	272	256	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	1.827	0.271	0.	0.	0.	0.

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	211	211	119	0	0	0	0	0
normalized size	1	1.	0.56	0.	0.	0.	0.	0.
time (sec)	N/A	0.251	0.339	1.404	0.	0.	0.	0.

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	285	285	10836	0	0	0	0	0
normalized size	1	1.	38.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.343	26.187	1.402	0.	0.	0.	0.

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.252	1.489	0.	0.	0.	0.

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.388	0.417	0.	0.	0.	0.

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	0.368	0.312	0.	0.	0.	0.

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	140	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.541	0.336	0.	0.	0.	0.

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.429	0.336	0.	0.	0.	0.

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.424	0.352	0.	0.	0.	0.

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.131	0.351	0.382	0.	0.	0.	0.

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	136	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.164	0.338	2.434	0.	0.	0.	0.

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.162	0.465	2.171	0.	0.	0.	0.

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	120	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.155	0.277	1.759	0.	0.	0.	0.

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	118	0	0	0	0	0
normalized size	1	1.	0.84	0.	0.	0.	0.	0.
time (sec)	N/A	0.138	0.226	1.421	0.	0.	0.	0.

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	141	141	112	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.157	0.158	1.625	0.	0.	0.	0.

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	132	132	109	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.166	0.185	1.261	0.	0.	0.	0.

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	109	0	0	0	0	0
normalized size	1	1.	0.83	0.	0.	0.	0.	0.
time (sec)	N/A	0.171	0.177	1.461	0.	0.	0.	0.

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	118	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.183	0.158	1.709	0.	0.	0.	0.

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.253	0.691	0.	0.	0.	0.

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.393	0.653	0.	0.	0.	0.

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.317	0.646	0.	0.	0.	0.

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.128	0.237	0.788	0.	0.	0.	0.

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.13	0.22	0.779	0.	0.	0.	0.

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	133	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.246	0.765	0.	0.	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.127	0.213	0.729	0.	0.	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	138	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.124	0.214	0.731	0.	0.	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	356	0	0	0	0	0
normalized size	1	1.	2.06	0.	0.	0.	0.	0.
time (sec)	N/A	0.209	45.803	1.662	0.	0.	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	295	295	13480	0	0	0	0	0
normalized size	1	1.	45.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.363	26.346	1.539	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	290	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.344	2.799	0.293	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	284	284	289	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.335	2.754	0.379	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	263	0	0	0	0	0
normalized size	1	1.	0.94	0.	0.	0.	0.	0.
time (sec)	N/A	0.323	2.031	0.335	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	281	281	261	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.327	2.128	0.303	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	142	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.161	0.277	1.463	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	125	382	0	0	0	0
normalized size	1	1.	0.6	1.83	0.	0.	0.	0.
time (sec)	N/A	0.242	1.	4.052	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	111	351	0	0	0	0
normalized size	1	1.	0.62	1.95	0.	0.	0.	0.
time (sec)	N/A	0.204	0.754	3.873	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	94	317	0	0	0	0
normalized size	1	1.	0.65	2.19	0.	0.	0.	0.
time (sec)	N/A	0.152	0.329	3.611	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	83	283	0	0	0	0
normalized size	1	1.	0.74	2.53	0.	0.	0.	0.
time (sec)	N/A	0.154	0.206	3.71	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	78	259	0	0	0	0
normalized size	1	1.	0.72	2.38	0.	0.	0.	0.
time (sec)	N/A	0.187	0.275	3.895	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	90	505	0	0	0	0
normalized size	1	1.	0.64	3.61	0.	0.	0.	0.
time (sec)	N/A	0.215	0.362	8.23	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	122	804	0	0	0	0
normalized size	1	1.	0.67	4.44	0.	0.	0.	0.
time (sec)	N/A	0.248	0.553	10.465	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	143	725	0	0	0	0
normalized size	1	1.	0.68	3.45	0.	0.	0.	0.
time (sec)	N/A	0.269	0.916	12.198	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	128	384	0	0	0	0
normalized size	1	1.	0.61	1.83	0.	0.	0.	0.
time (sec)	N/A	0.232	0.876	3.683	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	108	353	0	0	0	0
normalized size	1	1.	0.6	1.95	0.	0.	0.	0.
time (sec)	N/A	0.183	0.094	3.533	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	95	319	0	0	0	0
normalized size	1	1.	0.65	2.18	0.	0.	0.	0.
time (sec)	N/A	0.175	0.232	3.819	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	85	285	0	0	0	0
normalized size	1	1.	0.73	2.46	0.	0.	0.	0.
time (sec)	N/A	0.178	0.165	3.479	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	80	261	0	0	0	0
normalized size	1	1.	0.7	2.29	0.	0.	0.	0.
time (sec)	N/A	0.185	0.251	3.801	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	92	506	0	0	0	0
normalized size	1	1.	0.63	3.49	0.	0.	0.	0.
time (sec)	N/A	0.211	0.295	8.06	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	122	805	0	0	0	0
normalized size	1	1.	0.66	4.33	0.	0.	0.	0.
time (sec)	N/A	0.241	0.484	9.945	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	134	727	0	0	0	0
normalized size	1	1.	0.62	3.38	0.	0.	0.	0.
time (sec)	N/A	0.269	1.616	11.725	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	125	384	0	0	0	0
normalized size	1	1.	0.59	1.81	0.	0.	0.	0.
time (sec)	N/A	0.201	0.243	3.772	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	109	353	0	0	0	0
normalized size	1	1.	0.6	1.93	0.	0.	0.	0.
time (sec)	N/A	0.21	0.1	3.893	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	151	151	97	319	0	0	0	0
normalized size	1	1.	0.64	2.11	0.	0.	0.	0.
time (sec)	N/A	0.193	0.196	3.619	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	79	285	0	0	0	0
normalized size	1	1.	0.66	2.38	0.	0.	0.	0.
time (sec)	N/A	0.17	0.194	3.121	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	261	0	0	0	0
normalized size	1	1.	0.69	2.25	0.	0.	0.	0.
time (sec)	N/A	0.19	0.269	3.858	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	92	508	0	0	0	0
normalized size	1	1.	0.63	3.46	0.	0.	0.	0.
time (sec)	N/A	0.223	0.277	9.055	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	121	807	0	0	0	0
normalized size	1	1.	0.64	4.29	0.	0.	0.	0.
time (sec)	N/A	0.25	0.37	11.293	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	134	727	0	0	0	0
normalized size	1	1.	0.62	3.35	0.	0.	0.	0.
time (sec)	N/A	0.268	0.912	11.339	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	214	214	127	381	0	0	0	0
normalized size	1	1.	0.59	1.78	0.	0.	0.	0.
time (sec)	N/A	0.221	0.676	3.726	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	108	350	0	0	0	0
normalized size	1	1.	0.58	1.89	0.	0.	0.	0.
time (sec)	N/A	0.194	0.623	3.917	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	97	316	0	0	0	0
normalized size	1	1.	0.65	2.11	0.	0.	0.	0.
time (sec)	N/A	0.156	0.182	3.245	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	82	282	0	0	0	0
normalized size	1	1.	0.7	2.41	0.	0.	0.	0.
time (sec)	N/A	0.124	0.086	3.313	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	110	110	803	258	0	0	0	0
normalized size	1	1.	7.3	2.35	0.	0.	0.	0.
time (sec)	N/A	0.16	6.282	3.992	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	757	508	0	0	0	0
normalized size	1	1.	5.45	3.65	0.	0.	0.	0.
time (sec)	N/A	0.203	6.297	8.132	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	116	807	0	0	0	0
normalized size	1	1.	0.64	4.48	0.	0.	0.	0.
time (sec)	N/A	0.231	0.426	10.779	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	209	209	133	726	0	0	0	0
normalized size	1	1.	0.64	3.47	0.	0.	0.	0.
time (sec)	N/A	0.259	0.656	11.517	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	130	384	0	0	0	0
normalized size	1	1.	0.6	1.77	0.	0.	0.	0.
time (sec)	N/A	0.221	0.714	3.568	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	108	353	0	0	0	0
normalized size	1	1.	0.57	1.88	0.	0.	0.	0.
time (sec)	N/A	0.201	0.624	3.393	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	94	319	0	0	0	0
normalized size	1	1.	0.61	2.08	0.	0.	0.	0.
time (sec)	N/A	0.161	0.341	3.703	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	285	0	0	0	0
normalized size	1	1.	0.71	2.38	0.	0.	0.	0.
time (sec)	N/A	0.141	0.165	3.579	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	80	261	0	0	0	0
normalized size	1	1.	0.69	2.25	0.	0.	0.	0.
time (sec)	N/A	0.141	0.177	3.946	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	144	144	761	508	0	0	0	0
normalized size	1	1.	5.28	3.53	0.	0.	0.	0.
time (sec)	N/A	0.192	6.264	7.868	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	119	807	0	0	0	0
normalized size	1	1.	0.65	4.41	0.	0.	0.	0.
time (sec)	N/A	0.241	0.433	10.656	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	136	729	0	0	0	0
normalized size	1	1.	0.64	3.44	0.	0.	0.	0.
time (sec)	N/A	0.266	0.692	11.339	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	130	384	0	0	0	0
normalized size	1	1.	0.6	1.77	0.	0.	0.	0.
time (sec)	N/A	0.225	0.688	3.701	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	111	353	0	0	0	0
normalized size	1	1.	0.59	1.88	0.	0.	0.	0.
time (sec)	N/A	0.196	0.591	3.27	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	153	153	97	319	0	0	0	0
normalized size	1	1.	0.63	2.08	0.	0.	0.	0.
time (sec)	N/A	0.158	0.313	3.638	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	85	285	0	0	0	0
normalized size	1	1.	0.71	2.38	0.	0.	0.	0.
time (sec)	N/A	0.143	0.165	3.432	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	807	261	0	0	0	0
normalized size	1	1.	6.96	2.25	0.	0.	0.	0.
time (sec)	N/A	0.154	6.207	3.516	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	92	508	0	0	0	0
normalized size	1	1.	0.63	3.46	0.	0.	0.	0.
time (sec)	N/A	0.163	0.377	8.615	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	185	185	119	807	0	0	0	0
normalized size	1	1.	0.64	4.36	0.	0.	0.	0.
time (sec)	N/A	0.219	0.389	10.203	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	212	212	136	729	0	0	0	0
normalized size	1	1.	0.64	3.44	0.	0.	0.	0.
time (sec)	N/A	0.271	0.451	10.934	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	119	807	0	0	0	0
normalized size	1	1.	0.63	4.29	0.	0.	0.	0.
time (sec)	N/A	0.189	0.118	10.546	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	109	134	215	815	0	0
normalized size	1	1.	0.49	0.6	0.96	3.65	0.	0.
time (sec)	N/A	0.125	0.285	0.354	2.331	2.031	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	92	114	157	753	0	0
normalized size	1	1.	0.5	0.62	0.85	4.09	0.	0.
time (sec)	N/A	0.107	0.261	0.542	2.341	2.077	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	75	83	108	655	0	0
normalized size	1	1.	0.52	0.58	0.76	4.58	0.	0.
time (sec)	N/A	0.06	0.185	0.468	2.265	2.009	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	61	63	86	591	0	0
normalized size	1	1.	0.5	0.51	0.7	4.8	0.	0.
time (sec)	N/A	0.038	0.099	0.458	2.081	2.049	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	63	140	859	0	0
normalized size	1	1.	1.	0.68	1.51	9.24	0.	0.
time (sec)	N/A	0.057	0.101	0.385	2.076	2.397	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	72	194	875	0	0
normalized size	1	1.	0.65	0.77	2.09	9.41	0.	0.
time (sec)	N/A	0.066	0.078	0.401	2.081	2.389	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	69	150	1053	648	0	0
normalized size	1	1.	0.62	1.35	9.49	5.84	0.	0.
time (sec)	N/A	0.104	0.143	0.43	2.302	1.976	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	87	157	1362	722	0	0
normalized size	1	1.	0.57	1.03	8.96	4.75	0.	0.
time (sec)	N/A	0.116	0.413	0.444	2.386	1.972	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	110	248	3525	810	0	0
normalized size	1	1.	0.57	1.28	18.26	4.2	0.	0.
time (sec)	N/A	0.117	0.336	0.348	2.666	1.993	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	109	134	228	853	0	0
normalized size	1	1.	0.48	0.59	1.	3.72	0.	0.
time (sec)	N/A	0.127	0.317	0.293	2.33	2.07	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	92	114	170	778	0	0
normalized size	1	1.	0.49	0.6	0.9	4.12	0.	0.
time (sec)	N/A	0.119	0.214	0.485	2.294	2.037	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	76	83	116	684	0	0
normalized size	1	1.	0.52	0.56	0.79	4.65	0.	0.
time (sec)	N/A	0.059	0.072	0.418	2.323	1.968	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	61	63	90	605	0	0
normalized size	1	1.	0.48	0.5	0.71	4.76	0.	0.
time (sec)	N/A	0.038	0.124	0.283	2.091	1.992	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	93	63	144	869	0	0
normalized size	1	1.	0.97	0.66	1.5	9.05	0.	0.
time (sec)	N/A	0.055	0.122	0.278	2.128	2.404	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	60	72	198	886	0	0
normalized size	1	1.	0.62	0.75	2.06	9.23	0.	0.
time (sec)	N/A	0.061	0.075	0.247	2.108	2.416	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	69	150	1098	662	0	0
normalized size	1	1.	0.61	1.32	9.63	5.81	0.	0.
time (sec)	N/A	0.087	0.136	0.263	2.347	1.992	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	88	157	1409	741	0	0
normalized size	1	1.	0.56	1.01	9.03	4.75	0.	0.
time (sec)	N/A	0.106	0.055	0.275	2.361	2.045	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	198	198	111	248	3688	834	0	0
normalized size	1	1.	0.56	1.25	18.63	4.21	0.	0.
time (sec)	N/A	0.124	0.25	0.331	2.775	2.09	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	109	134	250	883	0	0
normalized size	1	1.	0.45	0.56	1.04	3.66	0.	0.
time (sec)	N/A	0.128	0.319	0.303	2.319	2.123	0.	0.

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	92	114	189	802	0	0
normalized size	1	1.	0.46	0.57	0.95	4.03	0.	0.
time (sec)	N/A	0.113	0.285	0.509	2.299	2.048	0.	0.

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	75	83	127	703	0	0
normalized size	1	1.	0.48	0.54	0.82	4.54	0.	0.
time (sec)	N/A	0.061	0.248	0.325	2.293	1.994	0.	0.

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	61	63	96	618	0	0
normalized size	1	1.	0.45	0.47	0.71	4.58	0.	0.
time (sec)	N/A	0.037	0.147	0.277	2.149	1.97	0.	0.

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	93	63	150	880	0	0
normalized size	1	1.	0.91	0.62	1.47	8.63	0.	0.
time (sec)	N/A	0.061	0.149	0.253	2.074	2.451	0.	0.

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	72	204	896	0	0
normalized size	1	1.	0.59	0.71	2.	8.78	0.	0.
time (sec)	N/A	0.064	0.122	0.25	2.125	2.439	0.	0.

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	151	1179	675	0	0
normalized size	1	1.	0.57	1.26	9.82	5.62	0.	0.
time (sec)	N/A	0.097	0.138	0.262	2.377	2.022	0.	0.

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	87	157	1501	760	0	0
normalized size	1	1.	0.53	0.96	9.15	4.63	0.	0.
time (sec)	N/A	0.11	0.422	0.293	2.434	1.997	0.	0.

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	110	248	4012	859	0	0
normalized size	1	1.	0.53	1.19	19.29	4.13	0.	0.
time (sec)	N/A	0.126	0.376	0.269	2.782	2.103	0.	0.

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	92	114	157	760	0	0
normalized size	1	1.	0.5	0.62	0.85	4.13	0.	0.
time (sec)	N/A	0.139	0.222	0.524	2.335	2.087	0.	0.

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	75	83	108	662	0	0
normalized size	1	1.	0.52	0.58	0.76	4.63	0.	0.
time (sec)	N/A	0.062	0.179	0.489	2.247	2.221	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	61	63	86	598	0	0
normalized size	1	1.	0.5	0.51	0.7	4.86	0.	0.
time (sec)	N/A	0.033	0.088	0.452	2.283	2.021	0.	0.

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	93	63	140	864	0	0
normalized size	1	1.	1.	0.68	1.51	9.29	0.	0.
time (sec)	N/A	0.058	0.096	0.441	2.13	2.413	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	93	93	60	72	201	880	0	0
normalized size	1	1.	0.65	0.77	2.16	9.46	0.	0.
time (sec)	N/A	0.073	0.071	0.421	2.15	2.411	0.	0.

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	69	150	1060	653	0	0
normalized size	1	1.	0.62	1.35	9.55	5.88	0.	0.
time (sec)	N/A	0.095	0.088	0.448	2.328	2.07	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	87	157	1369	728	0	0
normalized size	1	1.	0.57	1.03	9.01	4.79	0.	0.
time (sec)	N/A	0.119	0.175	0.479	2.377	2.236	0.	0.

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	110	248	3525	815	0	0
normalized size	1	1.	0.57	1.28	18.26	4.22	0.	0.
time (sec)	N/A	0.118	0.22	0.523	2.52	2.377	0.	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	92	114	157	765	0	0
normalized size	1	1.	0.46	0.57	0.79	3.84	0.	0.
time (sec)	N/A	0.109	0.179	0.373	2.426	2.521	0.	0.

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	75	83	108	667	0	0
normalized size	1	1.	0.48	0.54	0.7	4.3	0.	0.
time (sec)	N/A	0.06	0.169	0.313	2.334	2.052	0.	0.

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	61	63	86	603	0	0
normalized size	1	1.	0.45	0.47	0.64	4.47	0.	0.
time (sec)	N/A	0.033	0.106	0.28	2.186	1.704	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	93	63	140	869	0	0
normalized size	1	1.	0.91	0.62	1.37	8.52	0.	0.
time (sec)	N/A	0.06	0.105	0.377	2.198	2.102	0.	0.

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	72	212	886	0	0
normalized size	1	1.	0.59	0.71	2.08	8.69	0.	0.
time (sec)	N/A	0.084	0.079	0.378	2.385	2.091	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	150	1083	659	0	0
normalized size	1	1.	0.57	1.25	9.02	5.49	0.	0.
time (sec)	N/A	0.093	0.088	0.289	2.48	1.689	0.	0.

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	87	157	1415	733	0	0
normalized size	1	1.	0.53	0.96	8.63	4.47	0.	0.
time (sec)	N/A	0.124	0.149	0.3	2.461	1.681	0.	0.

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	110	248	3591	821	0	0
normalized size	1	1.	0.53	1.19	17.26	3.95	0.	0.
time (sec)	N/A	0.128	0.213	0.362	2.589	1.747	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	95	114	157	765	0	0
normalized size	1	1.	0.48	0.57	0.79	3.84	0.	0.
time (sec)	N/A	0.117	0.143	0.377	2.337	1.719	0.	0.

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	78	83	108	667	0	0
normalized size	1	1.	0.5	0.54	0.7	4.3	0.	0.
time (sec)	N/A	0.064	0.124	0.325	2.32	1.694	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	64	63	86	603	0	0
normalized size	1	1.	0.47	0.47	0.64	4.47	0.	0.
time (sec)	N/A	0.036	0.078	0.303	2.111	1.685	0.	0.

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	96	63	140	869	0	0
normalized size	1	1.	0.94	0.62	1.37	8.52	0.	0.
time (sec)	N/A	0.062	0.092	0.257	2.202	2.108	0.	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	60	72	212	886	0	0
normalized size	1	1.	0.59	0.71	2.08	8.69	0.	0.
time (sec)	N/A	0.062	0.069	0.384	2.082	2.198	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	69	151	1107	659	0	0
normalized size	1	1.	0.57	1.26	9.22	5.49	0.	0.
time (sec)	N/A	0.092	0.094	0.404	2.342	1.717	0.	0.

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	164	164	87	157	1482	733	0	0
normalized size	1	1.	0.53	0.96	9.04	4.47	0.	0.
time (sec)	N/A	0.11	0.296	0.434	2.387	1.601	0.	0.

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	110	248	3726	821	0	0
normalized size	1	1.	0.53	1.19	17.91	3.95	0.	0.
time (sec)	N/A	0.135	0.214	0.293	2.609	1.733	0.	0.

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.153	0.243	0.316	0.	0.	0.	0.

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.135	0.22	0.31	0.	0.	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	109	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.156	0.221	0.378	0.	0.	0.	0.

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	116	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.186	0.347	0.401	0.	0.	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	123	0	0	0	0	0
normalized size	1	1.	0.85	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.237	0.41	0.	0.	0.	0.

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	123	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.206	0.432	0.	0.	0.	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	111	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.15	0.352	0.322	0.	0.	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	0.22	0.278	0.	0.	0.	0.

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	109	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.224	0.362	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	108	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	0.204	0.376	0.	0.	0.	0.

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	117	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.193	0.229	0.421	0.	0.	0.	0.

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	124	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.194	0.209	0.418	0.	0.	0.	0.

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.146	0.27	0.319	0.	0.	0.	0.

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	109	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.148	0.212	0.294	0.	0.	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	108	0	0	0	0	0
normalized size	1	1.	0.7	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	0.142	0.246	0.	0.	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	149	149	268	0	0	0	0	0
normalized size	1	1.	1.8	0.	0.	0.	0.	0.
time (sec)	N/A	0.163	6.243	0.345	0.	0.	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	145	145	699	0	0	0	0	0
normalized size	1	1.	4.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.174	6.305	0.375	0.	0.	0.	0.

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	118	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.179	0.278	0.395	0.	0.	0.	0.

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.336	0.427	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	114	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.144	0.236	0.332	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	111	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.205	0.297	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	115	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.139	0.224	0.249	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	703	0	0	0	0	0
normalized size	1	1.	4.78	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	6.271	0.36	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	118	0	0	0	0	0
normalized size	1	1.	0.79	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.345	0.365	0.	0.	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	222	169	0	0	0	0	0
normalized size	1	0.96	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.652	0.319	0.	0.	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	219	166	0	0	0	0	0
normalized size	1	0.96	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.425	0.313	0.	0.	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	219	166	0	0	0	0	0
normalized size	1	0.96	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.403	0.32	0.	0.	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	219	166	0	0	0	0	0
normalized size	1	0.96	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.201	0.428	0.313	0.	0.	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	164	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.233	0.402	0.328	0.	0.	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	225	166	0	0	0	0	0
normalized size	1	0.96	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.537	0.307	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	227	227	161	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.232	0.256	2.476	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	144	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.219	0.482	2.026	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	144	0	0	0	0	0
normalized size	1	1.	0.77	0.	0.	0.	0.	0.
time (sec)	N/A	0.213	0.33	1.522	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	142	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.168	0.235	1.466	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	127	0	0	0	0	0
normalized size	1	1.	0.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.191	0.204	1.455	0.	0.	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	131	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.23	0.288	1.304	0.	0.	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	137	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.256	0.457	1.602	0.	0.	0.	0.

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	196	196	142	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	0.331	1.433	0.	0.	0.	0.

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	213	164	0	0	0	0	0
normalized size	1	0.96	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.237	0.525	0.674	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	213	164	0	0	0	0	0
normalized size	1	0.96	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.216	0.429	0.68	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	162	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.375	0.787	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	157	0	0	0	0	0
normalized size	1	1.	0.72	0.	0.	0.	0.	0.
time (sec)	N/A	0.202	0.446	0.76	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	163	0	0	0	0	0
normalized size	1	1.	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.217	0.408	0.648	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	213	164	0	0	0	0	0
normalized size	1	0.96	0.74	0.	0.	0.	0.	0.
time (sec)	N/A	0.212	0.41	0.653	0.	0.	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	183	183	557	0	0	0	0	0
normalized size	1	1.	3.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.248	3.666	1.628	0.	0.	0.	0.

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	137	0	0	0	0	0
normalized size	1	1.	0.95	0.	0.	0.	0.	0.
time (sec)	N/A	0.189	0.834	0.341	0.	0.	0.	0.

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.178	0.118	0.338	0.	0.	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	105	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.176	0.586	0.312	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	144	144	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.197	0.196	0.315	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	296	0	0	0	0	0
normalized size	1	1.	1.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.368	3.398	0.303	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	290	290	294	0	0	0	0	0
normalized size	1	1.	1.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.337	3.424	0.292	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	287	287	268	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.326	2.378	0.256	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	286	286	266	0	0	0	0	0
normalized size	1	1.	0.93	0.	0.	0.	0.	0.
time (sec)	N/A	0.324	2.39	0.277	0.	0.	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	215	215	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.253	3.8	1.612	0.	0.	0.	0.

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	303	303	16189	0	0	0	0	0
normalized size	1	1.	53.43	0.	0.	0.	0.	0.
time (sec)	N/A	0.375	26.839	1.438	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [241] had the largest ratio of [0.2121]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.	21	0.095
2	A	3	2	1.	21	0.095
3	A	3	2	1.	21	0.095
4	A	2	1	1.	19	0.053
5	A	2	2	1.	19	0.105
6	A	2	2	1.	21	0.095
7	A	3	3	1.	21	0.143
8	A	4	3	1.	21	0.143
9	A	5	3	1.	21	0.143
10	A	4	3	1.	21	0.143
11	A	3	3	1.	21	0.143
12	A	2	2	1.	21	0.095
13	A	3	3	1.	21	0.143
14	A	3	2	1.	21	0.095
15	A	3	2	1.	21	0.095
16	A	4	4	1.	25	0.16
17	A	4	4	1.	25	0.16
18	A	3	3	1.	25	0.12
19	A	3	3	1.	25	0.12
20	A	3	3	1.	25	0.12
21	A	3	3	1.	25	0.12
22	A	4	4	1.	25	0.16
23	A	4	4	1.	25	0.16
24	A	1	1	1.	23	0.043
25	A	1	1	1.	23	0.043
26	A	5	5	1.	25	0.2
27	A	5	5	1.	25	0.2
28	A	4	4	1.	25	0.16
29	A	4	4	1.	25	0.16
30	A	4	4	1.	25	0.16
31	A	4	4	1.	25	0.16
32	A	5	5	1.	25	0.2
33	A	5	5	1.	25	0.2
34	A	2	2	1.	23	0.087
35	A	1	1	1.	33	0.03
36	A	1	1	1.	32	0.031
37	A	5	5	1.	33	0.152
38	A	5	5	1.	31	0.161
39	A	3	3	1.	25	0.12
40	A	4	4	1.	31	0.129
41	A	4	4	1.	33	0.121
42	A	4	4	1.	33	0.121
43	A	5	5	1.	33	0.152

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	5	5	1.	33	0.152
45	A	5	5	1.	31	0.161
46	A	4	4	1.	25	0.16
47	A	4	4	1.	31	0.129
48	A	4	4	1.	33	0.121
49	A	4	4	1.	33	0.121
50	A	4	4	1.	33	0.121
51	A	5	5	1.	33	0.152
52	A	5	5	1.	33	0.152
53	A	4	4	1.	25	0.16
54	A	5	5	1.	31	0.161
55	A	4	4	1.	33	0.121
56	A	4	4	1.	33	0.121
57	A	4	4	1.	33	0.121
58	A	4	4	1.	33	0.121
59	A	5	5	1.	33	0.152
60	A	5	5	1.	33	0.152
61	A	6	5	1.	33	0.152
62	A	5	5	1.	33	0.152
63	A	5	5	1.	33	0.152
64	A	4	4	1.	31	0.129
65	A	3	3	1.	25	0.12
66	A	4	4	1.	31	0.129
67	A	4	4	1.	33	0.121
68	A	5	5	1.	33	0.152
69	A	5	5	1.	33	0.152
70	A	6	5	1.	33	0.152
71	A	5	5	1.	33	0.152
72	A	5	5	1.	33	0.152
73	A	4	4	1.	33	0.121
74	A	4	4	1.	31	0.129
75	A	3	3	1.	25	0.12
76	A	4	4	1.	31	0.129
77	A	5	5	1.	33	0.152
78	A	5	5	1.	33	0.152
79	A	5	5	1.	33	0.152
80	A	5	5	1.	33	0.152
81	A	4	4	1.	33	0.121
82	A	4	4	1.	33	0.121
83	A	4	4	1.	31	0.129
84	A	3	3	1.	25	0.12
85	A	5	5	1.	31	0.161
86	A	5	5	1.	33	0.152
87	A	4	4	1.	25	0.16

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	4	4	1.	25	0.16
89	A	4	3	1.	35	0.086
90	A	4	4	1.	35	0.114
91	A	3	2	1.	35	0.057
92	A	4	3	1.	35	0.086
93	A	3	3	1.	35	0.086
94	A	3	3	1.	35	0.086
95	A	3	3	1.	35	0.086
96	A	4	4	1.	35	0.114
97	A	4	4	1.	35	0.114
98	A	4	3	1.	35	0.086
99	A	4	4	1.	35	0.114
100	A	3	2	1.	35	0.057
101	A	4	3	1.	35	0.086
102	A	3	3	1.	35	0.086
103	A	3	3	1.	35	0.086
104	A	3	3	1.	35	0.086
105	A	4	4	1.	35	0.114
106	A	4	4	1.	35	0.114
107	A	4	3	1.	35	0.086
108	A	4	4	1.	35	0.114
109	A	3	2	1.	35	0.057
110	A	4	3	1.	35	0.086
111	A	3	3	1.	35	0.086
112	A	3	3	1.	35	0.086
113	A	3	3	1.	35	0.086
114	A	4	4	1.	35	0.114
115	A	4	4	1.	35	0.114
116	A	4	4	1.	35	0.114
117	A	3	2	1.	35	0.057
118	A	4	3	1.	35	0.086
119	A	3	3	1.	35	0.086
120	A	3	3	1.	35	0.086
121	A	3	3	1.	35	0.086
122	A	4	4	1.	35	0.114
123	A	4	4	1.	35	0.114
124	A	4	4	1.	35	0.114
125	A	3	2	1.	35	0.057
126	A	4	3	1.	35	0.086
127	A	3	3	1.	35	0.086
128	A	3	3	1.	35	0.086
129	A	3	3	1.	35	0.086
130	A	4	4	1.	35	0.114
131	A	4	4	1.	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	4	4	1.	35	0.114
133	A	3	2	1.	35	0.057
134	A	4	3	1.	35	0.086
135	A	3	3	1.	35	0.086
136	A	3	3	1.	35	0.086
137	A	3	3	1.	35	0.086
138	A	4	4	1.	35	0.114
139	A	4	4	1.	35	0.114
140	A	3	3	1.	33	0.091
141	A	3	3	1.	31	0.097
142	A	2	2	1.	25	0.08
143	A	3	3	1.	31	0.097
144	A	3	3	1.	33	0.091
145	A	3	3	1.	33	0.091
146	A	3	3	1.	33	0.091
147	A	3	3	1.	31	0.097
148	A	2	2	1.	25	0.08
149	A	3	3	1.	31	0.097
150	A	3	3	1.	33	0.091
151	A	3	3	1.	33	0.091
152	A	3	3	1.	33	0.091
153	A	3	3	1.	31	0.097
154	A	2	2	1.	25	0.08
155	A	3	3	1.	31	0.097
156	A	3	3	1.	33	0.091
157	A	3	3	1.	33	0.091
158	A	3	3	1.	33	0.091
159	A	3	3	1.	31	0.097
160	A	2	2	1.	25	0.08
161	A	3	3	1.	31	0.097
162	A	3	3	1.	33	0.091
163	A	3	3	1.	33	0.091
164	A	3	3	1.	33	0.091
165	A	3	3	1.	31	0.097
166	A	2	2	1.	25	0.08
167	A	3	3	1.	31	0.097
168	A	3	3	1.	33	0.091
169	A	3	3	1.	33	0.091
170	A	3	3	1.	33	0.091
171	A	3	3	1.	31	0.097
172	A	2	2	1.	25	0.08
173	A	3	3	1.	31	0.097
174	A	3	3	1.	33	0.091
175	A	3	3	1.	33	0.091

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	3	3	0.93	33	0.091
177	A	3	3	0.93	33	0.091
178	A	3	3	0.93	33	0.091
179	A	3	3	0.93	33	0.091
180	A	3	3	1.	33	0.091
181	A	3	3	0.93	33	0.091
182	A	3	3	1.	33	0.091
183	A	3	3	1.	31	0.097
184	A	3	3	1.	29	0.103
185	A	2	2	1.	23	0.087
186	A	3	3	1.	29	0.103
187	A	3	3	1.	31	0.097
188	A	3	3	1.	31	0.097
189	A	3	3	1.	31	0.097
190	A	3	3	0.93	33	0.091
191	A	3	3	0.93	33	0.091
192	A	3	3	0.93	33	0.091
193	A	3	3	1.	33	0.091
194	A	3	3	1.	33	0.091
195	A	3	3	0.94	33	0.091
196	A	3	3	0.93	33	0.091
197	A	3	3	0.93	33	0.091
198	A	4	4	1.	25	0.16
199	A	4	4	1.	27	0.148
200	A	4	4	1.	27	0.148
201	A	4	4	1.	27	0.148
202	A	4	4	1.	27	0.148
203	A	8	5	1.	27	0.185
204	A	8	5	1.	27	0.185
205	A	8	5	1.	27	0.185
206	A	8	5	1.	27	0.185
207	A	7	5	1.	26	0.192
208	A	8	5	1.	25	0.2
209	A	4	3	1.	30	0.1
210	A	5	4	1.	40	0.1
211	A	5	4	1.	40	0.1
212	A	5	4	1.	40	0.1
213	A	5	4	1.	40	0.1
214	A	5	4	1.	40	0.1
215	A	5	4	1.	40	0.1
216	A	5	4	1.	40	0.1
217	A	5	4	1.	38	0.105
218	A	5	4	1.	36	0.111
219	A	4	3	1.	30	0.1

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	5	4	1.	36	0.111
221	A	5	4	1.	38	0.105
222	A	5	4	1.	38	0.105
223	A	5	4	1.	38	0.105
224	A	5	4	1.	40	0.1
225	A	5	4	1.	40	0.1
226	A	5	4	1.	40	0.1
227	A	5	4	1.	40	0.1
228	A	5	4	1.	40	0.1
229	A	5	4	1.	40	0.1
230	A	5	4	1.	40	0.1
231	A	5	4	1.	40	0.1
232	A	4	4	1.	32	0.125
233	A	8	5	1.	32	0.156
234	A	8	5	1.	34	0.147
235	A	8	5	1.	34	0.147
236	A	8	5	1.	34	0.147
237	A	8	5	1.	34	0.147
238	A	4	3	1.	31	0.097
239	A	10	8	1.	41	0.195
240	A	9	8	1.	39	0.205
241	A	7	7	1.	33	0.212
242	A	7	7	1.	39	0.18
243	A	7	7	1.	41	0.171
244	A	8	8	1.	41	0.195
245	A	9	8	1.	41	0.195
246	A	10	8	1.	41	0.195
247	A	10	8	1.	39	0.205
248	A	8	7	1.	33	0.212
249	A	8	8	1.	39	0.205
250	A	7	7	1.	41	0.171
251	A	7	7	1.	41	0.171
252	A	8	8	1.	41	0.195
253	A	9	8	1.	41	0.195
254	A	10	8	1.	41	0.195
255	A	9	7	1.	33	0.212
256	A	9	8	1.	39	0.205
257	A	8	8	1.	41	0.195
258	A	7	7	1.	41	0.171
259	A	7	7	1.	41	0.171
260	A	8	8	1.	41	0.195
261	A	9	8	1.	41	0.195
262	A	10	8	1.	41	0.195
263	A	10	8	1.	41	0.195

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	9	8	1.	41	0.195
265	A	8	8	1.	39	0.205
266	A	6	6	1.	33	0.182
267	A	7	7	1.	39	0.18
268	A	8	8	1.	41	0.195
269	A	9	8	1.	41	0.195
270	A	10	8	1.	41	0.195
271	A	10	8	1.	41	0.195
272	A	9	8	1.	41	0.195
273	A	8	8	1.	41	0.195
274	A	7	7	1.	39	0.18
275	A	6	6	1.	33	0.182
276	A	8	8	1.	39	0.205
277	A	9	8	1.	41	0.195
278	A	10	8	1.	41	0.195
279	A	10	8	1.	41	0.195
280	A	9	8	1.	41	0.195
281	A	8	8	1.	41	0.195
282	A	7	7	1.	41	0.171
283	A	7	7	1.	39	0.18
284	A	7	7	1.	33	0.212
285	A	9	8	1.	39	0.205
286	A	10	8	1.	41	0.195
287	A	8	7	1.	33	0.212
288	A	8	6	1.	43	0.14
289	A	7	6	1.	43	0.14
290	A	3	3	1.	43	0.07
291	A	5	4	1.	43	0.093
292	A	4	4	1.	43	0.093
293	A	4	4	1.	43	0.093
294	A	6	6	1.	43	0.14
295	A	7	7	1.	43	0.163
296	A	7	6	1.	43	0.14
297	A	8	6	1.	43	0.14
298	A	7	6	1.	43	0.14
299	A	3	3	1.	43	0.07
300	A	5	4	1.	43	0.093
301	A	4	4	1.	43	0.093
302	A	4	4	1.	43	0.093
303	A	6	6	1.	43	0.14
304	A	7	7	1.	43	0.163
305	A	7	6	1.	43	0.14
306	A	8	6	1.	43	0.14
307	A	7	6	1.	43	0.14

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
308	A	3	3	1.	43	0.07
309	A	5	4	1.	43	0.093
310	A	4	4	1.	43	0.093
311	A	4	4	1.	43	0.093
312	A	6	6	1.	43	0.14
313	A	7	7	1.	43	0.163
314	A	7	6	1.	43	0.14
315	A	7	6	1.	43	0.14
316	A	3	3	1.	43	0.07
317	A	5	4	1.	43	0.093
318	A	4	4	1.	43	0.093
319	A	4	4	1.	43	0.093
320	A	6	6	1.	43	0.14
321	A	7	7	1.	43	0.163
322	A	7	6	1.	43	0.14
323	A	7	6	1.	43	0.14
324	A	3	3	1.	43	0.07
325	A	5	4	1.	43	0.093
326	A	4	4	1.	43	0.093
327	A	4	4	1.	43	0.093
328	A	6	6	1.	43	0.14
329	A	7	7	1.	43	0.163
330	A	7	6	1.	43	0.14
331	A	7	6	1.	43	0.14
332	A	3	3	1.	43	0.07
333	A	5	4	1.	43	0.093
334	A	4	4	1.	43	0.093
335	A	4	4	1.	43	0.093
336	A	6	6	1.	43	0.14
337	A	7	7	1.	43	0.163
338	A	7	6	1.	43	0.14
339	A	5	4	1.	39	0.103
340	A	4	3	1.	33	0.091
341	A	5	4	1.	39	0.103
342	A	5	4	1.	41	0.098
343	A	5	4	1.	41	0.098
344	A	5	4	1.	41	0.098
345	A	5	4	1.	39	0.103
346	A	4	3	1.	33	0.091
347	A	5	4	1.	39	0.103
348	A	5	4	1.	41	0.098
349	A	5	4	1.	41	0.098
350	A	5	4	1.	41	0.098
351	A	5	4	1.	41	0.098

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
352	A	5	4	1.	39	0.103
353	A	4	3	1.	33	0.091
354	A	5	4	1.	39	0.103
355	A	5	4	1.	41	0.098
356	A	5	4	1.	41	0.098
357	A	5	4	1.	41	0.098
358	A	5	4	1.	41	0.098
359	A	5	4	1.	39	0.103
360	A	4	3	1.	33	0.091
361	A	5	4	1.	39	0.103
362	A	5	4	1.	41	0.098
363	A	5	4	0.96	41	0.098
364	A	5	4	0.96	41	0.098
365	A	5	4	0.96	41	0.098
366	A	5	4	0.96	41	0.098
367	A	5	4	1.	41	0.098
368	A	5	4	0.96	41	0.098
369	A	5	4	1.	41	0.098
370	A	5	4	1.	39	0.103
371	A	5	4	1.	37	0.108
372	A	4	3	1.	31	0.097
373	A	5	4	1.	37	0.108
374	A	5	4	1.	39	0.103
375	A	5	4	1.	39	0.103
376	A	5	4	1.	39	0.103
377	A	5	4	0.96	41	0.098
378	A	5	4	0.96	41	0.098
379	A	5	4	1.	41	0.098
380	A	5	4	1.	41	0.098
381	A	5	4	1.	41	0.098
382	A	5	4	0.96	41	0.098
383	A	4	4	1.	33	0.121
384	A	4	4	1.	35	0.114
385	A	4	4	1.	35	0.114
386	A	4	4	1.	35	0.114
387	A	4	4	1.	35	0.114
388	A	8	5	1.	35	0.143
389	A	8	5	1.	35	0.143
390	A	8	5	1.	35	0.143
391	A	8	5	1.	35	0.143
392	A	7	5	1.	35	0.143
393	A	8	5	1.	33	0.152

Chapter 3

Listing of integrals

3.1 $\int \cos^7(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=92

$$-\frac{(A + 4C) \sin^7(c + dx)}{7d} + \frac{3(A + 2C) \sin^5(c + dx)}{5d} - \frac{(3A + 4C) \sin^3(c + dx)}{3d} + \frac{(A + C) \sin(c + dx)}{d} + \frac{C \sin^9(c + dx)}{9d}$$

```
[Out] ((A + C)*Sin[c + d*x])/d - ((3*A + 4*C)*Sin[c + d*x]^3)/(3*d) + (3*(A + 2*C)
)*Sin[c + d*x]^5)/(5*d) - ((A + 4*C)*Sin[c + d*x]^7)/(7*d) + (C*SIN[c + d*x
]^9)/(9*d)
```

Rubi [A] time = 0.070859, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3013, 373}

$$-\frac{(A + 4C) \sin^7(c + dx)}{7d} + \frac{3(A + 2C) \sin^5(c + dx)}{5d} - \frac{(3A + 4C) \sin^3(c + dx)}{3d} + \frac{(A + C) \sin(c + dx)}{d} + \frac{C \sin^9(c + dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2), x]
```

```
[Out] ((A + C)*Sin[c + d*x])/d - ((3*A + 4*C)*Sin[c + d*x]^3)/(3*d) + (3*(A + 2*C)
)*Sin[c + d*x]^5)/(5*d) - ((A + 4*C)*Sin[c + d*x]^7)/(7*d) + (C*SIN[c + d*x
]^9)/(9*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2),
x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cos^7(c + dx) (A + C \cos^2(c + dx)) dx = -\frac{\text{Subst}\left(\int (1 - x^2)^3 (A + C - Cx^2) dx, x, -\sin(c + dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(A\left(1 + \frac{C}{A}\right) - (3A + 4C)x^2 + 3(A + 2C)x^4 - (A + 4C)x^6 + Cx^8\right) dx, x, -\sin(c + dx)\right)}{d}$$

$$= \frac{(A + C) \sin(c + dx)}{d} - \frac{(3A + 4C) \sin^3(c + dx)}{3d} + \frac{3(A + 2C) \sin^5(c + dx)}{5d} - \frac{(A + 4C) \sin^7(c + dx)}{7d}$$

Mathematica [A] time = 0.0491361, size = 133, normalized size = 1.45

$$-\frac{A \sin^7(c + dx)}{7d} + \frac{3A \sin^5(c + dx)}{5d} - \frac{A \sin^3(c + dx)}{d} + \frac{A \sin(c + dx)}{d} + \frac{C \sin^9(c + dx)}{9d} - \frac{4C \sin^7(c + dx)}{7d} + \frac{6C \sin^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^7*(A + C*Cos[c + d*x]^2), x]

[Out] (A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (A*Sin[c + d*x]^3)/d - (4*C*Sin[c + d*x]^3)/(3*d) + (3*A*Sin[c + d*x]^5)/(5*d) + (6*C*Sin[c + d*x]^5)/(5*d) - (A*Sin[c + d*x]^7)/(7*d) - (4*C*Sin[c + d*x]^7)/(7*d) + (C*Sin[c + d*x]^9)/(9*d)

Maple [A] time = 0.044, size = 94, normalized size = 1.

$$\frac{1}{d} \left(\frac{C \sin(dx + c)}{9} \left(\frac{128}{35} + (\cos(dx + c))^8 + \frac{8 (\cos(dx + c))^6}{7} + \frac{48 (\cos(dx + c))^4}{35} + \frac{64 (\cos(dx + c))^2}{35} \right) + \frac{A \sin(dx + c)}{7} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^7*(A+C*cos(d*x+c)^2), x)

[Out] 1/d*(1/9*C*(128/35+cos(d*x+c)^8+8/7*cos(d*x+c)^6+48/35*cos(d*x+c)^4+64/35*cos(d*x+c)^2)*sin(d*x+c)+1/7*A*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c))

Maxima [A] time = 1.06933, size = 101, normalized size = 1.1

$$\frac{35C \sin(dx + c)^9 - 45(A + 4C) \sin(dx + c)^7 + 189(A + 2C) \sin(dx + c)^5 - 105(3A + 4C) \sin(dx + c)^3 + 315(A + C) \sin(dx + c)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] 1/315*(35*C*sin(d*x + c)^9 - 45*(A + 4*C)*sin(d*x + c)^7 + 189*(A + 2*C)*sin(d*x + c)^5 - 105*(3*A + 4*C)*sin(d*x + c)^3 + 315*(A + C)*sin(d*x + c))/d

Fricas [A] time = 1.64835, size = 207, normalized size = 2.25

$$\frac{(35 C \cos(dx + c)^8 + 5(9 A + 8 C) \cos(dx + c)^6 + 6(9 A + 8 C) \cos(dx + c)^4 + 8(9 A + 8 C) \cos(dx + c)^2 + 144 A + 128 C) \sin(dx + c)}{315 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] 1/315*(35*C*cos(d*x + c)^8 + 5*(9*A + 8*C)*cos(d*x + c)^6 + 6*(9*A + 8*C)*cos(d*x + c)^4 + 8*(9*A + 8*C)*cos(d*x + c)^2 + 144*A + 128*C)*sin(d*x + c)/d

Sympy [A] time = 34.2829, size = 199, normalized size = 2.16

$$\left\{ \begin{array}{l} \frac{16A \sin^7(c+dx)}{35d} + \frac{8A \sin^5(c+dx) \cos^2(c+dx)}{5d} + \frac{2A \sin^3(c+dx) \cos^4(c+dx)}{d} + \frac{A \sin(c+dx) \cos^6(c+dx)}{d} + \frac{128C \sin^9(c+dx)}{315d} + \frac{64C \sin^7(c+dx) \cos^2(c+dx)}{35d} \\ x(A + C \cos^2(c)) \cos^7(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**7*(A+C*cos(d*x+c)**2),x)

[Out] Piecewise((16*A*sin(c + d*x)**7/(35*d) + 8*A*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*A*sin(c + d*x)**3*cos(c + d*x)**4/d + A*sin(c + d*x)*cos(c + d*x)**6/d + 128*C*sin(c + d*x)**9/(315*d) + 64*C*sin(c + d*x)**7*cos(c + d*x)**2/(35*d) + 16*C*sin(c + d*x)**5*cos(c + d*x)**4/(5*d) + 8*C*sin(c + d*x)**3*cos(c + d*x)**6/(3*d) + C*sin(c + d*x)*cos(c + d*x)**8/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**7, True))

Giac [A] time = 1.15034, size = 126, normalized size = 1.37

$$\frac{C \sin(9 dx + 9 c)}{2304 d} + \frac{(4 A + 9 C) \sin(7 dx + 7 c)}{1792 d} + \frac{(7 A + 9 C) \sin(5 dx + 5 c)}{320 d} + \frac{7(A + C) \sin(3 dx + 3 c)}{64 d} + \frac{7(10 A + 9 C) \sin(dx + c)}{128 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^7*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] 1/2304*C*sin(9*d*x + 9*c)/d + 1/1792*(4*A + 9*C)*sin(7*d*x + 7*c)/d + 1/320*(7*A + 9*C)*sin(5*d*x + 5*c)/d + 7/64*(A + C)*sin(3*d*x + 3*c)/d + 7/128*(10*A + 9*C)*sin(d*x + c)/d

3.2 $\int \cos^5(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=72

$$\frac{(A + 3C) \sin^5(c + dx)}{5d} - \frac{(2A + 3C) \sin^3(c + dx)}{3d} + \frac{(A + C) \sin(c + dx)}{d} - \frac{C \sin^7(c + dx)}{7d}$$

[Out] ((A + C)*Sin[c + d*x])/d - ((2*A + 3*C)*Sin[c + d*x]^3)/(3*d) + ((A + 3*C)*Sin[c + d*x]^5)/(5*d) - (C*SIN[c + d*x]^7)/(7*d)

Rubi [A] time = 0.0656366, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3013, 373}

$$\frac{(A + 3C) \sin^5(c + dx)}{5d} - \frac{(2A + 3C) \sin^3(c + dx)}{3d} + \frac{(A + C) \sin(c + dx)}{d} - \frac{C \sin^7(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2), x]

[Out] ((A + C)*Sin[c + d*x])/d - ((2*A + 3*C)*Sin[c + d*x]^3)/(3*d) + ((A + 3*C)*Sin[c + d*x]^5)/(5*d) - (C*SIN[c + d*x]^7)/(7*d)

Rule 3013

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^(m - 1)/2]*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rule 373

Int[((a_.) + (b_.)*(x_)^(n_))^(p_.)*((c_.) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned} \int \cos^5(c + dx) (A + C \cos^2(c + dx)) dx &= -\frac{\text{Subst}\left(\int (1 - x^2)^2 (A + C - Cx^2) dx, x, -\sin(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(A\left(1 + \frac{C}{A}\right) - (2A + 3C)x^2 + (A + 3C)x^4 - Cx^6\right) dx, x, -\sin(c + dx)\right)}{d} \\ &= \frac{(A + C) \sin(c + dx)}{d} - \frac{(2A + 3C) \sin^3(c + dx)}{3d} + \frac{(A + 3C) \sin^5(c + dx)}{5d} - \frac{C \sin^7(c + dx)}{7d} \end{aligned}$$

Mathematica [A] time = 0.026855, size = 101, normalized size = 1.4

$$\frac{A \sin^5(c + dx)}{5d} - \frac{2A \sin^3(c + dx)}{3d} + \frac{A \sin(c + dx)}{d} - \frac{C \sin^7(c + dx)}{7d} + \frac{3C \sin^5(c + dx)}{5d} - \frac{C \sin^3(c + dx)}{d} + \frac{C \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2), x]

[Out] (A*Sin[c + d*x])/d + (C*Sin[c + d*x])/d - (2*A*Sin[c + d*x]^3)/(3*d) - (C*Sin[c + d*x]^3)/d + (A*Sin[c + d*x]^5)/(5*d) + (3*C*Sin[c + d*x]^5)/(5*d) - (C*Sin[c + d*x]^7)/(7*d)

Maple [A] time = 0.037, size = 74, normalized size = 1.

$$\frac{1}{d} \left(\frac{C \sin(dx + c)}{7} \left(\frac{16}{5} + (\cos(dx + c))^6 + \frac{6 (\cos(dx + c))^4}{5} + \frac{8 (\cos(dx + c))^2}{5} \right) + \frac{A \sin(dx + c)}{5} \left(\frac{8}{3} + (\cos(dx + c))^4 \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2), x)

[Out] 1/d*(1/7*C*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d*x+c)^2)*sin(d*x+c)+1/5*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c))

Maxima [A] time = 1.04842, size = 81, normalized size = 1.12

$$\frac{15 C \sin(dx + c)^7 - 21 (A + 3 C) \sin(dx + c)^5 + 35 (2 A + 3 C) \sin(dx + c)^3 - 105 (A + C) \sin(dx + c)}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] -1/105*(15*C*sin(d*x + c)^7 - 21*(A + 3*C)*sin(d*x + c)^5 + 35*(2*A + 3*C)*sin(d*x + c)^3 - 105*(A + C)*sin(d*x + c))/d

Fricas [A] time = 1.61571, size = 162, normalized size = 2.25

$$\frac{(15 C \cos(dx + c)^6 + 3 (7 A + 6 C) \cos(dx + c)^4 + 4 (7 A + 6 C) \cos(dx + c)^2 + 56 A + 48 C) \sin(dx + c)}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] 1/105*(15*C*cos(d*x + c)^6 + 3*(7*A + 6*C)*cos(d*x + c)^4 + 4*(7*A + 6*C)*cos(d*x + c)^2 + 56*A + 48*C)*sin(d*x + c)/d

Sympy [A] time = 11.1329, size = 151, normalized size = 2.1

$$\left\{ \begin{array}{l} \frac{8 A \sin^5(c+dx)}{15 d} + \frac{4 A \sin^3(c+dx) \cos^2(c+dx)}{3 d} + \frac{A \sin(c+dx) \cos^4(c+dx)}{d} + \frac{16 C \sin^7(c+dx)}{35 d} + \frac{8 C \sin^5(c+dx) \cos^2(c+dx)}{5 d} + \frac{2 C \sin^3(c+dx) \cos^4(c+dx)}{d} \\ x (A + C \cos^2(c)) \cos^5(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Piecewise((8*A*sin(c + d*x)**5/(15*d) + 4*A*sin(c + d*x)**3*cos(c + d*x)**2
/(3*d) + A*sin(c + d*x)*cos(c + d*x)**4/d + 16*C*sin(c + d*x)**7/(35*d) + 8
*C*sin(c + d*x)**5*cos(c + d*x)**2/(5*d) + 2*C*sin(c + d*x)**3*cos(c + d*x)
**4/d + C*sin(c + d*x)*cos(c + d*x)**6/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*c
os(c)**5, True))
```

Giac [A] time = 1.17715, size = 103, normalized size = 1.43

$$\frac{C \sin(7dx + 7c)}{448d} + \frac{(4A + 7C) \sin(5dx + 5c)}{320d} + \frac{(20A + 21C) \sin(3dx + 3c)}{192d} + \frac{5(8A + 7C) \sin(dx + c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/448*C*sin(7*d*x + 7*c)/d + 1/320*(4*A + 7*C)*sin(5*d*x + 5*c)/d + 1/192*(
20*A + 21*C)*sin(3*d*x + 3*c)/d + 5/64*(8*A + 7*C)*sin(d*x + c)/d
```


3.3 $\int \cos^3(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=50

$$\frac{(A + 2C) \sin^3(c + dx)}{3d} + \frac{(A + C) \sin(c + dx)}{d} + \frac{C \sin^5(c + dx)}{5d}$$

[Out] $((A + C) \sin[c + d*x])/d - ((A + 2*C) \sin[c + d*x]^3)/(3*d) + (C \sin[c + d*x]^5)/(5*d)$

Rubi [A] time = 0.0524978, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3013, 373}

$$\frac{(A + 2C) \sin^3(c + dx)}{3d} + \frac{(A + C) \sin(c + dx)}{d} + \frac{C \sin^5(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] `Int[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2), x]`

[Out] $((A + C) \sin[c + d*x])/d - ((A + 2*C) \sin[c + d*x]^3)/(3*d) + (C \sin[c + d*x]^5)/(5*d)$

Rule 3013

`Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]`

Rule 373

`Int[((a_.) + (b_.)*(x_)^(n_.))^(p_.)*((c_.) + (d_.)*(x_)^(n_.))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]`

Rubi steps

$$\begin{aligned} \int \cos^3(c + dx) (A + C \cos^2(c + dx)) dx &= -\frac{\text{Subst}\left(\int (1 - x^2) (A + C - Cx^2) dx, x, -\sin(c + dx)\right)}{d} \\ &= -\frac{\text{Subst}\left(\int \left(A \left(1 + \frac{C}{A}\right) - (A + 2C)x^2 + Cx^4\right) dx, x, -\sin(c + dx)\right)}{d} \\ &= \frac{(A + C) \sin(c + dx)}{d} - \frac{(A + 2C) \sin^3(c + dx)}{3d} + \frac{C \sin^5(c + dx)}{5d} \end{aligned}$$

Mathematica [A] time = 0.017613, size = 71, normalized size = 1.42

$$-\frac{A \sin^3(c + dx)}{3d} + \frac{A \sin(c + dx)}{d} + \frac{C \sin^5(c + dx)}{5d} - \frac{2C \sin^3(c + dx)}{3d} + \frac{C \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] `Integrate[Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2), x]`

[Out] $(A \sin[c + d*x])/d + (C \sin[c + d*x])/d - (A \sin[c + d*x]^3)/(3*d) - (2*C \sin[c + d*x]^3)/(3*d) + (C \sin[c + d*x]^5)/(5*d)$

Maple [A] time = 0.04, size = 54, normalized size = 1.1

$$\frac{1}{d} \left(\frac{C \sin(dx + c)}{5} \left(\frac{8}{3} + (\cos(dx + c))^4 + \frac{4 (\cos(dx + c))^2}{3} \right) + \frac{A (2 + (\cos(dx + c))^2) \sin(dx + c)}{3} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x)`

[Out] $1/d*(1/5*C*(8/3+\cos(d*x+c)^4+4/3*\cos(d*x+c)^2)*\sin(d*x+c)+1/3*A*(2+\cos(d*x+c)^2)*\sin(d*x+c)$

Maxima [A] time = 1.04346, size = 58, normalized size = 1.16

$$\frac{3 C \sin(dx + c)^5 - 5(A + 2 C) \sin(dx + c)^3 + 15(A + C) \sin(dx + c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="maxima")`

[Out] $1/15*(3*C*\sin(d*x + c)^5 - 5*(A + 2*C)*\sin(d*x + c)^3 + 15*(A + C)*\sin(d*x + c))/d$

Fricas [A] time = 1.62552, size = 113, normalized size = 2.26

$$\frac{(3 C \cos(dx + c)^4 + (5 A + 4 C) \cos(dx + c)^2 + 10 A + 8 C) \sin(dx + c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="fricas")`

[Out] $1/15*(3*C*\cos(d*x + c)^4 + (5*A + 4*C)*\cos(d*x + c)^2 + 10*A + 8*C)*\sin(d*x + c)/d$

Sympy [A] time = 3.17319, size = 105, normalized size = 2.1

$$\begin{cases} \frac{2A \sin^3(c+dx)}{3d} + \frac{A \sin(c+dx) \cos^2(c+dx)}{d} + \frac{8C \sin^5(c+dx)}{15d} + \frac{4C \sin^3(c+dx) \cos^2(c+dx)}{3d} + \frac{C \sin(c+dx) \cos^4(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + C \cos^2(c)) \cos^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2),x)`

```
[Out] Piecewise((2*A*sin(c + d*x)**3/(3*d) + A*sin(c + d*x)*cos(c + d*x)**2/d + 8
*C*sin(c + d*x)**5/(15*d) + 4*C*sin(c + d*x)**3*cos(c + d*x)**2/(3*d) + C*s
in(c + d*x)*cos(c + d*x)**4/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**3, T
rue))
```

Giac [A] time = 1.13065, size = 77, normalized size = 1.54

$$\frac{3C \sin(dx + c)^5 - 5A \sin(dx + c)^3 - 10C \sin(dx + c)^3 + 15A \sin(dx + c) + 15C \sin(dx + c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*C*sin(d*x + c)^5 - 5*A*sin(d*x + c)^3 - 10*C*sin(d*x + c)^3 + 15*A*
sin(d*x + c) + 15*C*sin(d*x + c))/d
```

3.4 $\int \cos(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=30

$$\frac{(A + C) \sin(c + dx)}{d} - \frac{C \sin^3(c + dx)}{3d}$$

[Out] ((A + C)*Sin[c + d*x])/d - (C*SIN[c + d*x]^3)/(3*d)

Rubi [A] time = 0.0232144, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {3013}

$$\frac{(A + C) \sin(c + dx)}{d} - \frac{C \sin^3(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]

[Out] ((A + C)*Sin[c + d*x])/d - (C*SIN[c + d*x]^3)/(3*d)

Rule 3013

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \cos(c + dx) (A + C \cos^2(c + dx)) dx &= -\frac{\text{Subst}\left(\int (A + C - Cx^2) dx, x, -\sin(c + dx)\right)}{d} \\ &= \frac{(A + C) \sin(c + dx)}{d} - \frac{C \sin^3(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0154197, size = 50, normalized size = 1.67

$$\frac{A \sin(c) \cos(dx)}{d} + \frac{A \cos(c) \sin(dx)}{d} - \frac{C \sin^3(c + dx)}{3d} + \frac{C \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(A + C*Cos[c + d*x]^2),x]

[Out] (A*Cos[d*x]*Sin[c])/d + (A*Cos[c]*Sin[d*x])/d + (C*SIN[c + d*x])/d - (C*SIN[c + d*x]^3)/(3*d)

Maple [A] time = 0.039, size = 33, normalized size = 1.1

$$\frac{1}{d} \left(\frac{C(2 + (\cos(dx + c))^2) \sin(dx + c)}{3} + A \sin(dx + c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+C*cos(d*x+c)^2),x)`

[Out] `1/d*(1/3*C*(2+cos(d*x+c)^2)*sin(d*x+c)+A*sin(d*x+c))`

Maxima [A] time = 1.01083, size = 46, normalized size = 1.53

$$\frac{(\sin(dx+c)^3 - 3 \sin(dx+c))C - 3A \sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")`

[Out] `-1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d`

Fricas [A] time = 1.65117, size = 69, normalized size = 2.3

$$\frac{(C \cos(dx+c)^2 + 3A + 2C) \sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")`

[Out] `1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sin(d*x + c)/d`

Sympy [A] time = 0.717456, size = 56, normalized size = 1.87

$$\begin{cases} \frac{A \sin(c+dx)}{d} + \frac{2C \sin^3(c+dx)}{3d} + \frac{C \sin(c+dx) \cos^2(c+dx)}{d} & \text{for } d \neq 0 \\ x(A + C \cos^2(c)) \cos(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2),x)`

[Out] `Piecewise((A*sin(c + d*x)/d + 2*C*sin(c + d*x)**3/(3*d) + C*sin(c + d*x)*cos(c + d*x)**2/d, Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c), True))`

Giac [A] time = 1.16038, size = 46, normalized size = 1.53

$$\frac{(\sin(dx+c)^3 - 3 \sin(dx+c))C - 3A \sin(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2),x, algorithm="giac")`

[Out] `-1/3*((sin(d*x + c)^3 - 3*sin(d*x + c))*C - 3*A*sin(d*x + c))/d`

3.5 $\int (A + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=24

$$\frac{A \tanh^{-1}(\sin(c + dx))}{d} + \frac{C \sin(c + dx)}{d}$$

[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d

Rubi [A] time = 0.0310902, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {3014, 3770}

$$\frac{A \tanh^{-1}(\sin(c + dx))}{d} + \frac{C \sin(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Sin[c + d*x])/d

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec(c + dx) dx &= \frac{C \sin(c + dx)}{d} + A \int \sec(c + dx) dx \\ &= \frac{A \tanh^{-1}(\sin(c + dx))}{d} + \frac{C \sin(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0163272, size = 35, normalized size = 1.46

$$\frac{A \tanh^{-1}(\sin(c + dx))}{d} + \frac{C \sin(c) \cos(dx)}{d} + \frac{C \cos(c) \sin(dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (A*ArcTanh[Sin[c + d*x]])/d + (C*Cos[d*x]*Sin[c])/d + (C*Cos[c]*Sin[d*x])/d

Maple [A] time = 0.059, size = 32, normalized size = 1.3

$$\frac{A \ln(\sec(dx + c) + \tan(dx + c))}{d} + \frac{C \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c),x)

[Out] 1/d*A*ln(sec(d*x+c)+tan(d*x+c))+C*sin(d*x+c)/d

Maxima [A] time = 1.03149, size = 51, normalized size = 2.12

$$\frac{A \log(\sin(dx + c) + 1) - A \log(\sin(dx + c) - 1) + 2 C \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")

[Out] 1/2*(A*log(sin(d*x + c) + 1) - A*log(sin(d*x + c) - 1) + 2*C*sin(d*x + c))/d

Fricas [A] time = 1.6919, size = 107, normalized size = 4.46

$$\frac{A \log(\sin(dx + c) + 1) - A \log(-\sin(dx + c) + 1) + 2 C \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")

[Out] 1/2*(A*log(sin(d*x + c) + 1) - A*log(-sin(d*x + c) + 1) + 2*C*sin(d*x + c))/d

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c),x)

[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x), x)

Giac [A] time = 1.20299, size = 54, normalized size = 2.25

$$\frac{A \log(|\sin(dx + c) + 1|) - A \log(|\sin(dx + c) - 1|) + 2 C \sin(dx + c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(A*log(abs(sin(d*x + c) + 1)) - A*log(abs(sin(d*x + c) - 1)) + 2*C*sin(d*x + c))/d
```


3.6 $\int (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$

Optimal. Leaf size=40

$$\frac{(A + 2C) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{A \tan(c + dx) \sec(c + dx)}{2d}$$

[Out] $((A + 2C) \text{ArcTanh}[\text{Sin}[c + d*x]])/(2*d) + (A \text{Sec}[c + d*x] \text{Tan}[c + d*x])/(2*d)$

Rubi [A] time = 0.0373045, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3012, 3770}

$$\frac{(A + 2C) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{A \tan(c + dx) \sec(c + dx)}{2d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C \text{Cos}[c + d*x]^2) \text{Sec}[c + d*x]^3, x]$

[Out] $((A + 2C) \text{ArcTanh}[\text{Sin}[c + d*x]])/(2*d) + (A \text{Sec}[c + d*x] \text{Tan}[c + d*x])/(2*d)$

Rule 3012

$\text{Int}[(b \cdot \sin(e \cdot x) + f \cdot x)^m (A + C \cdot \sin(e \cdot x) + f \cdot x)^2], x_Symbol] \rightarrow \text{Simp}[A \cdot \text{Cos}[e + f \cdot x] \cdot (b \cdot \text{Sin}[e + f \cdot x])^{m+1} / (b \cdot f \cdot (m+1)), x] + \text{Dist}[(A \cdot (m+2) + C \cdot (m+1)) / (b^2 \cdot (m+1)), \text{Int}[(b \cdot \text{Sin}[e + f \cdot x])^{m+2}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

$\text{Int}[\text{csc}(c \cdot x + d \cdot x), x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d \cdot x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^3(c + dx) dx &= \frac{A \sec(c + dx) \tan(c + dx)}{2d} + \frac{1}{2}(A + 2C) \int \sec(c + dx) dx \\ &= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx))}{2d} + \frac{A \sec(c + dx) \tan(c + dx)}{2d} \end{aligned}$$

Mathematica [A] time = 0.0282361, size = 48, normalized size = 1.2

$$\frac{A \tanh^{-1}(\sin(c + dx))}{2d} + \frac{A \tan(c + dx) \sec(c + dx)}{2d} + \frac{C \tanh^{-1}(\sin(c + dx))}{d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(A + C \text{Cos}[c + d*x]^2) \text{Sec}[c + d*x]^3, x]$

[Out] $(A \cdot \text{ArcTanh}[\text{Sin}[c + d \cdot x]]) / (2 \cdot d) + (C \cdot \text{ArcTanh}[\text{Sin}[c + d \cdot x]]) / d + (A \cdot \text{Sec}[c + d \cdot x] \cdot \text{Tan}[c + d \cdot x]) / (2 \cdot d)$

Maple [A] time = 0.064, size = 59, normalized size = 1.5

$$\frac{A \sec(dx + c) \tan(dx + c)}{2d} + \frac{A \ln(\sec(dx + c) + \tan(dx + c))}{2d} + \frac{C \ln(\sec(dx + c) + \tan(dx + c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)`

[Out] $1/2 \cdot A \cdot \sec(dx + c) \cdot \tan(dx + c) / d + 1/2 / d \cdot A \cdot \ln(\sec(dx + c) + \tan(dx + c)) + 1/d \cdot C \cdot \ln(\sec(dx + c) + \tan(dx + c))$

Maxima [A] time = 1.07052, size = 78, normalized size = 1.95

$$\frac{(A + 2C) \log(\sin(dx + c) + 1) - (A + 2C) \log(\sin(dx + c) - 1) - \frac{2A \sin(dx + c)}{\sin(dx + c)^2 - 1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")`

[Out] $1/4 \cdot ((A + 2C) \cdot \log(\sin(dx + c) + 1) - (A + 2C) \cdot \log(\sin(dx + c) - 1) - 2 \cdot A \cdot \sin(dx + c) / (\sin(dx + c)^2 - 1)) / d$

Fricas [A] time = 1.70487, size = 192, normalized size = 4.8

$$\frac{(A + 2C) \cos(dx + c)^2 \log(\sin(dx + c) + 1) - (A + 2C) \cos(dx + c)^2 \log(-\sin(dx + c) + 1) + 2A \sin(dx + c)}{4d \cos(dx + c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")`

[Out] $1/4 \cdot ((A + 2C) \cdot \cos(dx + c)^2 \cdot \log(\sin(dx + c) + 1) - (A + 2C) \cdot \cos(dx + c)^2 \cdot \log(-\sin(dx + c) + 1) + 2 \cdot A \cdot \sin(dx + c)) / (d \cdot \cos(dx + c)^2)$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)`

[Out] Timed out

Giac [A] time = 1.18859, size = 81, normalized size = 2.02

$$\frac{(A + 2C) \log(|\sin(dx + c) + 1|) - (A + 2C) \log(|\sin(dx + c) - 1|) - \frac{2A \sin(dx+c)}{\sin(dx+c)^2 - 1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] 1/4*((A + 2*C)*log(abs(sin(d*x + c) + 1)) - (A + 2*C)*log(abs(sin(d*x + c) - 1)) - 2*A*sin(d*x + c)/(sin(d*x + c)^2 - 1))/d

3.7 $\int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$

Optimal. Leaf size=70

$$\frac{(3A + 4C) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(3A + 4C) \tan(c + dx) \sec(c + dx)}{8d} + \frac{A \tan(c + dx) \sec^3(c + dx)}{4d}$$

[Out] $((3A + 4C) \text{ArcTanh}[\text{Sin}[c + d*x]])/(8*d) + ((3A + 4C) \text{Sec}[c + d*x] \text{Tan}[c + d*x])/(8*d) + (A \text{Sec}[c + d*x]^3 \text{Tan}[c + d*x])/(4*d)$

Rubi [A] time = 0.0468188, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3012, 3768, 3770}

$$\frac{(3A + 4C) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(3A + 4C) \tan(c + dx) \sec(c + dx)}{8d} + \frac{A \tan(c + dx) \sec^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C \text{Cos}[c + d*x]^2) \text{Sec}[c + d*x]^5, x]$

[Out] $((3A + 4C) \text{ArcTanh}[\text{Sin}[c + d*x]])/(8*d) + ((3A + 4C) \text{Sec}[c + d*x] \text{Tan}[c + d*x])/(8*d) + (A \text{Sec}[c + d*x]^3 \text{Tan}[c + d*x])/(4*d)$

Rule 3012

$\text{Int}[(b \cdot \sin[e + f \cdot x] + (f \cdot x))^{(m)} \cdot ((A) + (C) \cdot \sin[e + f \cdot x] + (f \cdot x)^2), x_Symbol] \rightarrow \text{Simp}[A \cdot \text{Cos}[e + f \cdot x] \cdot (b \cdot \text{Sin}[e + f \cdot x])^{(m+1)} / (b \cdot f \cdot (m+1)), x] + \text{Dist}[(A \cdot (m+2) + C \cdot (m+1)) / (b^2 \cdot (m+1)), \text{Int}[(b \cdot \text{Sin}[e + f \cdot x])^{(m+2)}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3768

$\text{Int}[(\text{csc}[c + d \cdot x] + (d \cdot x) \cdot (b \cdot \cos[c + d \cdot x]))^{(n)}, x_Symbol] \rightarrow -\text{Simp}[(b \cdot \text{Cos}[c + d \cdot x] \cdot (b \cdot \text{Csc}[c + d \cdot x])^{(n-1)}) / (d \cdot (n-1)), x] + \text{Dist}[(b^2 \cdot (n-2)) / (n-1), \text{Int}[(b \cdot \text{Csc}[c + d \cdot x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\text{Int}[\text{csc}[c + d \cdot x] + (d \cdot x) \cdot (b \cdot \cos[c + d \cdot x]), x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d \cdot x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx &= \frac{A \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{4}(3A + 4C) \int \sec^3(c + dx) dx \\ &= \frac{(3A + 4C) \sec(c + dx) \tan(c + dx)}{8d} + \frac{A \sec^3(c + dx) \tan(c + dx)}{4d} + \frac{1}{8}(3A + 4C) \int \sec(c + dx) dx \\ &= \frac{(3A + 4C) \tanh^{-1}(\sin(c + dx))}{8d} + \frac{(3A + 4C) \sec(c + dx) \tan(c + dx)}{8d} + \frac{A \sec^3(c + dx) \tan(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.114558, size = 54, normalized size = 0.77

$$\frac{(3A + 4C) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx) (2A \sec^2(c + dx) + 3A + 4C)}{8d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3*A + 4*C + 2*A*Sec[c + d*x]^2)*Tan[c + d*x])/(8*d)

Maple [A] time = 0.079, size = 98, normalized size = 1.4

$$\frac{A(\sec(dx+c))^3 \tan(dx+c)}{4d} + \frac{3A \sec(dx+c) \tan(dx+c)}{8d} + \frac{3A \ln(\sec(dx+c) + \tan(dx+c))}{8d} + \frac{C \tan(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)

[Out] 1/4*A*sec(d*x+c)^3*tan(d*x+c)/d+3/8*A*sec(d*x+c)*tan(d*x+c)/d+3/8/d*A*ln(sec(d*x+c)+tan(d*x+c))+1/2/d*C*tan(d*x+c)*sec(d*x+c)+1/2/d*C*ln(sec(d*x+c)+tan(d*x+c))

Maxima [A] time = 1.03508, size = 131, normalized size = 1.87

$$\frac{(3A + 4C) \log(\sin(dx+c) + 1) - (3A + 4C) \log(\sin(dx+c) - 1) - \frac{2((3A+4C)\sin(dx+c)^3 - (5A+4C)\sin(dx+c))}{\sin(dx+c)^4 - 2\sin(dx+c)^2 + 1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")

[Out] 1/16*((3*A + 4*C)*log(sin(d*x + c) + 1) - (3*A + 4*C)*log(sin(d*x + c) - 1) - 2*((3*A + 4*C)*sin(d*x + c)^3 - (5*A + 4*C)*sin(d*x + c)))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1))/d

Fricas [A] time = 1.66661, size = 243, normalized size = 3.47

$$\frac{(3A + 4C) \cos(dx+c)^4 \log(\sin(dx+c) + 1) - (3A + 4C) \cos(dx+c)^4 \log(-\sin(dx+c) + 1) + 2((3A + 4C) \cos(dx+c)^4 \log(\sin(dx+c) + 1) - (3A + 4C) \cos(dx+c)^4 \log(-\sin(dx+c) + 1) + 2((3A + 4C) \cos(dx+c)^2 + 2A) \sin(dx+c))}{16d \cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")

[Out] 1/16*((3*A + 4*C)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - (3*A + 4*C)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sin(d*x + c))/(d*cos(d*x + c)^4)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)

[Out] Timed out

Giac [A] time = 1.23869, size = 132, normalized size = 1.89

$$(3A + 4C) \log(|\sin(dx + c) + 1|) - (3A + 4C) \log(|\sin(dx + c) - 1|) - \frac{2(3A \sin(dx+c)^3 + 4C \sin(dx+c)^3 - 5A \sin(dx+c) - 4C \sin(dx+c))}{(\sin(dx+c)^2 - 1)^2}$$

$$16d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")

[Out] 1/16*((3*A + 4*C)*log(abs(sin(d*x + c) + 1)) - (3*A + 4*C)*log(abs(sin(d*x + c) - 1)) - 2*(3*A*sin(d*x + c)^3 + 4*C*sin(d*x + c)^3 - 5*A*sin(d*x + c) - 4*C*sin(d*x + c))/(sin(d*x + c)^2 - 1)^2)/d

3.8 $\int (A + C \cos^2(c + dx)) \sec^7(c + dx) dx$

Optimal. Leaf size=98

$$\frac{(5A + 6C) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{(5A + 6C) \tan(c + dx) \sec^3(c + dx)}{24d} + \frac{(5A + 6C) \tan(c + dx) \sec(c + dx)}{16d} + \frac{A \tan(c + dx)}{6d}$$

[Out] $((5*A + 6*C)*\text{ArcTanh}[\text{Sin}[c + d*x]])/(16*d) + ((5*A + 6*C)*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(16*d) + ((5*A + 6*C)*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/(24*d) + (A*\text{Sec}[c + d*x]^5*\text{Tan}[c + d*x])/(6*d)$

Rubi [A] time = 0.0601627, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3012, 3768, 3770}

$$\frac{(5A + 6C) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{(5A + 6C) \tan(c + dx) \sec^3(c + dx)}{24d} + \frac{(5A + 6C) \tan(c + dx) \sec(c + dx)}{16d} + \frac{A \tan(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^7, x]$

[Out] $((5*A + 6*C)*\text{ArcTanh}[\text{Sin}[c + d*x]])/(16*d) + ((5*A + 6*C)*\text{Sec}[c + d*x]*\text{Tan}[c + d*x])/(16*d) + ((5*A + 6*C)*\text{Sec}[c + d*x]^3*\text{Tan}[c + d*x])/(24*d) + (A*\text{Sec}[c + d*x]^5*\text{Tan}[c + d*x])/(6*d)$

Rule 3012

$\text{Int}[(b*\sin[e + f*x] + (f*x))^{m+1} * ((A + C*\sin[e + f*x] + (f*x))^2), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x] * (b*\sin[e + f*x])^{m+1}) / (b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1)) / (b^2*(m+1)), \text{Int}[(b*\sin[e + f*x])^{m+2}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3768

$\text{Int}[(\text{csc}[c + d*x] + (d*x)) * (b*x)^n, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Csc}[c + d*x])^{n-1}) / (d*(n-1)), x] + \text{Dist}[(b^2*(n-2)) / (n-1), \text{Int}[(b*\text{Csc}[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

$\text{Int}[\text{csc}[c + d*x] * (d*x), x_Symbol] \rightarrow -\text{Simp}[\text{ArcTanh}[\text{Cos}[c + d*x]] / d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^7(c + dx) dx &= \frac{A \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{1}{6}(5A + 6C) \int \sec^5(c + dx) dx \\ &= \frac{(5A + 6C) \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{A \sec^5(c + dx) \tan(c + dx)}{6d} + \frac{1}{8}(5A + 6C) \int \sec^3(c + dx) dx \\ &= \frac{(5A + 6C) \sec(c + dx) \tan(c + dx)}{16d} + \frac{(5A + 6C) \sec^3(c + dx) \tan(c + dx)}{24d} + \frac{1}{8}(5A + 6C) \int \sec(c + dx) dx \\ &= \frac{(5A + 6C) \tanh^{-1}(\sin(c + dx))}{16d} + \frac{(5A + 6C) \sec(c + dx) \tan(c + dx)}{16d} + \frac{1}{8}(5A + 6C) \int \sec(c + dx) dx \end{aligned}$$

Mathematica [A] time = 0.300422, size = 75, normalized size = 0.77

$$\frac{3(5A + 6C) \tanh^{-1}(\sin(c + dx)) + \tan(c + dx) \sec(c + dx) (2(5A + 6C) \sec^2(c + dx) + 8A \sec^4(c + dx) + 3(5A + 6C))}{48d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]

[Out] (3*(5*A + 6*C)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(3*(5*A + 6*C) + 2*(5*A + 6*C)*Sec[c + d*x]^2 + 8*A*Sec[c + d*x]^4)*Tan[c + d*x])/(48*d)

Maple [A] time = 0.067, size = 138, normalized size = 1.4

$$\frac{A(\sec(dx + c))^5 \tan(dx + c)}{6d} + \frac{5A(\sec(dx + c))^3 \tan(dx + c)}{24d} + \frac{5A \sec(dx + c) \tan(dx + c)}{16d} + \frac{5A \ln(\sec(dx + c) + \tan(dx + c))}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)

[Out] 1/6*A*sec(d*x+c)^5*tan(d*x+c)/d+5/24*A*sec(d*x+c)^3*tan(d*x+c)/d+5/16*A*sec(d*x+c)*tan(d*x+c)/d+5/16/d*A*ln(sec(d*x+c)+tan(d*x+c))+1/4/d*C*tan(d*x+c)*sec(d*x+c)^3+3/8/d*C*tan(d*x+c)*sec(d*x+c)+3/8/d*C*ln(sec(d*x+c)+tan(d*x+c))

Maxima [A] time = 1.06679, size = 170, normalized size = 1.73

$$\frac{3(5A + 6C) \log(\sin(dx + c) + 1) - 3(5A + 6C) \log(\sin(dx + c) - 1) - \frac{2(3(5A+6C) \sin(dx+c)^5 - 8(5A+6C) \sin(dx+c)^3 + 3(11A+10C) \sin(dx+c))}{\sin(dx+c)^6 - 3 \sin(dx+c)^4 + 3 \sin(dx+c)^2 - 1}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="maxima")

[Out] 1/96*(3*(5*A + 6*C)*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*log(sin(d*x + c) - 1) - 2*(3*(5*A + 6*C)*sin(d*x + c)^5 - 8*(5*A + 6*C)*sin(d*x + c)^3 + 3*(11*A + 10*C)*sin(d*x + c)))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1)/d

Fricas [A] time = 1.73048, size = 293, normalized size = 2.99

$$\frac{3(5A + 6C) \cos(dx + c)^6 \log(\sin(dx + c) + 1) - 3(5A + 6C) \cos(dx + c)^6 \log(-\sin(dx + c) + 1) + 2(3(5A + 6C) \cos(dx + c)^4 + 2(5A + 6C) \cos(dx + c)^2 + 3(5A + 6C)) \log(\sin(dx + c) + 1)}{96d \cos(dx + c)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas")

[Out] 1/96*(3*(5*A + 6*C)*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 3*(5*A + 6*C)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(3*(5*A + 6*C)*cos(d*x + c)^4 + 2*(5*A + 6*C)*cos(d*x + c)^2 + 3*(5*A + 6*C))*log(sin(d*x + c) + 1)

$$5*A + 6*C)*\cos(d*x + c)^2 + 8*A)*\sin(d*x + c))/(d*\cos(d*x + c)^6)$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)

[Out] Timed out

Giac [A] time = 1.20344, size = 163, normalized size = 1.66

$$3(5A + 6C) \log(|\sin(dx + c) + 1|) - 3(5A + 6C) \log(|\sin(dx + c) - 1|) - \frac{2(15A \sin(dx+c)^5 + 18C \sin(dx+c)^5 - 40A \sin(dx+c)^3 - 48C \sin(dx+c)^3 + 33A \sin(dx+c) + 30C \sin(dx+c))}{96d \sin(dx+c)^2 - 1} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="giac")

[Out] 1/96*(3*(5*A + 6*C)*log(abs(sin(d*x + c) + 1)) - 3*(5*A + 6*C)*log(abs(sin(d*x + c) - 1)) - 2*(15*A*sin(d*x + c)^5 + 18*C*sin(d*x + c)^5 - 40*A*sin(d*x + c)^3 - 48*C*sin(d*x + c)^3 + 33*A*sin(d*x + c) + 30*C*sin(d*x + c)))/(sin(d*x + c)^2 - 1)^3)/d

3.9 $\int \cos^6(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=117

$$\frac{(8A + 7C) \sin(c + dx) \cos^5(c + dx)}{48d} + \frac{5(8A + 7C) \sin(c + dx) \cos^3(c + dx)}{192d} + \frac{5(8A + 7C) \sin(c + dx) \cos(c + dx)}{128d} + \frac{5}{128}$$

[Out] (5*(8*A + 7*C)*x)/128 + (5*(8*A + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*(8*A + 7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + ((8*A + 7*C)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (C*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)

Rubi [A] time = 0.0670029, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$\frac{(8A + 7C) \sin(c + dx) \cos^5(c + dx)}{48d} + \frac{5(8A + 7C) \sin(c + dx) \cos^3(c + dx)}{192d} + \frac{5(8A + 7C) \sin(c + dx) \cos(c + dx)}{128d} + \frac{5}{128}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2), x]

[Out] (5*(8*A + 7*C)*x)/128 + (5*(8*A + 7*C)*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (5*(8*A + 7*C)*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) + ((8*A + 7*C)*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (C*Cos[c + d*x]^7*Sin[c + d*x])/(8*d)

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \cos^6(c + dx) (A + C \cos^2(c + dx)) dx &= \frac{C \cos^7(c + dx) \sin(c + dx)}{8d} + \frac{1}{8}(8A + 7C) \int \cos^6(c + dx) dx \\ &= \frac{(8A + 7C) \cos^5(c + dx) \sin(c + dx)}{48d} + \frac{C \cos^7(c + dx) \sin(c + dx)}{8d} + \frac{1}{48}(5(8A + 7C) \cos^3(c + dx) \sin(c + dx) \\ &+ (8A + 7C) \cos^5(c + dx) \sin(c + dx) + 5(8A + 7C) \cos(c + dx) \sin(c + dx) \\ &+ 5(8A + 7C) \cos^3(c + dx) \sin(c + dx)) \\ &= \frac{5(8A + 7C) \cos^3(c + dx) \sin(c + dx)}{192d} + \frac{(8A + 7C) \cos^5(c + dx) \sin(c + dx)}{48d} \\ &+ \frac{5(8A + 7C) \cos(c + dx) \sin(c + dx)}{128d} + \frac{5(8A + 7C) \cos^3(c + dx) \sin(c + dx)}{192d} \\ &+ \frac{5}{128}(8A + 7C)x + \frac{5(8A + 7C) \cos(c + dx) \sin(c + dx)}{128d} + \frac{5(8A + 7C) \cos^3(c + dx) \sin(c + dx)}{192d} \end{aligned}$$

Mathematica [A] time = 0.156183, size = 93, normalized size = 0.79

$$\frac{48(15A + 14C)\sin(2(c + dx)) + 24(6A + 7C)\sin(4(c + dx)) + 16A\sin(6(c + dx)) + 960Ac + 960Adx + 32C\sin(6(c + dx))}{3072d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^6*(A + C*Cos[c + d*x]^2), x]

[Out] (960*A*c + 840*c*C + 960*A*d*x + 840*C*d*x + 48*(15*A + 14*C)*Sin[2*(c + d*x)] + 24*(6*A + 7*C)*Sin[4*(c + d*x)] + 16*A*Sin[6*(c + d*x)] + 32*C*Sin[6*(c + d*x)] + 3*C*Sin[8*(c + d*x)])/(3072*d)

Maple [A] time = 0.037, size = 106, normalized size = 0.9

$$\frac{1}{d} \left(C \left(\frac{\sin(dx+c)}{8} \left((\cos(dx+c))^7 + \frac{7(\cos(dx+c))^5}{6} + \frac{35(\cos(dx+c))^3}{24} + \frac{35\cos(dx+c)}{16} \right) + \frac{35dx}{128} + \frac{35c}{128} \right) + A \left(\frac{\sin(dx+c)}{6} \left((\cos(dx+c))^5 + \frac{5\cos(dx+c)}{4} \right) + \frac{15dx}{8} + \frac{15c}{8} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^6*(A+C*cos(d*x+c)^2), x)

[Out] 1/d*(C*(1/8*(cos(d*x+c)^7+7/6*cos(d*x+c)^5+35/24*cos(d*x+c)^3+35/16*cos(d*x+c))*sin(d*x+c)+35/128*d*x+35/128*c)+A*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c))

Maxima [A] time = 1.56528, size = 176, normalized size = 1.5

$$\frac{15(dx+c)(8A+7C) + \frac{15(8A+7C)\tan(dx+c)^7 + 55(8A+7C)\tan(dx+c)^5 + 73(8A+7C)\tan(dx+c)^3 + 3(88A+93C)\tan(dx+c)}{\tan(dx+c)^8 + 4\tan(dx+c)^6 + 6\tan(dx+c)^4 + 4\tan(dx+c)^2 + 1}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] 1/384*(15*(d*x + c)*(8*A + 7*C) + (15*(8*A + 7*C)*tan(d*x + c)^7 + 55*(8*A + 7*C)*tan(d*x + c)^5 + 73*(8*A + 7*C)*tan(d*x + c)^3 + 3*(88*A + 93*C)*tan(d*x + c))/(tan(d*x + c)^8 + 4*tan(d*x + c)^6 + 6*tan(d*x + c)^4 + 4*tan(d*x + c)^2 + 1))/d

Fricas [A] time = 1.69715, size = 216, normalized size = 1.85

$$\frac{15(8A+7C)dx + (48C\cos(dx+c)^7 + 8(8A+7C)\cos(dx+c)^5 + 10(8A+7C)\cos(dx+c)^3 + 15(8A+7C)\cos(dx+c))\sin(dx+c)}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] 1/384*(15*(8*A + 7*C)*d*x + (48*C*cos(d*x + c)^7 + 8*(8*A + 7*C)*cos(d*x + c)^5 + 10*(8*A + 7*C)*cos(d*x + c)^3 + 15*(8*A + 7*C)*cos(d*x + c))*sin(d*x + c)

+ c))/d

Sympy [A] time = 19.1781, size = 354, normalized size = 3.03

$$\left\{ \begin{array}{l} \frac{5Ax \sin^6(c+dx)}{16} + \frac{15Ax \sin^4(c+dx) \cos^2(c+dx)}{16} + \frac{15Ax \sin^2(c+dx) \cos^4(c+dx)}{16} + \frac{5Ax \cos^6(c+dx)}{16} + \frac{5A \sin^5(c+dx) \cos(c+dx)}{16d} + \frac{5A \sin^3(c+dx) \cos^3(c+dx)}{6d} \\ x(A + C \cos^2(c)) \cos^6(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**6*(A+C*cos(d*x+c)**2),x)

[Out] Piecewise(((5*A*x*sin(c + d*x)**6/16 + 15*A*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*A*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*A*x*cos(c + d*x)**6/16 + 5*A*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*A*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*A*sin(c + d*x)*cos(c + d*x)**5/(16*d) + 35*C*x*sin(c + d*x)**8/128 + 35*C*x*sin(c + d*x)**6*cos(c + d*x)**2/32 + 105*C*x*sin(c + d*x)**4*cos(c + d*x)**4/64 + 35*C*x*sin(c + d*x)**2*cos(c + d*x)**6/32 + 35*C*x*cos(c + d*x)**8/128 + 35*C*sin(c + d*x)**7*cos(c + d*x)/(128*d) + 385*C*sin(c + d*x)**5*cos(c + d*x)**3/(384*d) + 511*C*sin(c + d*x)**3*cos(c + d*x)**5/(384*d) + 93*C*sin(c + d*x)*cos(c + d*x)**7/(128*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**6, True))

Giac [A] time = 1.15112, size = 117, normalized size = 1.

$$\frac{5}{128}(8A + 7C)x + \frac{C \sin(8dx + 8c)}{1024d} + \frac{(A + 2C) \sin(6dx + 6c)}{192d} + \frac{(6A + 7C) \sin(4dx + 4c)}{128d} + \frac{(15A + 14C) \sin(2dx + 2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] 5/128*(8*A + 7*C)*x + 1/1024*C*sin(8*d*x + 8*c)/d + 1/192*(A + 2*C)*sin(6*d*x + 6*c)/d + 1/128*(6*A + 7*C)*sin(4*d*x + 4*c)/d + 1/64*(15*A + 14*C)*sin(2*d*x + 2*c)/d

3.10 $\int \cos^4(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=89

$$\frac{(6A + 5C) \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{(6A + 5C) \sin(c + dx) \cos(c + dx)}{16d} + \frac{1}{16}x(6A + 5C) + \frac{C \sin(c + dx) \cos^5(c + dx)}{6d}$$

[Out] $((6*A + 5*C)*x)/16 + ((6*A + 5*C)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(16*d) + ((6*A + 5*C)*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(24*d) + (C*\text{Cos}[c + d*x]^5*\text{Sin}[c + d*x])/ (6*d)$

Rubi [A] time = 0.0532344, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3014, 2635, 8}

$$\frac{(6A + 5C) \sin(c + dx) \cos^3(c + dx)}{24d} + \frac{(6A + 5C) \sin(c + dx) \cos(c + dx)}{16d} + \frac{1}{16}x(6A + 5C) + \frac{C \sin(c + dx) \cos^5(c + dx)}{6d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^4*(A + C*\text{Cos}[c + d*x]^2), x]$

[Out] $((6*A + 5*C)*x)/16 + ((6*A + 5*C)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(16*d) + ((6*A + 5*C)*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(24*d) + (C*\text{Cos}[c + d*x]^5*\text{Sin}[c + d*x])/ (6*d)$

Rule 3014

$\text{Int}[(b*\sin[e + f*x] + (f*x))^{m+1} * ((A + C*\sin[e + f*x] + (f*x))^2), x_Symbol] :> -\text{Simp}[(C*\text{Cos}[e + f*x] * (b*\text{Sin}[e + f*x])^{m+1}) / (b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1)) / (m+2), \text{Int}[(b*\text{Sin}[e + f*x])^{m+1}, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

$\text{Int}[(b*\sin[c + d*x] + (d*x))^n, x_Symbol] :> -\text{Simp}[(b*\text{Cos}[c + d*x] * (b*\text{Sin}[c + d*x])^{n-1}) / (d*n), x] + \text{Dist}[(b^2*(n-1)) / n, \text{Int}[(b*\text{Sin}[c + d*x])^{n-2}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a, x_Symbol] :> \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \cos^4(c + dx) (A + C \cos^2(c + dx)) dx &= \frac{C \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{6}(6A + 5C) \int \cos^4(c + dx) dx \\ &= \frac{(6A + 5C) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{C \cos^5(c + dx) \sin(c + dx)}{6d} + \frac{1}{8}(6A + 5C)x \\ &= \frac{(6A + 5C) \cos(c + dx) \sin(c + dx)}{16d} + \frac{(6A + 5C) \cos^3(c + dx) \sin(c + dx)}{24d} + \frac{1}{16}(6A + 5C)x \\ &= \frac{1}{16}(6A + 5C)x + \frac{(6A + 5C) \cos(c + dx) \sin(c + dx)}{16d} + \frac{(6A + 5C) \cos^3(c + dx) \sin(c + dx)}{24d} \end{aligned}$$

Mathematica [A] time = 0.0959939, size = 68, normalized size = 0.76

$$\frac{(48A + 45C) \sin(2(c + dx)) + (6A + 9C) \sin(4(c + dx)) + 72Ac + 72Adx + C \sin(6(c + dx)) + 60cC + 60Cdx}{192d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2), x]

[Out] (72*A*c + 60*c*C + 72*A*d*x + 60*C*d*x + (48*A + 45*C)*Sin[2*(c + d*x)] + (6*A + 9*C)*Sin[4*(c + d*x)] + C*Ssin[6*(c + d*x)])/(192*d)

Maple [A] time = 0.052, size = 86, normalized size = 1.

$$\frac{1}{d} \left(C \left(\frac{\sin(dx+c)}{6} \left((\cos(dx+c))^5 + \frac{5(\cos(dx+c))^3}{4} + \frac{15\cos(dx+c)}{8} \right) + \frac{5dx}{16} + \frac{5c}{16} \right) + A \left(\frac{\sin(dx+c)}{4} \left((\cos(dx+c))^5 + \frac{5(\cos(dx+c))^3}{4} + \frac{15\cos(dx+c)}{8} \right) + \frac{5dx}{16} + \frac{5c}{16} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2), x)

[Out] 1/d*(C*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c))

Maxima [A] time = 1.58612, size = 139, normalized size = 1.56

$$\frac{3(dx+c)(6A+5C) + \frac{3(6A+5C)\tan(dx+c)^5 + 8(6A+5C)\tan(dx+c)^3 + 3(10A+11C)\tan(dx+c)}{\tan(dx+c)^6 + 3\tan(dx+c)^4 + 3\tan(dx+c)^2 + 1}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] 1/48*(3*(d*x + c)*(6*A + 5*C) + (3*(6*A + 5*C)*tan(d*x + c)^5 + 8*(6*A + 5*C)*tan(d*x + c)^3 + 3*(10*A + 11*C)*tan(d*x + c)))/(tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1)/d

Fricas [A] time = 1.65714, size = 167, normalized size = 1.88

$$\frac{3(6A+5C)dx + (8C \cos(dx+c)^5 + 2(6A+5C) \cos(dx+c)^3 + 3(6A+5C) \cos(dx+c)) \sin(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] 1/48*(3*(6*A + 5*C)*d*x + (8*C*cos(d*x + c)^5 + 2*(6*A + 5*C)*cos(d*x + c)^3 + 3*(6*A + 5*C)*cos(d*x + c))*sin(d*x + c)/d

Sympy [A] time = 6.59781, size = 258, normalized size = 2.9

$$\left\{ \begin{array}{l} \frac{3Ax \sin^4(c+dx)}{8} + \frac{3Ax \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3Ax \cos^4(c+dx)}{8} + \frac{3A \sin^3(c+dx) \cos(c+dx)}{8d} + \frac{5A \sin(c+dx) \cos^3(c+dx)}{8d} + \frac{5Cx \sin^6(c+dx)}{16} + \\ x(A + C \cos^2(c)) \cos^4(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2),x)

[Out] Piecewise((3*A*x*sin(c + d*x)**4/8 + 3*A*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*A*x*cos(c + d*x)**4/8 + 3*A*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*A*sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*C*x*sin(c + d*x)**6/16 + 15*C*x*sin(c + d*x)**4*cos(c + d*x)**2/16 + 15*C*x*sin(c + d*x)**2*cos(c + d*x)**4/16 + 5*C*x*cos(c + d*x)**6/16 + 5*C*sin(c + d*x)**5*cos(c + d*x)/(16*d) + 5*C*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) + 11*C*sin(c + d*x)*cos(c + d*x)**5/(16*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**4, True))

Giac [A] time = 1.16127, size = 92, normalized size = 1.03

$$\frac{1}{16}(6A + 5C)x + \frac{C \sin(6dx + 6c)}{192d} + \frac{(2A + 3C) \sin(4dx + 4c)}{64d} + \frac{(16A + 15C) \sin(2dx + 2c)}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] 1/16*(6*A + 5*C)*x + 1/192*C*sin(6*d*x + 6*c)/d + 1/64*(2*A + 3*C)*sin(4*d*x + 4*c)/d + 1/64*(16*A + 15*C)*sin(2*d*x + 2*c)/d

3.11 $\int \cos^2(c + dx) (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=61

$$\frac{(4A + 3C) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}x(4A + 3C) + \frac{C \sin(c + dx) \cos^3(c + dx)}{4d}$$

[Out] $((4*A + 3*C)*x)/8 + ((4*A + 3*C)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (C*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d)$

Rubi [A] time = 0.0412907, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3014, 2635, 8}

$$\frac{(4A + 3C) \sin(c + dx) \cos(c + dx)}{8d} + \frac{1}{8}x(4A + 3C) + \frac{C \sin(c + dx) \cos^3(c + dx)}{4d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^2*(A + C*\text{Cos}[c + d*x]^2), x]$

[Out] $((4*A + 3*C)*x)/8 + ((4*A + 3*C)*\text{Cos}[c + d*x]*\text{Sin}[c + d*x])/(8*d) + (C*\text{Cos}[c + d*x]^3*\text{Sin}[c + d*x])/(4*d)$

Rule 3014

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)})^{(m_*)}, x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(m+2), \text{Int}[(b*\text{Sin}[e + f*x])^{(m)}, x], x] /; \text{FreeQ}\{b, e, f, A, C, m\}, x] \&\& !\text{LtQ}[m, -1]$

Rule 2635

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{GtQ}[n, 1] \&\& \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx) (A + C \cos^2(c + dx)) dx &= \frac{C \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{4}(4A + 3C) \int \cos^2(c + dx) dx \\ &= \frac{(4A + 3C) \cos(c + dx) \sin(c + dx)}{8d} + \frac{C \cos^3(c + dx) \sin(c + dx)}{4d} + \frac{1}{8}(4A + 3C)x \\ &= \frac{1}{8}(4A + 3C)x + \frac{(4A + 3C) \cos(c + dx) \sin(c + dx)}{8d} + \frac{C \cos^3(c + dx) \sin(c + dx)}{4d} \end{aligned}$$

Mathematica [A] time = 0.0855863, size = 45, normalized size = 0.74

$$\frac{4(4A + 3C)(c + dx) + 8(A + C) \sin(2(c + dx)) + C \sin(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2), x]

[Out] $(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*\text{Sin}[2*(c + d*x)] + C*\text{Sin}[4*(c + d*x)]) / (32*d)$

Maple [A] time = 0.037, size = 65, normalized size = 1.1

$$\frac{1}{d} \left(C \left(\frac{\sin(dx+c)}{4} \left((\cos(dx+c))^3 + \frac{3 \cos(dx+c)}{2} \right) + \frac{3dx}{8} + \frac{3c}{8} \right) + A \left(\frac{\cos(dx+c) \sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2), x)

[Out] $1/d*(C*(1/4*(\cos(d*x+c))^3+3/2*\cos(d*x+c))*\sin(d*x+c)+3/8*d*x+3/8*c)+A*(1/2*\cos(d*x+c)*\sin(d*x+c)+1/2*d*x+1/2*c)$

Maxima [A] time = 1.68262, size = 99, normalized size = 1.62

$$\frac{(dx+c)(4A+3C) + \frac{(4A+3C)\tan(dx+c)^3 + (4A+5C)\tan(dx+c)}{\tan(dx+c)^4 + 2\tan(dx+c)^2 + 1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] $1/8*((d*x + c)*(4*A + 3*C) + ((4*A + 3*C)*\tan(d*x + c)^3 + (4*A + 5*C)*\tan(d*x + c)) / (\tan(d*x + c)^4 + 2*\tan(d*x + c)^2 + 1)) / d$

Fricas [A] time = 1.608, size = 119, normalized size = 1.95

$$\frac{(4A+3C)dx + (2C \cos(dx+c)^3 + (4A+3C) \cos(dx+c)) \sin(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] $1/8*((4*A + 3*C)*d*x + (2*C*\cos(d*x + c)^3 + (4*A + 3*C)*\cos(d*x + c))*\sin(d*x + c)) / d$

Sympy [A] time = 1.72031, size = 158, normalized size = 2.59

$$\left\{ \begin{array}{l} \frac{Ax \sin^2(c+dx)}{2} + \frac{Ax \cos^2(c+dx)}{2} + \frac{A \sin(c+dx) \cos(c+dx)}{2d} + \frac{3Cx \sin^4(c+dx)}{8} + \frac{3Cx \sin^2(c+dx) \cos^2(c+dx)}{4} + \frac{3Cx \cos^4(c+dx)}{8} + \frac{3C \sin^3(c+dx)}{8d} \\ x(A + C \cos^2(c)) \cos^2(c) \end{array} \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2),x)

[Out] Piecewise((A*x*sin(c + d*x)**2/2 + A*x*cos(c + d*x)**2/2 + A*sin(c + d*x)*cos(c + d*x)/(2*d) + 3*C*x*sin(c + d*x)**4/8 + 3*C*x*sin(c + d*x)**2*cos(c + d*x)**2/4 + 3*C*x*cos(c + d*x)**4/8 + 3*C*sin(c + d*x)**3*cos(c + d*x)/(8*d) + 5*C*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(A + C*cos(c)**2)*cos(c)**2, True))

Giac [A] time = 1.12214, size = 58, normalized size = 0.95

$$\frac{1}{8}(4A + 3C)x + \frac{C \sin(4dx + 4c)}{32d} + \frac{(A + C) \sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] 1/8*(4*A + 3*C)*x + 1/32*C*sin(4*d*x + 4*c)/d + 1/4*(A + C)*sin(2*d*x + 2*c)/d

3.12 $\int (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$

Optimal. Leaf size=15

$$\frac{A \tan(c + dx)}{d} + Cx$$

[Out] C*x + (A*Tan[c + d*x])/d

Rubi [A] time = 0.0244259, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3012, 8}

$$\frac{A \tan(c + dx)}{d} + Cx$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] C*x + (A*Tan[c + d*x])/d

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^2(c + dx) dx &= \frac{A \tan(c + dx)}{d} + C \int 1 dx \\ &= Cx + \frac{A \tan(c + dx)}{d} \end{aligned}$$

Mathematica [A] time = 0.0086647, size = 15, normalized size = 1.

$$\frac{A \tan(c + dx)}{d} + Cx$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] C*x + (A*Tan[c + d*x])/d

Maple [A] time = 0.064, size = 21, normalized size = 1.4

$$\frac{A \tan(dx + c) + C(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] 1/d*(A*tan(d*x+c)+C*(d*x+c))

Maxima [A] time = 1.49507, size = 27, normalized size = 1.8

$$\frac{(dx + c)C + A \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] ((d*x + c)*C + A*tan(d*x + c))/d

Fricas [B] time = 1.59324, size = 76, normalized size = 5.07

$$\frac{Cdx \cos(dx + c) + A \sin(dx + c)}{d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] (C*d*x*cos(d*x + c) + A*sin(d*x + c))/(d*cos(d*x + c))

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)

[Out] Integral((A + C*cos(c + d*x)**2)*sec(c + d*x)**2, x)

Giac [A] time = 1.16327, size = 27, normalized size = 1.8

$$\frac{(dx + c)C + A \tan(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*C + A*tan(d*x + c))/d
```

3.13 $\int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$

Optimal. Leaf size=43

$$\frac{(2A + 3C) \tan(c + dx)}{3d} + \frac{A \tan(c + dx) \sec^2(c + dx)}{3d}$$

[Out] $((2*A + 3*C)*\text{Tan}[c + d*x])/(3*d) + (A*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/(3*d)$

Rubi [A] time = 0.0375058, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3012, 3767, 8}

$$\frac{(2A + 3C) \tan(c + dx)}{3d} + \frac{A \tan(c + dx) \sec^2(c + dx)}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^4, x]$

[Out] $((2*A + 3*C)*\text{Tan}[c + d*x])/(3*d) + (A*\text{Sec}[c + d*x]^2*\text{Tan}[c + d*x])/(3*d)$

Rule 3012

$\text{Int}[(b_* \sin[e_*] + (f_*)*(x_*))^{(m_*)}*((A_*) + (C_*)*\sin[e_*] + (f_*)*(x_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 1)), x] + \text{Dist}[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 2)}, x], x] /;$ $\text{FreeQ}\{b, e, f, A, C\}, x \ \&\& \ \text{LtQ}[m, -1]$

Rule 3767

$\text{Int}[\text{csc}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ $\text{FreeQ}\{c, d\}, x \ \&\& \ \text{IGtQ}[n/2, 0]$

Rule 8

$\text{Int}[a_*, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ $\text{FreeQ}[a, x]$

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx &= \frac{A \sec^2(c + dx) \tan(c + dx)}{3d} + \frac{1}{3}(2A + 3C) \int \sec^2(c + dx) dx \\ &= \frac{A \sec^2(c + dx) \tan(c + dx)}{3d} - \frac{(2A + 3C) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{3d} \\ &= \frac{(2A + 3C) \tan(c + dx)}{3d} + \frac{A \sec^2(c + dx) \tan(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.0872789, size = 36, normalized size = 0.84

$$\frac{A \left(\frac{1}{3} \tan^3(c + dx) + \tan(c + dx) \right)}{d} + \frac{C \tan(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (C*Tan[c + d*x])/d + (A*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d

Maple [A] time = 0.067, size = 35, normalized size = 0.8

$$\frac{1}{d} \left(-A \left(-\frac{2}{3} - \frac{(\sec(dx+c))^2}{3} \right) \tan(dx+c) + C \tan(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] 1/d*(-A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+C*tan(d*x+c))

Maxima [A] time = 1.09077, size = 36, normalized size = 0.84

$$\frac{A \tan(dx+c)^3 + 3(A+C) \tan(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] 1/3*(A*tan(d*x + c)^3 + 3*(A + C)*tan(d*x + c))/d

Fricas [A] time = 1.545, size = 95, normalized size = 2.21

$$\frac{((2A + 3C) \cos(dx+c)^2 + A) \sin(dx+c)}{3d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sin(d*x + c)/(d*cos(d*x + c)^3)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)

[Out] Timed out

Giac [A] time = 1.15448, size = 46, normalized size = 1.07

$$\frac{A \tan(dx + c)^3 + 3 A \tan(dx + c) + 3 C \tan(dx + c)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] 1/3*(A*tan(d*x + c)^3 + 3*A*tan(d*x + c) + 3*C*tan(d*x + c))/d

3.14 $\int (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$

Optimal. Leaf size=65

$$\frac{(4A + 5C) \tan^3(c + dx)}{15d} + \frac{(4A + 5C) \tan(c + dx)}{5d} + \frac{A \tan(c + dx) \sec^4(c + dx)}{5d}$$

[Out] $((4*A + 5*C)*Tan[c + d*x])/(5*d) + (A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*A + 5*C)*Tan[c + d*x]^3)/(15*d)$

Rubi [A] time = 0.0435946, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3012, 3767}

$$\frac{(4A + 5C) \tan^3(c + dx)}{15d} + \frac{(4A + 5C) \tan(c + dx)}{5d} + \frac{A \tan(c + dx) \sec^4(c + dx)}{5d}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] $((4*A + 5*C)*Tan[c + d*x])/(5*d) + (A*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + ((4*A + 5*C)*Tan[c + d*x]^3)/(15*d)$

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^6(c + dx) dx &= \frac{A \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{1}{5}(4A + 5C) \int \sec^4(c + dx) dx \\ &= \frac{A \sec^4(c + dx) \tan(c + dx)}{5d} - \frac{(4A + 5C) \text{Subst}\left(\int (1 + x^2) dx, x, -\tan(c + dx)\right)}{5d} \\ &= \frac{(4A + 5C) \tan(c + dx)}{5d} + \frac{A \sec^4(c + dx) \tan(c + dx)}{5d} + \frac{(4A + 5C) \tan^3(c + dx)}{15d} \end{aligned}$$

Mathematica [A] time = 0.200543, size = 61, normalized size = 0.94

$$\frac{A \left(\frac{1}{5} \tan^5(c + dx) + \frac{2}{3} \tan^3(c + dx) + \tan(c + dx) \right)}{d} + \frac{C \left(\frac{1}{3} \tan^3(c + dx) + \tan(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] (C*(Tan[c + d*x] + Tan[c + d*x]^3/3))/d + (A*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d

Maple [A] time = 0.07, size = 58, normalized size = 0.9

$$\frac{1}{d} \left(-A \left(-\frac{8}{15} - \frac{(\sec(dx+c))^4}{5} - \frac{4(\sec(dx+c))^2}{15} \right) \tan(dx+c) - C \left(-\frac{2}{3} - \frac{(\sec(dx+c))^2}{3} \right) \tan(dx+c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)

[Out] 1/d*(-A*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-C*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))

Maxima [A] time = 1.01904, size = 58, normalized size = 0.89

$$\frac{3 A \tan(dx+c)^5 + 5(2 A + C) \tan(dx+c)^3 + 15(A+C) \tan(dx+c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")

[Out] 1/15*(3*A*tan(d*x + c)^5 + 5*(2*A + C)*tan(d*x + c)^3 + 15*(A + C)*tan(d*x + c))/d

Fricas [A] time = 1.68914, size = 140, normalized size = 2.15

$$\frac{(2(4A+5C)\cos(dx+c)^4 + (4A+5C)\cos(dx+c)^2 + 3A)\sin(dx+c)}{15d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")

[Out] 1/15*(2*(4*A + 5*C)*cos(d*x + c)^4 + (4*A + 5*C)*cos(d*x + c)^2 + 3*A)*sin(d*x + c)/(d*cos(d*x + c)^5)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)

[Out] Timed out

Giac [A] time = 1.15568, size = 77, normalized size = 1.18

$$\frac{3 A \tan (d x + c)^5 + 10 A \tan (d x + c)^3 + 5 C \tan (d x + c)^3 + 15 A \tan (d x + c) + 15 C \tan (d x + c)}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")

[Out] 1/15*(3*A*tan(d*x + c)^5 + 10*A*tan(d*x + c)^3 + 5*C*tan(d*x + c)^3 + 15*A*tan(d*x + c) + 15*C*tan(d*x + c))/d

3.15 $\int (A + C \cos^2(c + dx)) \sec^8(c + dx) dx$

Optimal. Leaf size=87

$$\frac{(6A + 7C) \tan^5(c + dx)}{35d} + \frac{2(6A + 7C) \tan^3(c + dx)}{21d} + \frac{(6A + 7C) \tan(c + dx)}{7d} + \frac{A \tan(c + dx) \sec^6(c + dx)}{7d}$$

[Out] $((6*A + 7*C)*\text{Tan}[c + d*x])/(7*d) + (A*\text{Sec}[c + d*x]^6*\text{Tan}[c + d*x])/(7*d) + (2*(6*A + 7*C)*\text{Tan}[c + d*x]^3)/(21*d) + ((6*A + 7*C)*\text{Tan}[c + d*x]^5)/(35*d)$

Rubi [A] time = 0.0496711, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3012, 3767}

$$\frac{(6A + 7C) \tan^5(c + dx)}{35d} + \frac{2(6A + 7C) \tan^3(c + dx)}{21d} + \frac{(6A + 7C) \tan(c + dx)}{7d} + \frac{A \tan(c + dx) \sec^6(c + dx)}{7d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^8, x]$

[Out] $((6*A + 7*C)*\text{Tan}[c + d*x])/(7*d) + (A*\text{Sec}[c + d*x]^6*\text{Tan}[c + d*x])/(7*d) + (2*(6*A + 7*C)*\text{Tan}[c + d*x]^3)/(21*d) + ((6*A + 7*C)*\text{Tan}[c + d*x]^5)/(35*d)$

Rule 3012

$\text{Int}[(b_* \sin[(e_*) + (f_*)*(x_*)])^{(m_*)} * ((A_*) + (C_*) * \sin[(e_*) + (f_*)*(x_*)])^{(n_*)}], x_Symbol] :> \text{Simp}[(A*\text{Cos}[e + f*x] * (b*\text{Sin}[e + f*x])^{(m + 1)}) / (b*f*(m + 1)), x] + \text{Dist}[(A*(m + 2) + C*(m + 1)) / (b^2*(m + 1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 2)}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

$\text{Int}[\text{csc}[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] :> -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{ExpandIntegrand}[(1 + x^2)^{(n/2 - 1)}, x], x], x, \text{Cot}[c + d*x]], x] /;$ FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) \sec^8(c + dx) dx &= \frac{A \sec^6(c + dx) \tan(c + dx)}{7d} + \frac{1}{7}(6A + 7C) \int \sec^6(c + dx) dx \\ &= \frac{A \sec^6(c + dx) \tan(c + dx)}{7d} - \frac{(6A + 7C) \text{Subst}\left(\int (1 + 2x^2 + x^4) dx, x, -\tan(c + dx)\right)}{7d} \\ &= \frac{(6A + 7C) \tan(c + dx)}{7d} + \frac{A \sec^6(c + dx) \tan(c + dx)}{7d} + \frac{2(6A + 7C) \tan^3(c + dx)}{21d} \end{aligned}$$

Mathematica [A] time = 0.30241, size = 81, normalized size = 0.93

$$\frac{A \left(\frac{1}{7} \tan^7(c + dx) + \frac{3}{5} \tan^5(c + dx) + \tan^3(c + dx) + \tan(c + dx) \right)}{d} + \frac{C \left(\frac{1}{5} \tan^5(c + dx) + \frac{2}{3} \tan^3(c + dx) + \tan(c + dx) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^8,x]

[Out] (C*(Tan[c + d*x] + (2*Tan[c + d*x]^3)/3 + Tan[c + d*x]^5/5))/d + (A*(Tan[c + d*x] + Tan[c + d*x]^3 + (3*Tan[c + d*x]^5)/5 + Tan[c + d*x]^7/7))/d

Maple [A] time = 0.069, size = 78, normalized size = 0.9

$$\frac{1}{d} \left(-A \left(-\frac{16}{35} - \frac{(\sec(dx+c))^6}{7} - \frac{6(\sec(dx+c))^4}{35} - \frac{8(\sec(dx+c))^2}{35} \right) \tan(dx+c) - C \left(-\frac{8}{15} - \frac{(\sec(dx+c))^4}{5} - \frac{4(\sec(dx+c))^2}{35} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x)

[Out] 1/d*(-A*(-16/35-1/7*sec(d*x+c)^6-6/35*sec(d*x+c)^4-8/35*sec(d*x+c)^2)*tan(d*x+c)-C*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c))

Maxima [A] time = 1.01291, size = 81, normalized size = 0.93

$$\frac{15 A \tan(dx+c)^7 + 21(3A+C)\tan(dx+c)^5 + 35(3A+2C)\tan(dx+c)^3 + 105(A+C)\tan(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="maxima")

[Out] 1/105*(15*A*tan(d*x + c)^7 + 21*(3*A + C)*tan(d*x + c)^5 + 35*(3*A + 2*C)*tan(d*x + c)^3 + 105*(A + C)*tan(d*x + c))/d

Fricas [A] time = 1.60903, size = 188, normalized size = 2.16

$$\frac{(8(6A+7C)\cos(dx+c)^6 + 4(6A+7C)\cos(dx+c)^4 + 3(6A+7C)\cos(dx+c)^2 + 15A)\sin(dx+c)}{105d\cos(dx+c)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="fricas")

[Out] 1/105*(8*(6*A + 7*C)*cos(d*x + c)^6 + 4*(6*A + 7*C)*cos(d*x + c)^4 + 3*(6*A + 7*C)*cos(d*x + c)^2 + 15*A)*sin(d*x + c)/(d*cos(d*x + c)^7)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**8,x)

[Out] Timed out

Giac [A] time = 1.19337, size = 107, normalized size = 1.23

$$\frac{15 A \tan(dx + c)^7 + 63 A \tan(dx + c)^5 + 21 C \tan(dx + c)^5 + 105 A \tan(dx + c)^3 + 70 C \tan(dx + c)^3 + 105 A \tan(dx + c)}{105 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^8,x, algorithm="giac")

[Out] 1/105*(15*A*tan(d*x + c)^7 + 63*A*tan(d*x + c)^5 + 21*C*tan(d*x + c)^5 + 105*A*tan(d*x + c)^3 + 70*C*tan(d*x + c)^3 + 105*A*tan(d*x + c) + 105*C*tan(d*x + c))/d

3.16 $\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=113

$$\frac{2b^2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C)\sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{5/2}}{9bd}$$

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.0822025, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3014, 2635, 2640, 2639}

$$\frac{2b^2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C)\sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{5/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2), x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 3014

```
Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} + \frac{1}{9}(9A + 7C) \int (b \cos(c + dx))^{5/2} dx \\
&= \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} + \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&= \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} + \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&= \frac{2b^2(9A + 7C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d}
\end{aligned}$$

Mathematica [A] time = 0.404401, size = 88, normalized size = 0.78

$$\frac{(b \cos(c + dx))^{5/2} \left(24(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin(2(c + dx))\sqrt{\cos(c + dx)}(18A + 5C \cos(2(c + dx)) + 19C) \right)}{180d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2), x]

[Out] ((b*cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*C + 5*C*cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*d*cos[c + d*x]^(5/2))

Maple [B] time = 3.893, size = 324, normalized size = 2.9

$$-\frac{2b^3}{45d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(-160C \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^{10} + 320C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^8 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2), x)

[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c))+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)

3.17 $\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=113

$$\frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b(7A + 5C)\sin(c + dx)\sqrt{b \cos(c + dx)}}{21d} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{5/2}}{7bd}$$

[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)

Rubi [A] time = 0.0832329, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3014, 2635, 2642, 2641}

$$\frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b(7A + 5C)\sin(c + dx)\sqrt{b \cos(c + dx)}}{21d} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{5/2}}{7bd}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)

Rule 3014

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} + \frac{1}{7}(7A + 5C) \int (b \cos(c + dx))^{3/2} dx \\
&= \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&= \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&= \frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)}}{21d}
\end{aligned}$$

Mathematica [A] time = 0.366993, size = 86, normalized size = 0.76

$$\frac{(b \cos(c + dx))^{3/2} \left(4(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin(c + dx) \sqrt{\cos(c + dx)} (14A + 3C \cos(2(c + dx))) + 13C \right)}{42d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Cos[c + d*x]^(3/2))

Maple [B] time = 3.369, size = 296, normalized size = 2.6

$$-\frac{2b^2}{21d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(48C (\sin(1/2 dx + c/2))^8 \cos(1/2 dx + c/2) - 72C (\sin(1/2 dx + c/2))^6 \cos(1/2 dx + c/2) + (28A + 56C) \sin(1/2 dx + c/2)^4 \cos(1/2 dx + c/2) + (-14A - 16C) \sin(1/2 dx + c/2)^2 \cos(1/2 dx + c/2) + 7A \left(\sin(1/2 dx + c/2)^2 \right)^{1/2} + 5C \left(\sin(1/2 dx + c/2)^2 \right)^{1/2} \right) / (-b \left(2 \sin(1/2 dx + c/2)^2 - 1 \right)^{1/2} \sin(1/2 dx + c/2) / (b \left(2 \cos(1/2 dx + c/2)^2 - 1 \right)^{1/2} / d)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2), x)

[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] Timed out

3.18 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=77

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5bd}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rubi [A] time = 0.060222, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3014, 2640, 2639}

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{1}{5}(5A + 3C) \int \sqrt{b \cos(c + dx)} dx \\ &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{\left((5A + 3C)\sqrt{b \cos(c + dx)}\right) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\ &= \frac{2(5A + 3C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \end{aligned}$$

Mathematica [A] time = 0.130978, size = 70, normalized size = 0.91

$$\frac{\sqrt{b \cos(c + dx)} \left(2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx))\sqrt{\cos(c + dx)} \right)}{5d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])

Maple [B] time = 3.912, size = 261, normalized size = 3.4

$$\frac{2b}{5d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(8C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^6 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 8C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^4 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c))+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)

$$3.19 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=75

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd}$$

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rubi [A] time = 0.0572765, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3014, 2642, 2641}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rule 3014

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx &= \frac{2C\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd} + \frac{1}{3}(3A+C) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx \\ &= \frac{2C\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd} + \frac{((3A+C)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c+dx)}} \\ &= \frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2C\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd} \end{aligned}$$

Mathematica [A] time = 0.148953, size = 58, normalized size = 0.77

$$\frac{2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right) + C\sin(2(c + dx))}{3d\sqrt{b\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.829, size = 236, normalized size = 3.2

$$-\frac{2}{3d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(4C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 3A\sqrt{\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] `integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="giac")`

[Out] `integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)`

$$3.20 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=74

$$\frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}}$$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0619689, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3012, 2640, 2639}

$$\frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)/(b*\text{Cos}[c + d*x])^{3/2}, x]$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)})^2, x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /;$ $\text{FreeQ}\{b, e, f, A, C\}, x] \ \&\& \ \text{LtQ}[m, -1]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ $\text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx &= \frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{(A-C) \int \sqrt{b \cos(c+dx)} dx}{b^2} \\ &= \frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{((A-C)\sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{b^2 \sqrt{\cos(c+dx)}} \\ &= -\frac{2(A-C)\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.14842, size = 57, normalized size = 0.77

$$\frac{2A \sin(c + dx) - 2(A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2), x]

[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 4.411, size = 216, normalized size = 2.9

$$-2 \frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE}(\cos(1/2 dx + c/2), 2) \right)}{b \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] -2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="fricas")

[Out] `integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="giac")`

[Out] `integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)`

$$3.21 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=78

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}}$$

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0638188, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3012, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx &= \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{(A+3C) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{3b^2} \\ &= \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{((A+3C)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^2\sqrt{b \cos(c+dx)}} \\ &= \frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.211813, size = 58, normalized size = 0.74

$$\frac{2 \left((A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \right)}{3b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.996, size = 294, normalized size = 3.8

$$-\frac{2}{3b^2d} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^3),
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)
```


3.22 $\int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{7/2}} dx$

Optimal. Leaf size=115

$$\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

[Out] $(-2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*b^4*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(5*b*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*b^3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0866652, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3012, 2636, 2640, 2639}

$$\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)/(b*\text{Cos}[c + d*x])^(7/2), x]$

[Out] $(-2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*b^4*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(5*b*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*b^3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^(m_*)*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^(2)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + \text{Dist}[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), \text{Int}[(b*\text{Sin}[e + f*x])^(m + 2), x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^(n_), x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^(n + 2), x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx &= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{(3A + 5C) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx}{5b^2} \\
&= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}} - \frac{(3A + 5C) \int \sqrt{b \cos(c + dx)} dx}{5b^4} \\
&= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}} - \frac{((3A + 5C) \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5b^4 \sqrt{\cos(c + dx)}} \\
&= -\frac{2(3A + 5C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^4 d \sqrt{\cos(c + dx)}} + \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.2641, size = 81, normalized size = 0.7

$$\frac{2 \left((3A + 5C) \sin(c + dx) - (3A + 5C) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \sec(c + dx) \right)}{5b^3 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2), x]

[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 9.671, size = 601, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-10*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^4*cos(d*x + c)^4), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)

3.23 $\int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{9/2}} dx$

Optimal. Leaf size=115

$$\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0896272, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3012, 2636, 2642, 2641}

$$\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2), x]

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx &= \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{(5A + 7C) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx}{7b^2} \\
&= \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21b^3d(b \cos(c + dx))^{3/2}} + \frac{(5A + 7C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{21b^4} \\
&= \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21b^3d(b \cos(c + dx))^{3/2}} + \frac{((5A + 7C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}}}{21b^4\sqrt{b \cos(c + dx)}} \\
&= \frac{2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21b^4d\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21b^3d(b \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.375375, size = 77, normalized size = 0.67

$$\frac{2 \left((5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + \tan(c + dx)(3A \sec^2(c + dx) + 5A + 7C) \right)}{21b^4d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2), x]

[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C + 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 7.382, size = 413, normalized size = 3.6

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)}(\sin(1/2 dx + c/2))^2}{b^4 \sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} d \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2), x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{9/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^5 \cos(dx + c)^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^5*cos(d*x + c)^5), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)

$$3.24 \quad \int \sqrt{\cos(c + dx)} (3 - 5 \cos^2(c + dx)) dx$$

Optimal. Leaf size=21

$$-\frac{2 \sin(c + dx) \cos^3(c + dx)}{d}$$

[Out] $(-2*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/d$

Rubi [A] time = 0.0228646, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3011}

$$-\frac{2 \sin(c + dx) \cos^3(c + dx)}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c + d*x]]*(3 - 5*\text{Cos}[c + d*x]^2), x]$

[Out] $(-2*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/d$

Rule 3011

$\text{Int}[(b_*\sin(e_*) + (f_*)*(x_*)])^{(m_*)}*((A_*) + (C_*)*\sin(e_*) + (f_*)*(x_*)^2), x_Symbol] :> \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 1)), x] /; \text{FreeQ}\{b, e, f, A, C, m\}, x] \ \&\& \ \text{EqQ}[A*(m + 2) + C*(m + 1), 0]$

Rubi steps

$$\int \sqrt{\cos(c + dx)} (3 - 5 \cos^2(c + dx)) dx = -\frac{2 \cos^3(c + dx) \sin(c + dx)}{d}$$

Mathematica [A] time = 0.0559298, size = 23, normalized size = 1.1

$$-\frac{\sin(2(c + dx))\sqrt{\cos(c + dx)}}{d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[\text{Sqrt}[\text{Cos}[c + d*x]]*(3 - 5*\text{Cos}[c + d*x]^2), x]$

[Out] $-((\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[2*(c + d*x)])/d)$

Maple [B] time = 1.698, size = 99, normalized size = 4.7

$$-4 \frac{\sqrt{(2 (\cos(1/2 dx + c/2))^2 - 1) (\sin(1/2 dx + c/2))^2 \cos(1/2 dx + c/2) \sqrt{-2 (\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2}}{\sin(1/2 dx + c/2) d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)`

[Out] $-4*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/\sin(1/2*d*x+1/2*c)/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (5 \cos(dx + c)^2 - 3) \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)`

Fricas [A] time = 1.58204, size = 51, normalized size = 2.43

$$-\frac{2 \cos(dx + c)^{\frac{3}{2}} \sin(dx + c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `-2*cos(d*x + c)^(3/2)*sin(d*x + c)/d`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3-5*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -(5 \cos(dx + c)^2 - 3) \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((3-5*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate(-(5*cos(d*x + c)^2 - 3)*sqrt(cos(d*x + c)), x)`

$$3.25 \quad \int \frac{1-3 \cos^2(c+dx)}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=21

$$-\frac{2 \sin(c+dx) \sqrt{\cos(c+dx)}}{d}$$

[Out] $(-2*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/d$

Rubi [A] time = 0.0227313, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {3011}

$$-\frac{2 \sin(c+dx) \sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(1 - 3*\text{Cos}[c + d*x]^2)/\text{Sqrt}[\text{Cos}[c + d*x]], x]$

[Out] $(-2*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/d$

Rule 3011

$\text{Int}[(b_*)\sin[(e_*) + (f_*)(x_*)]^{(m_*)}((A_*) + (C_*)\sin[(e_*) + (f_*)(x_*)]^2), x_Symbol] :> \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 1)), x] /; \text{FreeQ}\{b, e, f, A, C, m\}, x] \&\& \text{EqQ}[A*(m + 2) + C*(m + 1), 0]$

Rubi steps

$$\int \frac{1-3 \cos^2(c+dx)}{\sqrt{\cos(c+dx)}} dx = -\frac{2\sqrt{\cos(c+dx)} \sin(c+dx)}{d}$$

Mathematica [A] time = 0.0567009, size = 21, normalized size = 1.

$$-\frac{2 \sin(c+dx) \sqrt{\cos(c+dx)}}{d}$$

Antiderivative was successfully verified.

[In] $\text{Integrate}[(1 - 3*\text{Cos}[c + d*x]^2)/\text{Sqrt}[\text{Cos}[c + d*x]], x]$

[Out] $(-2*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sin}[c + d*x])/d$

Maple [B] time = 2.026, size = 99, normalized size = 4.7

$$-4 \frac{\sqrt{(2 (\cos(1/2 dx + c/2))^2 - 1) (\sin(1/2 dx + c/2))^2 \cos(1/2 dx + c/2) \sqrt{-2 (\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2}}}{\sin(1/2 dx + c/2) \sqrt{2 (\cos(1/2 dx + c/2))^2 - 1} d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)`

[Out] $-4*((2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\cos(1/2*d*x+1/2*c)*(-2*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(2*\cos(1/2*d*x+1/2*c)^2-1)^{(1/2)}/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{3 \cos(dx+c)^2 - 1}{\sqrt{\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")`

[Out] `-integrate((3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)`

Fricas [A] time = 1.62265, size = 51, normalized size = 2.43

$$-\frac{2 \sqrt{\cos(dx+c)} \sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")`

[Out] `-2*sqrt(cos(d*x + c))*sin(d*x + c)/d`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-3*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{3 \cos(dx+c)^2 - 1}{\sqrt{\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((1-3*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")`

[Out] `integrate(-(3*cos(d*x + c)^2 - 1)/sqrt(cos(d*x + c)), x)`

3.26 $\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{9/2} dx$

Optimal. Leaf size=115

$$\frac{2b^3(5A + 7C) \sin(c + dx)(b \sec(c + dx))^{3/2}}{21d} + \frac{2b^4(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \sec(c + dx)}}{21d} + \frac{2Ab^2 \tan(c + dx)}{7d}$$

```
[Out] (2*b^4*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]/(21*d) + (2*b^3*(5*A + 7*C)*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*A*b^2*(b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rubi [A] time = 0.124562, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3238, 4046, 3768, 3771, 2641}

$$\frac{2b^3(5A + 7C) \sin(c + dx)(b \sec(c + dx))^{3/2}}{21d} + \frac{2b^4(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \sec(c + dx)}}{21d} + \frac{2Ab^2 \tan(c + dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2), x]
```

```
[Out] (2*b^4*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]/(21*d) + (2*b^3*(5*A + 7*C)*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(21*d) + (2*A*b^2*(b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 3238

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]
```

Rule 4046

```
Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3771

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{9/2} dx &= b^2 \int (b \sec(c + dx))^{5/2} (C + A \sec^2(c + dx)) dx \\ &= \frac{2Ab^2(b \sec(c + dx))^{5/2} \tan(c + dx)}{7d} + \frac{1}{7} (b^2(5A + 7C)) \int (b \sec(c + dx))^{5/2} dx \\ &= \frac{2b^3(5A + 7C)(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d} + \frac{2Ab^2(b \sec(c + dx))^{5/2} \tan(c + dx)}{7d} \\ &= \frac{2b^3(5A + 7C)(b \sec(c + dx))^{3/2} \sin(c + dx)}{21d} + \frac{2Ab^2(b \sec(c + dx))^{5/2} \tan(c + dx)}{7d} \\ &= \frac{2b^4(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{21d} + \frac{2b^3(5A + 7C)\tan(c + dx)}{7d} \end{aligned}$$

Mathematica [A] time = 0.80782, size = 78, normalized size = 0.68

$$\frac{b^2(b \sec(c + dx))^{5/2} \left((5A + 7C) \sin(2(c + dx)) + 2(5A + 7C) \cos^2(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6A \tan(c + dx) \right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(9/2), x]
```

```
[Out] (b^2*(b*Sec[c + d*x])^(5/2)*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)
```

Maple [C] time = 0.6, size = 249, normalized size = 2.2

$$\frac{(-2 + 2 \cos(dx + c)) \cos(dx + c) (1 + \cos(dx + c))^2}{21 d (\sin(dx + c))^3} \left(5 i A (\cos(dx + c))^3 \sin(dx + c) \sqrt{(1 + \cos(dx + c))^{-1}} \sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2), x)
```

```
[Out] -2/21/d*(-1+cos(d*x+c))*(5*I*A*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)+7*I*C*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-5*A*cos(d*x+c)^3-7*C*cos(d*x+c)^3+5*A*cos(d*x+c)^2+7*C*cos(d*x+c)^2-3*A*cos(d*x+c)+3*A*cos(d*x+c)*(1+cos(d*x+c))^2*(b/cos(d*x+c))^(9/2)/sin(d*x+c)^3)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \sec(dx + c))^{9/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^4 \cos(dx + c)^2 + Ab^4\right)\sqrt{b \sec(dx + c)} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="fricas")

[Out] integral((C*b^4*cos(d*x + c)^2 + A*b^4)*sqrt(b*sec(d*x + c))*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \sec(dx + c))^{\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(9/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(9/2), x)

3.27 $\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{7/2} dx$

Optimal. Leaf size=115

$$\frac{2b^3(3A + 5C) \sin(c + dx) \sqrt{b \sec(c + dx)}}{5d} - \frac{2b^4(3A + 5C) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2Ab^2 \tan(c + dx) (b \sec(c + dx))^{3/2}}{5d}$$

[Out] $(-2*b^4*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d*\sqrt{\cos[c + d*x]}*\sqrt{b*\sec[c + d*x]}) + (2*b^3*(3*A + 5*C)*\sqrt{b*\sec[c + d*x]}*\sin[c + d*x])/(5*d) + (2*A*b^2*(b*\sec[c + d*x])^{3/2}*\tan[c + d*x])/(5*d)$

Rubi [A] time = 0.13362, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3238, 4046, 3768, 3771, 2639}

$$\frac{2b^3(3A + 5C) \sin(c + dx) \sqrt{b \sec(c + dx)}}{5d} - \frac{2b^4(3A + 5C) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2Ab^2 \tan(c + dx) (b \sec(c + dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\cos[c + d*x]^2)*(b*\sec[c + d*x])^{7/2}, x]$

[Out] $(-2*b^4*(3*A + 5*C)*EllipticE[(c + d*x)/2, 2])/(5*d*\sqrt{\cos[c + d*x]}*\sqrt{b*\sec[c + d*x]}) + (2*b^3*(3*A + 5*C)*\sqrt{b*\sec[c + d*x]}*\sin[c + d*x])/(5*d) + (2*A*b^2*(b*\sec[c + d*x])^{3/2}*\tan[c + d*x])/(5*d)$

Rule 3238

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(d_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)])^{(n_.)}]^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\csc[e + f*x])^{(m - n*p)}*(b + a*\csc[e + f*x]^n)^p, x], x] /;$ FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 4046

$\text{Int}[(\csc[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\csc[(e_.) + (f_.)*(x_.)]^{2*(C_.)} + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(C*\cot[e + f*x]*(b*\csc[e + f*x])^m)/(f*(m + 1)), x] + \text{Dist}[(C*m + A*(m + 1))/(m + 1), \text{Int}[(b*\csc[e + f*x])^m, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 3768

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow -\text{Simp}[(b*\cos[c + d*x]*(b*\csc[c + d*x])^{(n - 1)})/(d*(n - 1)), x] + \text{Dist}[(b^2*(n - 2))/(n - 1), \text{Int}[(b*\csc[c + d*x])^{(n - 2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3771

$\text{Int}[(\csc[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\csc[c + d*x])^n*\sin[c + d*x]^n, \text{Int}[1/\sin[c + d*x]^n, x], x] /;$ FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{7/2} dx &= b^2 \int (b \sec(c + dx))^{3/2} (C + A \sec^2(c + dx)) dx \\
 &= \frac{2Ab^2(b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} + \frac{1}{5} (b^2(3A + 5C)) \int (b \sec(c + dx))^{3/2} dx \\
 &= \frac{2b^3(3A + 5C)\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2Ab^2(b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} \\
 &= \frac{2b^3(3A + 5C)\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d} + \frac{2Ab^2(b \sec(c + dx))^{3/2} \tan(c + dx)}{5d} \\
 &= -\frac{2b^4(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^3(3A + 5C)\sqrt{b \sec(c + dx)} \sin(c + dx)}{5d}
 \end{aligned}$$

Mathematica [A] time = 0.467094, size = 79, normalized size = 0.69

$$\frac{b^2(b \sec(c + dx))^{3/2} \left(-(3A + 5C) \sin(2(c + dx)) + 2(3A + 5C) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) - 2A \tan(c + dx) \right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(7/2), x]

[Out] -(b^2*(b*Sec[c + d*x])^(3/2)*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x]))/(5*d)

Maple [C] time = 0.648, size = 668, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(7/2), x)

[Out] -2/5/d*(-1+cos(d*x+c))^2*(3*I*A*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-3*I*A*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)+5*I*C*cos(d*x+c)^3*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-5*I*C*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)+3*I*A*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-3*I*A*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)+5*I*C*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-5*I*C*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c))))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)+3*A*cos(d*x+c)^3+5*C*cos(d*x+c)^3-2*A*cos(d*x+c)^2-

$5C \cos(dx+c)^2 - A \cos(dx+c) (1+\cos(dx+c))^2 (b/\cos(dx+c))^{7/2} / \sin(dx+c)^5$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \sec(dx+c))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)*(b*sec(dx+c))^(7/2),x, algorithm="maxima")

[Out] integrate((C*cos(dx+c)^2 + A)*(b*sec(dx+c))^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb^3 \cos(dx+c)^2 + Ab^3) \sqrt{b \sec(dx+c)} \sec(dx+c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)*(b*sec(dx+c))^(7/2),x, algorithm="fricas")

[Out] integral((C*b^3*cos(dx+c)^2 + A*b^3)*sqrt(b*sec(dx+c))*sec(dx+c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)**2)*(b*sec(dx+c))**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \sec(dx+c))^{7/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)*(b*sec(dx+c))^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(dx+c)^2 + A)*(b*sec(dx+c))^(7/2), x)

3.28 $\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{5/2} dx$

Optimal. Leaf size=78

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \sec(c + dx)}}{3d} + \frac{2Ab^2 \tan(c + dx)\sqrt{b \sec(c + dx)}}{3d}$$

[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]/(3*d) + (2*A*b^2*Sqrt[b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)

Rubi [A] time = 0.0957784, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3238, 4046, 3771, 2641}

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \sec(c + dx)}}{3d} + \frac{2Ab^2 \tan(c + dx)\sqrt{b \sec(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2), x]

[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]/(3*d) + (2*A*b^2*Sqrt[b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)

Rule 3238

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 4046

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> -Simp[(C*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*(m + 1)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{5/2} dx &= b^2 \int \sqrt{b \sec(c + dx)} (C + A \sec^2(c + dx)) dx \\
&= \frac{2Ab^2 \sqrt{b \sec(c + dx)} \tan(c + dx)}{3d} + \frac{1}{3} (b^2(A + 3C)) \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{2Ab^2 \sqrt{b \sec(c + dx)} \tan(c + dx)}{3d} + \frac{1}{3} (b^2(A + 3C) \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}) \\
&= \frac{2b^2(A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2Ab^2 \sqrt{b \sec(c + dx)}}{3d}
\end{aligned}$$

Mathematica [A] time = 0.221161, size = 58, normalized size = 0.74

$$\frac{2b^2 \sqrt{b \sec(c + dx)} \left((A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(5/2), x]

[Out] (2*b^2*Sqrt[b*Sec[c + d*x]]*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]))/(3*d)

Maple [C] time = 0.587, size = 199, normalized size = 2.6

$$-\frac{(-2 + 2 \cos(dx + c)) \cos(dx + c) (1 + \cos(dx + c))^2}{3d (\sin(dx + c))^3} \left(iA \cos(dx + c) \sin(dx + c) \sqrt{(1 + \cos(dx + c))^{-1}} \sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2), x)

[Out] -2/3/d*(-1+cos(d*x+c))*(I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)+3*I*C*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-A*cos(d*x+c)+A)*cos(d*x+c)*(1+cos(d*x+c))^2*(b/cos(d*x+c))^(5/2)/sin(d*x+c)^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \sec(dx + c))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^2 + Ab^2\right)\sqrt{b \sec(dx + c)} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^2 + A*b^2)*sqrt(b*sec(d*x + c))*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \sec(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(5/2), x)

3.29 $\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{3/2} dx$

Optimal. Leaf size=74

$$\frac{2Ab^2 \tan(c + dx)}{d\sqrt{b \sec(c + dx)}} - \frac{2b^2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

[Out] $(-2*b^2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*A*b^2*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])$

Rubi [A] time = 0.0964418, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3238, 4046, 3771, 2639}

$$\frac{2Ab^2 \tan(c + dx)}{d\sqrt{b \sec(c + dx)}} - \frac{2b^2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)*(b*\text{Sec}[c + d*x])^{3/2}, x]$

[Out] $(-2*b^2*(A - C)*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*A*b^2*Tan[c + d*x])/(d*Sqrt[b*Sec[c + d*x]])$

Rule 3238

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(d_.))^{(m_.)}*((a_.) + (b_.)*\sin[(e_.) + (f_.)*(x_.)]^{(n_.)})^{(p_.)}, x_Symbol] \rightarrow \text{Dist}[d^{(n*p)}, \text{Int}[(d*\text{Csc}[e + f*x])^{(m - n*p)}*(b + a*\text{Csc}[e + f*x]^n)^p, x], x] /;$ FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 4046

$\text{Int}[(\text{csc}[(e_.) + (f_.)*(x_.)]*(b_.))^{(m_.)}*(\text{csc}[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cot}[e + f*x]*(b*\text{Csc}[e + f*x])^m)/(f*(m + 1)), x] + \text{Dist}[(C*m + A*(m + 1))/(m + 1), \text{Int}[(b*\text{Csc}[e + f*x])^m, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]

Rule 3771

$\text{Int}[(\text{csc}[(c_.) + (d_.)*(x_.)]*(b_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b*\text{Csc}[c + d*x])^n*\text{Sin}[c + d*x]^n, \text{Int}[1/\text{Sin}[c + d*x]^n, x], x] /;$ FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (A + C \cos^2(c + dx)) (b \sec(c + dx))^{3/2} dx &= b^2 \int \frac{C + A \sec^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2Ab^2 \tan(c + dx)}{d\sqrt{b \sec(c + dx)}} - (b^2(A - C)) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2Ab^2 \tan(c + dx)}{d\sqrt{b \sec(c + dx)}} - \frac{(b^2(A - C)) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= -\frac{2b^2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2Ab^2 \tan(c + dx)}{d\sqrt{b \sec(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.14087, size = 55, normalized size = 0.74

$$\frac{2b\sqrt{b \sec(c + dx)} \left(A \sin(c + dx) - (A - C)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*(b*Sec[c + d*x])^(3/2), x]

[Out] (2*b*Sqrt[b*Sec[c + d*x]]*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d*x]))/d

Maple [C] time = 0.513, size = 590, normalized size = 8.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2), x)

[Out] 2/d*(I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)-I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-I*C*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)+I*C*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)+I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)-I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)+I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)-C*cos(d*x+c)^2-A*cos(d*x+c)+C*cos(d*x+c)+A*cos(d*x+c)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \sec(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^2 + Ab\right)\sqrt{b \sec(dx + c)} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^2 + A*b)*sqrt(b*sec(d*x + c))*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \sec(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*sec(d*x + c))^(3/2), x)

3.30 $\int (A + C \cos^2(c + dx)) \sqrt{b \sec(c + dx)} dx$

Optimal. Leaf size=75

$$\frac{2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \sec(c + dx)}}{3d} + \frac{2b^2C \tan(c + dx)}{3d(b \sec(c + dx))^{3/2}}$$

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b^2*C*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2))

Rubi [A] time = 0.0995225, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3238, 4045, 3771, 2641}

$$\frac{2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \sec(c + dx)}}{3d} + \frac{2b^2C \tan(c + dx)}{3d(b \sec(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(3*d) + (2*b^2*C*Tan[c + d*x])/(3*d*(b*Sec[c + d*x])^(3/2))

Rule 3238

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] := Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] := Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (A + C \cos^2(c + dx)) \sqrt{b \sec(c + dx)} dx &= b^2 \int \frac{C + A \sec^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx \\
&= \frac{2b^2 C \tan(c + dx)}{3d(b \sec(c + dx))^{3/2}} + \frac{1}{3}(3A + C) \int \sqrt{b \sec(c + dx)} dx \\
&= \frac{2b^2 C \tan(c + dx)}{3d(b \sec(c + dx))^{3/2}} + \frac{1}{3}((3A + C)\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx \\
&= \frac{2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \sec(c + dx)}}{3d} + \frac{2b^2 C \tan(c + dx)}{3d(b \sec(c + dx))}
\end{aligned}$$

Mathematica [A] time = 0.127726, size = 58, normalized size = 0.77

$$\frac{\sqrt{b \sec(c + dx)} \left(2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx)) \right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)*Sqrt[b*Sec[c + d*x]],x]

[Out] (Sqrt[b*Sec[c + d*x]]*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d)

Maple [C] time = 0.652, size = 190, normalized size = 2.5

$$\frac{(-2 + 2 \cos(dx + c))(1 + \cos(dx + c))^2}{3d(\sin(dx + c))^3} \left(3iA\sqrt{(1 + \cos(dx + c))^{-1}} \sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}} \text{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right), \right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x)

[Out] -2/3/d*(-1+cos(d*x+c))*(3*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-C*cos(d*x+c)^2+C*cos(d*x+c)*(1+cos(d*x+c))^2*(b/cos(d*x+c))^(1/2)/sin(d*x+c)^3

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)\sqrt{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec(c + dx)} (A + C \cos^2(c + dx)) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*sec(d*x+c))**(1/2),x)

[Out] Integral(sqrt(b*sec(c + d*x))*(A + C*cos(c + d*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right)\sqrt{b \sec(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c)), x)

$$3.31 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt{b \sec(c+dx)}} dx$$

Optimal. Leaf size=77

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} + \frac{2b^2C \tan(c+dx)}{5d(b \sec(c+dx))^{5/2}}$$

[Out] (2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b^2*C*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2))

Rubi [A] time = 0.0991752, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3238, 4045, 3771, 2639}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b \sec(c+dx)}} + \frac{2b^2C \tan(c+dx)}{5d(b \sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]], x]

[Out] (2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*b^2*C*Tan[c + d*x])/(5*d*(b*Sec[c + d*x])^(5/2))

Rule 3238

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^m_)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^p_., x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^m_)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^n_., x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx &= b^2 \int \frac{C + A \sec^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx \\
&= \frac{2b^2 C \tan(c + dx)}{5d(b \sec(c + dx))^{5/2}} + \frac{1}{5}(5A + 3C) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx \\
&= \frac{2b^2 C \tan(c + dx)}{5d(b \sec(c + dx))^{5/2}} + \frac{(5A + 3C) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} \\
&= \frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}} + \frac{2b^2 C \tan(c + dx)}{5d(b \sec(c + dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.231961, size = 61, normalized size = 0.79

$$\frac{\frac{4(5A+3C)E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}} + 2C \sin(2(c + dx))}{10d\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Sec[c + d*x]],x]

[Out] ((4*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*C*Sin[2*(c + d*x)])/(10*d*Sqrt[b*Sec[c + d*x]])

Maple [C] time = 0.592, size = 608, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x)

[Out] $-2/5/d*(5*I*A*\cos(d*x+c)*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-5*I*A*\cos(d*x+c)*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)+3*I*C*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\cos(d*x+c)*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-3*I*C*\cos(d*x+c)*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)+5*I*A*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-5*I*A*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)+3*I*C*\sin(d*x+c)*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*EllipticE(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)-3*I*C*(1/(1+\cos(d*x+c)))^{1/2}*(\cos(d*x+c)/(1+\cos(d*x+c)))^{1/2}*\sin(d*x+c)*EllipticF(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)+C*\cos(d*x+c)^4+5*A*\cos(d*x+c)^2+2*C*\cos(d*x+c)^2-5*A*\cos(d*x+c)-3*C*\cos(d*x+c))*(b/\cos(d*x+c))^{1/2}/b/\sin(d*x+c)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \sec(dx + c)}}{b \sec(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c))/(b*sec(d*x + c)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt{b \sec(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(1/2),x)

[Out] Integral((A + C*cos(c + d*x)**2)/sqrt(b*sec(c + d*x)), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \sec(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*sec(d*x + c)), x)

$$3.32 \quad \int \frac{A+C \cos^2(c+dx)}{(b \sec(c+dx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}} + \frac{2b^2C \tan(c+dx)}{7d(b \sec(c+dx))^{7/2}}$$

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*b^2*d) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*b*d*Sqrt[b*Sec[c + d*x]]) + (2*b^2*C*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2))

Rubi [A] time = 0.133047, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3238, 4045, 3769, 3771, 2641}

$$\frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \sec(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sin(c+dx)}{21bd\sqrt{b \sec(c+dx)}} + \frac{2b^2C \tan(c+dx)}{7d(b \sec(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2), x]

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/(21*b^2*d) + (2*(7*A + 5*C)*Sin[c + d*x])/(21*b*d*Sqrt[b*Sec[c + d*x]]) + (2*b^2*C*Tan[c + d*x])/(7*d*(b*Sec[c + d*x])^(7/2))

Rule 3238

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p) * (b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegerQ[n, p]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{(b \sec(c + dx))^{3/2}} dx &= b^2 \int \frac{C + A \sec^2(c + dx)}{(b \sec(c + dx))^{7/2}} dx \\ &= \frac{2b^2 C \tan(c + dx)}{7d(b \sec(c + dx))^{7/2}} + \frac{1}{7}(7A + 5C) \int \frac{1}{(b \sec(c + dx))^{3/2}} dx \\ &= \frac{2(7A + 5C) \sin(c + dx)}{21bd\sqrt{b \sec(c + dx)}} + \frac{2b^2 C \tan(c + dx)}{7d(b \sec(c + dx))^{7/2}} + \frac{(7A + 5C) \int \sqrt{b \sec(c + dx)} dx}{21b^2} \\ &= \frac{2(7A + 5C) \sin(c + dx)}{21bd\sqrt{b \sec(c + dx)}} + \frac{2b^2 C \tan(c + dx)}{7d(b \sec(c + dx))^{7/2}} + \frac{((7A + 5C)\sqrt{\cos(c + dx)}\sqrt{b \sec(c + dx)}) \int}{21b^2} \\ &= \frac{2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \sec(c + dx)}}{21b^2 d} + \frac{2(7A + 5C) \sin(c + dx)}{21bd\sqrt{b \sec(c + dx)}} + \frac{2b^2 C}{7d(b \sec(c + dx))^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.524948, size = 79, normalized size = 0.69

$$\frac{2 \sin(c + dx)(14A + 3C \cos(2(c + dx))) + 13C + \frac{4(7A+5C)F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{42bd\sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(3/2), x]
```

```
[Out] ((4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 2*(14*A + 1
3*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(42*b*d*Sqrt[b*Sec[c + d*x]])
```

Maple [C] time = 0.533, size = 241, normalized size = 2.1

$$\frac{2(1 + \cos(dx + c))^2(-1 + \cos(dx + c))}{21d(\cos(dx + c))^2(\sin(dx + c))^3} \left(-7iA\sqrt{(1 + \cos(dx + c))^{-1}}\sqrt{\frac{\cos(dx + c)}{1 + \cos(dx + c)}} \operatorname{EllipticF}\left(\frac{i(-1 + \cos(dx + c))}{\sin(dx + c)}\right), \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2), x)
```

```
[Out] 2/21/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))*(-7*I*A*(1/(1+cos(d*x+c)))^(1/2)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)*s
in(d*x+c)-5*I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)+3*C*cos(d*x+c)^4-3*C*c
os(d*x+c)^3+7*A*cos(d*x+c)^2+5*C*cos(d*x+c)^2-7*A*cos(d*x+c)-5*C*cos(d*x+c)
)/cos(d*x+c)^2/(b/cos(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \sec(dx + c)}}{b^2 \sec(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c))/(b^2*sec(d*x + c)^2), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{A + C \cos^2(c + dx)}{(b \sec(c + dx))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(3/2),x)

[Out] Integral((A + C*cos(c + d*x)**2)/(b*sec(c + d*x))**(3/2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \sec(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(3/2), x)

3.33 $\int \frac{A+C \cos^2(c+dx)}{(b \sec(c+dx))^{5/2}} dx$

Optimal. Leaf size=115

$$\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

[Out] (2*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*(9*A + 7*C)*Sin[c + d*x])/(45*b*d*(b*Sec[c + d*x])^(3/2)) + (2*b^2*C*Tan[c + d*x])/(9*d*(b*Sec[c + d*x])^(9/2))

Rubi [A] time = 0.127748, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3238, 4045, 3769, 3771, 2639}

$$\frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2(9A+7C)\sin(c+dx)}{45bd(b\sec(c+dx))^{3/2}} + \frac{2b^2C\tan(c+dx)}{9d(b\sec(c+dx))^{9/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2), x]

[Out] (2*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]]) + (2*(9*A + 7*C)*Sin[c + d*x])/(45*b*d*(b*Sec[c + d*x])^(3/2)) + (2*b^2*C*Tan[c + d*x])/(9*d*(b*Sec[c + d*x])^(9/2))

Rule 3238

Int[(csc[(e_.) + (f_.)*(x_.)]*(d_.))^(m_.)*((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)]^(n_.))^(p_.), x_Symbol] :> Dist[d^(n*p), Int[(d*Csc[e + f*x])^(m - n*p)*(b + a*Csc[e + f*x]^n)^p, x], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && !IntegerQ[m] && IntegersQ[n, p]

Rule 4045

Int[(csc[(e_.) + (f_.)*(x_.)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_.)]^2*(C_.) + (A_.)), x_Symbol] :> Simp[(A*Cot[e + f*x]*(b*Csc[e + f*x])^m)/(f*m), x] + Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]

Rule 3769

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Csc[c + d*x])^(n + 1))/(b*d*n), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 3771

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> Dist[(b*Csc[c + d*x])^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && EqQ[n^2, 1/4]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{(b \sec(c + dx))^{5/2}} dx &= b^2 \int \frac{C + A \sec^2(c + dx)}{(b \sec(c + dx))^{9/2}} dx \\ &= \frac{2b^2 C \tan(c + dx)}{9d(b \sec(c + dx))^{9/2}} + \frac{1}{9}(9A + 7C) \int \frac{1}{(b \sec(c + dx))^{5/2}} dx \\ &= \frac{2(9A + 7C) \sin(c + dx)}{45bd(b \sec(c + dx))^{3/2}} + \frac{2b^2 C \tan(c + dx)}{9d(b \sec(c + dx))^{9/2}} + \frac{(9A + 7C) \int \frac{1}{\sqrt{b \sec(c + dx)}} dx}{15b^2} \\ &= \frac{2(9A + 7C) \sin(c + dx)}{45bd(b \sec(c + dx))^{3/2}} + \frac{2b^2 C \tan(c + dx)}{9d(b \sec(c + dx))^{9/2}} + \frac{(9A + 7C) \int \sqrt{\cos(c + dx)} dx}{15b^2 \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} \\ &= \frac{2(9A + 7C) E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^2 d \sqrt{\cos(c + dx)} \sqrt{b \sec(c + dx)}} + \frac{2(9A + 7C) \sin(c + dx)}{45bd(b \sec(c + dx))^{3/2}} + \frac{2b^2 C \tan(c + dx)}{9d(b \sec(c + dx))^{9/2}} \end{aligned}$$

Mathematica [A] time = 0.622843, size = 81, normalized size = 0.7

$$\frac{4 \sin(2(c + dx))(18A + 5C \cos(2(c + dx)) + 19C) + \frac{48(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{\sqrt{\cos(c+dx)}}}{360b^2 d \sqrt{b \sec(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Sec[c + d*x])^(5/2), x]

[Out] ((48*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2])/Sqrt[Cos[c + d*x]] + 4*(18*A + 19*C + 5*C*Cos[2*(c + d*x)]*Sin[2*(c + d*x)])/(360*b^2*d*Sqrt[b*Sec[c + d*x]]))

Maple [C] time = 0.547, size = 636, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2), x)

[Out] -2/45/d*(27*I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)-27*I*A*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)+5*C*cos(d*x+c)^6+21*I*C*cos(d*x+c)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*I*C*cos(d*x+c)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)+27*I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)-27*I*A*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c), I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+21*I*C*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c), I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-21*I*C*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*

$x+c)*\text{EllipticF}(I*(-1+\cos(d*x+c))/\sin(d*x+c),I)+9*A*\cos(d*x+c)^4+2*C*\cos(d*x+c)^4+18*A*\cos(d*x+c)^2+14*C*\cos(d*x+c)^2-27*A*\cos(d*x+c)-21*C*\cos(d*x+c))/\cos(d*x+c)^3/(b/\cos(d*x+c))^{(5/2)}/\sin(d*x+c)$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx+c)^2 + A}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx+c)^2 + A)\sqrt{b \sec(dx+c)}}{b^3 \sec(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*sec(d*x + c))/(b^3*sec(d*x + c)^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*sec(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx+c)^2 + A}{(b \sec(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*sec(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*sec(d*x + c))^(5/2), x)

3.34 $\int (b \cos(c + dx))^m (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=117

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{m+1}}{bd(m+2)} - \frac{(A(m+2) + C(m+1)) \sin(c + dx)(b \cos(c + dx))^{m+1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(c + dx)\right)}{bd(m+1)(m+2)\sqrt{\sin^2(c + dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m)) - ((C*(1 + m) + A*(2 + m))*(b*Cos[c + d*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0721987, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3014, 2643}

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{m+1}}{bd(m+2)} - \frac{(A(m+2) + C(m+1)) \sin(c + dx)(b \cos(c + dx))^{m+1} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(c + dx)\right)}{bd(m+1)(m+2)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^m*(A + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m)) - ((C*(1 + m) + A*(2 + m))*(b*Cos[c + d*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + m)*(2 + m)*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^m (A + C \cos^2(c + dx)) dx &= \frac{C(b \cos(c + dx))^{1+m} \sin(c + dx)}{bd(2 + m)} + \left(A + \frac{C(1 + m)}{2 + m}\right) \int (b \cos(c + dx))^m \\ &= \frac{C(b \cos(c + dx))^{1+m} \sin(c + dx)}{bd(2 + m)} - \frac{\left(A + \frac{C(1+m)}{2+m}\right) (b \cos(c + dx))^{1+m} {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(c + dx)\right)}{bd(1 + m)} \end{aligned}$$

Mathematica [A] time = 0.173454, size = 114, normalized size = 0.97

$$\frac{\sqrt{\sin^2(c + dx)} \cot(c + dx)(b \cos(c + dx))^m \left(A(m + 3) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(c + dx)\right) + C(m + 1) \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(c + dx)\right)\right)}{d(m + 1)(m + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^m*(A + C*cos[c + d*x]^2),x]

[Out] -(((b*cos[c + d*x])^m*Cot[c + d*x]*(A*(3 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] + C*(1 + m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m)*(3 + m)))

Maple [F] time = 1.754, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^m (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) (b \cos(dx + c))^m, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**m*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^m, x)

$$3.35 \quad \int (b \cos(c + dx))^m \left(-\frac{C(1+m)}{2+m} + C \cos^2(c + dx) \right) dx$$

Optimal. Leaf size=31

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{m+1}}{bd(m + 2)}$$

[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m))

Rubi [A] time = 0.0416858, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.03$, Rules used = {3011}

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{m+1}}{bd(m + 2)}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(2 + m))

Rule 3011

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0]

Rubi steps

$$\int (b \cos(c + dx))^m \left(-\frac{C(1+m)}{2+m} + C \cos^2(c + dx) \right) dx = \frac{C(b \cos(c + dx))^{1+m} \sin(c + dx)}{bd(2 + m)}$$

Mathematica [C] time = 0.193215, size = 113, normalized size = 3.65

$$\frac{C \sqrt{\sin^2(c + dx)} \cot(c + dx) (b \cos(c + dx))^m \left((m + 3) {}_2F_1 \left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(c + dx) \right) - (m + 2) \cos^2(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{m+3}{2}; \right) \right)}{d(m + 2)(m + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^m*(-((C*(1 + m))/(2 + m)) + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^m*Cot[c + d*x]*((3 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2] - (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m)*(3 + m))

Maple [F] time = 1.934, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^m \left(-\frac{C(1+m)}{2+m} + C(\cos(dx + c))^2 \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x)

Maxima [B] time = 2.21787, size = 236, normalized size = 7.61

$$\frac{(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{\frac{1}{2}m} C b^m \sin(-(dx + c)(m + 2) + m \arctan(\sin(2dx + 2c)))}{2^m d (m + 2)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] -1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*m)*C*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*m)*C*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(2^m*d*(m + 2))

Fricas [A] time = 1.39454, size = 81, normalized size = 2.61

$$\frac{(b \cos(dx + c))^m C \cos(dx + c) \sin(dx + c)}{dm + 2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] (b*cos(d*x + c))^m*C*cos(d*x + c)*sin(d*x + c)/(d*m + 2*d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**m*(-C*(1+m)/(2+m)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(-C*(1+m)/(2+m)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


$$3.36 \quad \int (b \cos(c + dx))^m \left(A - \frac{A(2+m) \cos^2(c+dx)}{1+m} \right) dx$$

Optimal. Leaf size=32

$$-\frac{A \sin(c + dx)(b \cos(c + dx))^{m+1}}{bd(m + 1)}$$

[Out] -((A*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(1 + m)))

Rubi [A] time = 0.0499524, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = {3011}

$$-\frac{A \sin(c + dx)(b \cos(c + dx))^{m+1}}{bd(m + 1)}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]

[Out] -((A*(b*Cos[c + d*x])^(1 + m)*Sin[c + d*x])/(b*d*(1 + m)))

Rule 3011

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] /; FreeQ[{b, e, f, A, C, m}, x] && EqQ[A*(m + 2) + C*(m + 1), 0]

Rubi steps

$$\int (b \cos(c + dx))^m \left(A - \frac{A(2 + m) \cos^2(c + dx)}{1 + m} \right) dx = -\frac{A(b \cos(c + dx))^{1+m} \sin(c + dx)}{bd(1 + m)}$$

Mathematica [C] time = 0.197603, size = 119, normalized size = 3.72

$$\frac{A \sin(c + dx) \cos(c + dx) (b \cos(c + dx))^m \left((m + 2) \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; \cos^2(c + dx)\right) - (m + 3) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \dots\right) \right)}{d(m + 1)(m + 3)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^m*(A - (A*(2 + m)*Cos[c + d*x]^2)/(1 + m)),x]

[Out] (A*Cos[c + d*x]*(b*Cos[c + d*x])^m*(-((3 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[c + d*x]^2]) + (2 + m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[c + d*x]^2])*Sin[c + d*x])/(d*(1 + m)*(3 + m)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 1.523, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^m \left(A - \frac{A(2+m)(\cos(dx + c))^2}{1+m} \right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x)

[Out] int((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x)

Maxima [B] time = 2.15901, size = 236, normalized size = 7.38

$$\frac{(\cos(2dx + 2c)^2 + \sin(2dx + 2c)^2 + 2\cos(2dx + 2c) + 1)^{\frac{1}{2}m} Ab^m \sin(-(dx + c)(m + 2) + m \arctan(\sin(2dx + 2c)),$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="maxima")

[Out] 1/4*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*m)*A*b^m*sin(-(d*x + c)*(m + 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/2*m)*A*b^m*sin(-(d*x + c)*(m - 2) + m*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(2^m*d*(m + 1))

Fricas [A] time = 1.32814, size = 80, normalized size = 2.5

$$-\frac{(b \cos(dx + c))^m A \cos(dx + c) \sin(dx + c)}{dm + d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="fricas")

[Out] -(b*cos(d*x + c))^m*A*cos(d*x + c)*sin(d*x + c)/(d*m + d)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**m*(A-A*(2+m)*cos(d*x+c)**2/(1+m)),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^m*(A-A*(2+m)*cos(d*x+c)^2/(1+m)),x, algorithm="gias")
```

```
[Out] Exception raised: NotImplementedError
```

3.37 $\int \cos^2(c+dx)\sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=112

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45bd} + \frac{2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{7/2}}{9b^3d}$$

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^3*d)

Rubi [A] time = 0.0988935, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2640, 2639}

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45bd} + \frac{2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{7/2}}{9b^3d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(n_), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c+d*x]*(b*Sin[c+d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c+d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_)+(d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx))dx}{b^2} \\
&= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^3d} + \frac{(9A+7C)\int(b\cos(c+dx))^{5/2}dx}{9b^2} \\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C(b\cos(c+dx))^{5/2}}{9b^2} \\
&= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45bd} + \frac{2C(b\cos(c+dx))^{5/2}}{9b^2} \\
&= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{5/2}}{9b^2}
\end{aligned}$$

Mathematica [A] time = 0.303698, size = 88, normalized size = 0.79

$$\frac{\sqrt{b\cos(c+dx)}\left(24(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right) + 2\sin(2(c+dx))\sqrt{\cos(c+dx)}(18A+5C\cos(2(c+dx))+19C)\right)}{180d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)]*Sin[2*(c + d*x)]))/(180*d*Sqrt[Cos[c + d*x]])

Maple [B] time = 3.27, size = 322, normalized size = 2.9

$$-\frac{2b}{45d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-160C\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^{10}+320C(\sin(1/2dx+c/2))^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x)

[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c))+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C\cos(dx+c)^2 + A)\sqrt{b\cos(dx+c)}\cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^4 + A \cos(dx + c)^2\right) \sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

3.38 $\int \cos(c+dx)\sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=110

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d} + \frac{2b(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^5}{7b^2d}$$

```
[Out] (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rubi [A] time = 0.0919654, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3014, 2635, 2642, 2641}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21d} + \frac{2b(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^5}{7b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]
```

```
[Out] (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b
*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3014

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(
x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))dx &= \frac{\int (b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx))dx}{b} \\
&= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} + \frac{(7A+5C)\int (b\cos(c+dx))^3}{7b} \\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} \\
&= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^2d} \\
&= \frac{2b(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{7b^2d}
\end{aligned}$$

Mathematica [A] time = 0.332497, size = 89, normalized size = 0.81

$$\frac{(b\cos(c+dx))^{3/2}\left(4(7A+5C)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2\sin(c+dx)\sqrt{\cos(c+dx)}(14A+3C\cos(2(c+dx))+13C)\right)}{42bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*b*d*Cos[c + d*x]^(3/2))

Maple [B] time = 3.734, size = 294, normalized size = 2.7

$$-\frac{2b}{21d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48C\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^8\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)-72C\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^6\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)+\left(28A+56C\right)\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)^4\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)+\left(-14A-16C\right)\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)^2\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)+7A\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2\right)^{1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x)

[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C\cos(dx+c)^2+A)\sqrt{b\cos(dx+c)}\cos(dx+c)dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^3 + A \cos(dx + c)\right)\sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

3.39 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=77

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5bd}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rubi [A] time = 0.0554759, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3014, 2640, 2639}

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rule 3014

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{1}{5}(5A + 3C) \int \sqrt{b \cos(c + dx)} dx \\ &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{((5A + 3C)\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5\sqrt{\cos(c + dx)}} \\ &= \frac{2(5A + 3C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \end{aligned}$$

Mathematica [A] time = 0.080073, size = 70, normalized size = 0.91

$$\frac{\sqrt{b \cos(c + dx)} \left(2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx))\sqrt{\cos(c + dx)} \right)}{5d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])

Maple [B] time = 3.621, size = 261, normalized size = 3.4

$$\frac{2b}{5d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(8C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^6 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 8C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^4 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 5A \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 \right) \left(2 \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 \text{EllipticE}\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right), 2\right) + 3C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 \left(2 \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 \text{EllipticE}\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right), 2\right) + 2C \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right) / \left(-b \left(2 \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^4 - \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 \right)^{1/2} / \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) / \left(b \left(2 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 - 1 \right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+2*C*sin(1/2*d*x+1/2*c)*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(1/2)*(A+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/2)*(A+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)), x)

3.40 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=73

$$\frac{2b(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rubi [A] time = 0.0726957, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3014, 2642, 2641}

$$\frac{2b(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(2), x_Symbol] :=> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \frac{A + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}(b(3A + C)) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b(3A + C) \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3 \sqrt{b \cos(c + dx)}} \\
&= \frac{2b(3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.133265, size = 59, normalized size = 0.81

$$\frac{b \left(2(3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx)) \right)}{3d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (b*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 2.95, size = 237, normalized size = 3.3

$$-\frac{2b}{3d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2} dx + \frac{c}{2}\right) \right)^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(4C \left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right) \right)^4 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right) + 3A \sqrt{\left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right) \right)^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)\sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right)\sqrt{b \cos(dx + c)} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

3.41 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$

Optimal. Leaf size=69

$$\frac{2Ab \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0880731, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2640, 2639}

$$\frac{2Ab \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[b*\text{Cos}[c + d*x]]*(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^2, x]$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_)]^2), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^2(c+dx) dx &= b^2 \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx \\
&= \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + (-A + C) \int \sqrt{b \cos(c+dx)} dx \\
&= \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{((-A + C)\sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{2(A - C)\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d\sqrt{\cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.200728, size = 55, normalized size = 0.8

$$\frac{2b \left(A \sin(c+dx) - (A - C)\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) \right)}{d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.233, size = 214, normalized size = 3.1

$$\frac{b\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A\sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \right)}{-2\sqrt{-b(2(\sin(1/2 dx + c/2))^4 - 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x)

[Out]
$$\frac{-2*b*(-2*b*\sin(1/2*d*x+1/2*c)^4 + \sin(1/2*d*x+1/2*c)^2*b)^{(1/2)}*(A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2 - C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}))}{(-b*(2*\sin(1/2*d*x+1/2*c)^4 - \sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A)\sqrt{b \cos(dx+c)} \sec(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

3.42 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$

Optimal. Leaf size=76

$$\frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

[Out] (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0892076, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2642, 2641}

$$\frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (2*b*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^3(c+dx) dx &= b^3 \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx \\
&= \frac{2Ab^2 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{1}{3}(b(A+3C)) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx \\
&= \frac{2Ab^2 \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{(b(A+3C)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c+dx)}} \\
&= \frac{2b(A+3C)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2Ab^2 \sin(c+dx)}{3d(b \cos(c+dx))^3}
\end{aligned}$$

Mathematica [A] time = 0.174358, size = 56, normalized size = 0.74

$$\frac{2b \left((A+3C)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) + A \tan(c+dx) \right)}{3d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (2*b*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 4.084, size = 292, normalized size = 3.8

$$-\frac{2b}{3d} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} (\sin(1/2 dx + c/2)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) \sqrt{b \cos(dx+c)} \sec(dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

3.43 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$

Optimal. Leaf size=110

$$\frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^{(5/2)}) + (2*b*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.113597, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[b*\text{Cos}[c + d*x]]*(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^4, x]$

[Out] $(-2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^{(5/2)}) + (2*b*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x \ \&\& \ \text{IntegerQ}[m]$

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)})^2, x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /; \text{FreeQ}\{b, e, f, A, C\}, x \ \&\& \ \text{LtQ}[m, -1]$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^4(c+dx) dx &= b^4 \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{7/2}} dx \\
&= \frac{2Ab^3 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{1}{5} (b^2(3A+5C)) \int \frac{1}{(b \cos(c+dx))^{3/2}} dx \\
&= \frac{2Ab^3 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2b(3A+5C) \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}} + \frac{1}{5} (-3A - 5C) \tan(c+dx) \\
&= \frac{2Ab^3 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2b(3A+5C) \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}} + \frac{((-3A - 5C) \tan(c+dx))}{5} \\
&= -\frac{2(3A+5C)\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{5d\sqrt{\cos(c+dx)}} + \frac{2Ab^3 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}}
\end{aligned}$$

Mathematica [A] time = 0.306681, size = 84, normalized size = 0.76

$$\frac{\sec^2(c+dx)\sqrt{b \cos(c+dx)} \left(-(3A+5C) \sin(2(c+dx)) + 2(3A+5C) \cos^{\frac{3}{2}}(c+dx) E\left(\frac{1}{2}(c+dx) \middle| 2\right) - 2A \tan(c+dx) \right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] -(Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^2*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x]))/(5*d)

Maple [B] time = 8.442, size = 598, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-10*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

3.44 $\int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$

Optimal. Leaf size=113

$$\frac{2b^2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}}$$

[Out] (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b *Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.113661, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2642, 2641}

$$\frac{2b^2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b *Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^5(c+dx) dx &= b^5 \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{9/2}} dx \\
&= \frac{2Ab^4 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{1}{7} (b^3(5A+7C)) \int \frac{1}{(b \cos(c+dx))^{5/2}} dx \\
&= \frac{2Ab^4 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{1}{21} (b(5A+7C)) \\
&= \frac{2Ab^4 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2b^2(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{(b(5A+7C))}{21} \\
&= \frac{2b(5A+7C)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2Ab^4 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}}
\end{aligned}$$

Mathematica [A] time = 0.48046, size = 83, normalized size = 0.73

$$\frac{\sec^3(c+dx)\sqrt{b \cos(c+dx)} \left((5A+7C) \sin(2(c+dx)) + 2(5A+7C) \cos^2(c+dx) F\left(\frac{1}{2}(c+dx) \middle| 2\right) + 6A \tan(c+dx) \right)}{21d}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] (Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)

Maple [B] time = 7.637, size = 411, normalized size = 3.6

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} (\sin(1/2 dx + c/2))^2 b}{\sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} d} \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)

3.45 $\int \cos(c+dx)(b \cos(c+dx))^{3/2} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=110

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2b(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d \sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))}{9b^2d}$$

[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^2*d)

Rubi [A] time = 0.0934267, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3014, 2635, 2640, 2639}

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2b(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d \sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))}{9b^2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^2*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(n_), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c+d*x]*(b*Sin[c+d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c+d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_)+(d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \cos(c+dx)(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx)) dx}{b} \\
&= \frac{2C(b \cos(c+dx))^{7/2} \sin(c+dx)}{9b^2d} + \frac{(9A+7C) \int (b \cos(c+dx))^{3/2} \sin(c+dx) dx}{9b} \\
&= \frac{2(9A+7C)(b \cos(c+dx))^{3/2} \sin(c+dx)}{45d} + \frac{2C(b \cos(c+dx))^{5/2} \sin(c+dx)}{9b} \\
&= \frac{2(9A+7C)(b \cos(c+dx))^{3/2} \sin(c+dx)}{45d} + \frac{2C(b \cos(c+dx))^{5/2} \sin(c+dx)}{9b} \\
&= \frac{2b(9A+7C)\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{15d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b \cos(c+dx))^{5/2} \sin(c+dx)}{9b}
\end{aligned}$$

Mathematica [A] time = 0.242085, size = 91, normalized size = 0.83

$$\frac{(b \cos(c+dx))^{5/2} \left(24(9A+7C) E\left(\frac{1}{2}(c+dx) \middle| 2\right) + 2 \sin(2(c+dx)) \sqrt{\cos(c+dx)} (18A+5C \cos(2(c+dx)) + 19C) \right)}{180bd \cos^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*b*d*Cos[c + d*x]^(5/2))

Maple [B] time = 3.417, size = 324, normalized size = 3.

$$-\frac{2b^2}{45d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(-160 C \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^{10} + 320 C (\sin(1/2 dx + c/2))^{10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2), x)

[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^2 \cos(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^4 + Ab \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)

3.46 $\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=113

$$\frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{21d\sqrt{b\cos(c + dx)}} + \frac{2b(7A + 5C)\sin(c + dx)\sqrt{b\cos(c + dx)}}{21d} + \frac{2C\sin(c + dx)(b\cos(c + dx))}{7bd}$$

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rubi [A] time = 0.0812235, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3014, 2635, 2642, 2641}

$$\frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{21d\sqrt{b\cos(c + dx)}} + \frac{2b(7A + 5C)\sin(c + dx)\sqrt{b\cos(c + dx)}}{21d} + \frac{2C\sin(c + dx)(b\cos(c + dx))}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt
[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21
*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} + \frac{1}{7}(7A + 5C) \int (b \cos(c + dx))^{3/2} dx \\
&= \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&= \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} \\
&= \frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)}}{21d}
\end{aligned}$$

Mathematica [A] time = 0.0674589, size = 86, normalized size = 0.76

$$\frac{(b \cos(c + dx))^{3/2} \left(4(7A + 5C)F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin(c + dx)\sqrt{\cos(c + dx)}(14A + 3C \cos(2(c + dx))) + 13C\right)}{42d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(3/2)*(A + C*cos[c + d*x]^2), x]

[Out] ((b*cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[cos[c + d*x]]*(14*A + 13*C + 3*C*cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*cos[c + d*x]^(3/2))

Maple [B] time = 3.625, size = 296, normalized size = 2.6

$$-\frac{2b^2}{21d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} \left(48C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^8 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 72C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^6 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + (28A + 56C) \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + (-14A - 16C) \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 7A \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 5C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 5C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right) / \left(-b \left(2 \sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4 - \sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^{1/2} / \sin\left(\frac{1}{2}dx + \frac{c}{2}\right) / \left(b \left(2 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2), x)

[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] Timed out

3.47 $\int (b \cos(c+dx))^{3/2} (A + C \cos^2(c + dx)) \sec(c+dx) dx$

Optimal. Leaf size=75

$$\frac{2b(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d}$$

[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rubi [A] time = 0.0800262, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3014, 2640, 2639}

$$\frac{2b(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(m + 2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)] + (d_)*(x_)], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_)] + (d_)*(x_)], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx \\
&= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{1}{5}(b(5A + 3C)) \int \sqrt{b \cos(c + dx)} dx \\
&= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{(b(5A + 3C)\sqrt{b \cos(c + dx)})}{5\sqrt{\cos(c + dx)}} \\
&= \frac{2b(5A + 3C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.065546, size = 71, normalized size = 0.95

$$\frac{b\sqrt{b \cos(c + dx)} \left(2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx))\sqrt{\cos(c + dx)} \right)}{5d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (b*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])

Maple [B] time = 3.659, size = 263, normalized size = 3.5

$$\frac{2b^2}{5d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(8C (\sin(1/2 dx + c/2))^6 \cos(1/2 dx + c/2) - 8C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) + 5A (\sin(1/2 dx + c/2))^2 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)

$$3.48 \quad \int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal. Leaf size=76

$$\frac{2b^2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bC \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rubi [A] time = 0.104869, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3014, 2642, 2641}

$$\frac{2b^2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bC \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} (b^2(3A + C)) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b^2(3A + C)\sqrt{\cos(c + dx)})}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2b^2(3A + C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0824603, size = 61, normalized size = 0.8

$$\frac{b^2 \left(2(3A + C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx)) \right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(3/2)*(A + C*cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (b^2*(2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)]))/(3*d*Sqrt[b*cos[c + d*x]])

Maple [B] time = 3.189, size = 239, normalized size = 3.1

$$-\frac{2b^2}{3d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(4C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) + 3A \sqrt{(\sin(1/2 dx + c/2))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)

3.49 $\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$

Optimal. Leaf size=72

$$\frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*b*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^2*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0984745, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2640, 2639}

$$\frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{3/2}*(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^3, x]$

[Out] $(-2*b*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^2*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\
&= \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - (b(A - C)) \int \sqrt{b \cos(c + dx)} dx \\
&= \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{(b(A - C)\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}} \\
&= -\frac{2b(A - C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.137032, size = 57, normalized size = 0.79

$$\frac{2b^2 \left(A \sin(c + dx) - (A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 4.014, size = 216, normalized size = 3.

$$\frac{b^2 \sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \left(\frac{1}{2}(c + dx) \middle| 2 \right) - \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} \right)}{\sqrt{-b(2(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] -2*b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)

$$3.50 \quad \int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal. Leaf size=78

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0990916, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2642, 2641}

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (2*b^2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{1}{3} (b^2(A + 3C)) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{(b^2(A + 3C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2b^2(A + 3C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.169828, size = 58, normalized size = 0.74

$$\frac{2b^2 \left((A + 3C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (2*b^2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.707, size = 294, normalized size = 3.8

$$-\frac{2b^2}{3d} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} (\sin(1/2 dx + c/2)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{3/2} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

3.51 $\int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$

Optimal. Leaf size=113

$$\frac{2b^2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} - \frac{2b(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*b*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^4*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*b^2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.126973, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2b^2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} - \frac{2b(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^(3/2)*(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^5, x]$

[Out] $(-2*b*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^4*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*b^2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^(m_*)*((b_*)*(v_*))^(n_), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m+n), x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_)]^(m_)*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_)]^(n_)), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^(m+1))/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^(m+2), x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]^(n_), x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^(n+1))/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^(n+2), x], x] /;$ FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx &= b^5 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5} (b^3(3A + 5C)) \int \frac{1}{(b \cos(c + dx))} \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{1}{5} (b(3A \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{(b(3A \\ &= -\frac{2b(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx)\middle|2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.228633, size = 84, normalized size = 0.74

$$\frac{\sec^3(c + dx)(b \cos(c + dx))^{3/2} \left(-(3A + 5C) \sin(2(c + dx)) + 2(3A + 5C) \cos^3(c + dx) E\left(\frac{1}{2}(c + dx)\middle|2\right) - 2A \tan(c + dx) \right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] -((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(2*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - (3*A + 5*C)*Sin[2*(c + d*x)] - 2*A*Tan[c + d*x]))/(5*d)

Maple [B] time = 8.875, size = 599, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1

)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-10*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb \cos(dx + c)^3 + Ab \cos(dx + c))\sqrt{b \cos(dx + c)} \sec(dx + c)^5, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)

$$3.52 \quad \int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$$

Optimal. Leaf size=115

$$\frac{2b^3(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

[Out] (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^3*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.123082, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2642, 2641}

$$\frac{2b^3(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^3*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^(2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx &= b^6 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\ &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7} (b^4(5A + 7C)) \int \frac{1}{(b \cos(c + dx))^{5/2}} \\ &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^3(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{1}{21} (b^2(5A + \\ &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^3(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{(b^2(5A + \\ &= \frac{2b^2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.30436, size = 83, normalized size = 0.72

$$\frac{\sec^4(c + dx)(b \cos(c + dx))^{3/2} \left((5A + 7C) \sin(2(c + dx)) + 2(5A + 7C) \cos^2(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6A \tan(c + dx) \right)}{21d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] ((b*Cos[c + d*x])^(3/2)*Sec[c + d*x]^4*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)

Maple [B] time = 7., size = 413, normalized size = 3.6

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} (\sin(1/2 dx + c/2))^2 b^2}{\sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} d} \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((Cb \cos(dx + c)^3 + Ab \cos(dx + c)) \sqrt{b \cos(dx + c)} \sec(dx + c)^6, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)

3.53 $\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=113

$$\frac{2b^2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{5/2}}{9bd}$$

[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)

Rubi [A] time = 0.0804051, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3014, 2635, 2640, 2639}

$$\frac{2b^2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{5/2}}{9bd}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2), x]

[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)

Rule 3014

Int[((b_)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_) + (C_)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_.) + (d_.)*(x_)]^(n_)), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} + \frac{1}{9}(9A + 7C) \int (b \cos(c + dx))^{5/2} dx \\
&= \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} + \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&= \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} + \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} \\
&= \frac{2b^2(9A + 7C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d}
\end{aligned}$$

Mathematica [A] time = 0.0631868, size = 88, normalized size = 0.78

$$\frac{(b \cos(c + dx))^{5/2} \left(24(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin(2(c + dx))\sqrt{\cos(c + dx)}(18A + 5C \cos(2(c + dx)) + 19C) \right)}{180d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(5/2)*(24*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]))/(180*d*Cos[c + d*x]^(5/2))

Maple [B] time = 3.396, size = 324, normalized size = 2.9

$$-\frac{2b^3}{45d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(-160 C \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^{10} + 320 C (\sin(1/2 dx + c/2))^{10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2), x)

[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2), x)

3.54 $\int (b \cos(c+dx))^{5/2} (A + C \cos^2(c + dx)) \sec(c+dx) dx$

Optimal. Leaf size=112

$$\frac{2b^2(7A + 5C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2b^3(7A + 5C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))}{7d}$$

[Out] (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rubi [A] time = 0.108595, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3014, 2635, 2642, 2641}

$$\frac{2b^2(7A + 5C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2b^3(7A + 5C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))}{7d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^(n_), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int (b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx)) dx \\
&= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{1}{7}(b(7A + 5C)) \int (b \cos(c + dx))^{3/2} dx \\
&= \frac{2b^2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2C(b \cos(c + dx))^{3/2}}{7d} \\
&= \frac{2b^2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2C(b \cos(c + dx))^{3/2}}{7d} \\
&= \frac{2b^3(7A + 5C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b^2(7A + 5C)\sqrt{b \cos(c + dx)}}{7d}
\end{aligned}$$

Mathematica [A] time = 0.0749331, size = 87, normalized size = 0.78

$$\frac{b(b \cos(c + dx))^{3/2} \left(4(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \sin(c + dx) \sqrt{\cos(c + dx)} (14A + 3C \cos(2(c + dx)) + 13C) \right)}{42d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (b*(b*Cos[c + d*x])^(3/2)*(4*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + 2*Sqrt[Cos[c + d*x]]*(14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(42*d*Cos[c + d*x]^(3/2))

Maple [B] time = 4.09, size = 296, normalized size = 2.6

$$-\frac{2b^3}{21d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(48 C (\sin(1/2 dx + c/2))^8 \cos(1/2 dx + c/2) - 72 C (\sin(1/2 dx + c/2))^6 \cos(1/2 dx + c/2) + (28A + 56C) \sin(1/2 dx + c/2)^4 \cos(1/2 dx + c/2) + (-14A - 16C) \sin(1/2 dx + c/2)^2 \cos(1/2 dx + c/2) + 7A (\sin(1/2 dx + c/2)^2)^{(1/2)} + 5C (\sin(1/2 dx + c/2)^2)^{(1/2)} \right) / (-b (2 \sin(1/2 dx + c/2)^4 - \sin(1/2 dx + c/2)^2))^{(1/2)} / \sin(1/2 dx + c/2) / (b (2 \cos(1/2 dx + c/2)^2 - 1))^{(1/2)} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)

[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{5/2} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)

3.55 $\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$

Optimal. Leaf size=78

$$\frac{2b^2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2bC \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d}$$

[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rubi [A] time = 0.0961418, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3014, 2640, 2639}

$$\frac{2b^2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2bC \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \sqrt{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx \\
&= \frac{2bC(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{1}{5} (b^2(5A + 3C)) \int \sqrt{b \cos(c + dx)} dx \\
&= \frac{2bC(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{(b^2(5A + 3C)) \sqrt{b \cos(c + dx)}}{5\sqrt{c}} \\
&= \frac{2b^2(5A + 3C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{2bC(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d}
\end{aligned}$$

Mathematica [A] time = 0.052817, size = 73, normalized size = 0.94

$$\frac{b^2 \sqrt{b \cos(c + dx)} \left(2(5A + 3C) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx)) \sqrt{\cos(c + dx)} \right)}{5d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (b^2*Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*d*Sqrt[Cos[c + d*x]])

Maple [B] time = 3.398, size = 263, normalized size = 3.4

$$\frac{2b^3}{5d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(8C (\sin(1/2 dx + c/2))^6 \cos(1/2 dx + c/2) - 8C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^5 \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)

$$3.56 \quad \int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal. Leaf size=78

$$\frac{2b^3(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2b^2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rubi [A] time = 0.0920315, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3014, 2642, 2641}

$$\frac{2b^3(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2b^2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} (b^3 (3A + C)) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b^3 (3A + C) \sqrt{\cos(c + dx)})}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2b^3 (3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d \sqrt{b \cos(c + dx)}} + \frac{2b^2 C \sqrt{b \cos(c + dx)}}{3d}
\end{aligned}$$

Mathematica [A] time = 0.153938, size = 65, normalized size = 0.83

$$\frac{2(b \cos(c + dx))^{5/2} \left((3A + C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(c + dx) \sqrt{\cos(c + dx)} \right)}{3d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (2*(b*Cos[c + d*x])^(5/2)*((3*A + C)*EllipticF[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(5/2))

Maple [B] time = 3.631, size = 239, normalized size = 3.1

$$-\frac{2b^3}{3d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(4C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) + 3A \sqrt{(\sin(1/2 dx + c/2))^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A \right) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)

$$3.57 \quad \int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal. Leaf size=74

$$\frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b^2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*b^2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0974184, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2640, 2639}

$$\frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b^2(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{5/2}*(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^4, x]$

[Out] $(-2*b^2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}), x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\
&= \frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - (b^2(A - C)) \int \sqrt{b \cos(c + dx)} dx \\
&= \frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{(b^2(A - C)\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{\sqrt{\cos(c + dx)}} \\
&= -\frac{2b^2(A - C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.134037, size = 57, normalized size = 0.77

$$\frac{2b^3 \left(A \sin(c + dx) - (A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.762, size = 216, normalized size = 2.9

$$\frac{b^3 \sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \left(\frac{1}{2}(c + dx) \middle| 2 \right) - 2 \right)}{\sqrt{-b(2(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] -2*b^3*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)

$$3.58 \quad \int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx$$

Optimal. Leaf size=78

$$\frac{2b^3(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

[Out] (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0974828, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2642, 2641}

$$\frac{2b^3(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^5(c + dx) dx &= b^5 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{1}{3} (b^3(A + 3C)) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{(b^3(A + 3C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2b^3(A + 3C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.170999, size = 58, normalized size = 0.74

$$\frac{2b^3 \left((A + 3C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] (2*b^3*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.911, size = 294, normalized size = 3.8

$$-\frac{2b^3}{3d} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} (\sin(1/2 dx + c/2)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*b^3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{5/2} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx + c)^4 + Ab^2 \cos(dx + c)^2\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)

3.59 $\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx$

Optimal. Leaf size=115

$$\frac{2b^3(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2b^2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}}$$

[Out] $(-2*b^2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^5*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^{(5/2)}) + (2*b^3*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.120856, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2b^3(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2b^2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{(5/2)}*(A + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^6, x]$

[Out] $(-2*b^2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*b^5*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^{(5/2)}) + (2*b^3*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n, x\} \ \&\& \ \text{IntegerQ}[m]$

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(n_*)})^2, x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /; \text{FreeQ}\{b, e, f, A, C\}, x\} \ \&\& \ \text{LtQ}[m, -1]$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x\} \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^6(c + dx) dx &= b^6 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5} (b^4(3A + 5C)) \int \frac{1}{(b \cos(c + dx))} \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{1}{5} (b^2(3A + 5C)) \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{(b^2(3A + 5C))}{5d} \\ &= -\frac{2b^2(3A + 5C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \end{aligned}$$

Mathematica [A] time = 0.226301, size = 80, normalized size = 0.7

$$\frac{2b^4 \left(-\frac{1}{2}(3A + 5C) \sin(2(c + dx)) + (3A + 5C) \cos^3(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) - A \tan(c + dx) \right)}{5d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] (-2*b^4*((3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - ((3*A + 5*C)*Sin[2*(c + d*x)]/2 - A*Tan[c + d*x]))/(5*d*(b*Cos[c + d*x])^(3/2))

Maple [B] time = 9.168, size = 601, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-10*C*sin(1/2*d*x+1/2*c)^2*c

$$\frac{\cos(1/2*d*x+1/2*c)*(-2*b*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2*b)^{(1/2)}}{(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d}$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Ab^2 \cos(dx+c)^2\right)\sqrt{b \cos(dx+c)} \sec(dx+c)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{\frac{5}{2}} \sec(dx+c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^6, x)

$$3.60 \quad \int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^7(c + dx) dx$$

Optimal. Leaf size=115

$$\frac{2b^4(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^3(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

[Out] (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^4*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.126589, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2642, 2641}

$$\frac{2b^4(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^3(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]

[Out] (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^4*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) \sec^7(c + dx) dx &= b^7 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7} (b^5(5A + 7C)) \int \frac{1}{(b \cos(c + dx))^{5/2}} \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^4(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{1}{21} (b^3(5A + 7C)) \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^4(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{(b^3(5A + 7C)) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} \end{aligned}$$

Mathematica [A] time = 0.391943, size = 83, normalized size = 0.72

$$\frac{\sec^5(c + dx)(b \cos(c + dx))^{5/2} \left((5A + 7C) \sin(2(c + dx)) + 2(5A + 7C) \cos^2(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6A \tan(c + dx) \right)}{21d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^7,x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*Sec[c + d*x]^5*(2*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C)*Sin[2*(c + d*x)] + 6*A*Tan[c + d*x]))/(21*d)
```

Maple [B] time = 7.737, size = 413, normalized size = 3.6

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} (\sin(1/2 dx + c/2))^2 b^3}{\sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} d} \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)
```

```
[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Ab^2 \cos(dx+c)^2\right)\sqrt{b \cos(dx+c)} \sec(dx+c)^7, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^7, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**7,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{\frac{5}{2}} \sec(dx+c)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)

$$3.61 \quad \int \frac{\cos^4(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=147

$$\frac{2(11A + 9C) \sin(c + dx)(b \cos(c + dx))^{5/2}}{77b^3d} + \frac{10(11A + 9C) \sin(c + dx)\sqrt{b \cos(c + dx)}}{231bd} + \frac{10(11A + 9C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}\right)}{231d\sqrt{b \cos(c + dx)}}$$

[Out] (10*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[b*Cos[c + d*x]]) + (10*(11*A + 9*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(231*b*d) + (2*(11*A + 9*C)*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*b^3*d) + (2*C*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*b^5*d)

Rubi [A] time = 0.126337, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2642, 2641}

$$\frac{2(11A + 9C) \sin(c + dx)(b \cos(c + dx))^{5/2}}{77b^3d} + \frac{10(11A + 9C) \sin(c + dx)\sqrt{b \cos(c + dx)}}{231bd} + \frac{10(11A + 9C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}\right)}{231d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (10*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(231*d*Sqrt[b*Cos[c + d*x]]) + (10*(11*A + 9*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(231*b*d) + (2*(11*A + 9*C)*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(77*b^3*d) + (2*C*(b*Cos[c + d*x])^(9/2)*Sin[c + d*x])/(11*b^5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(n_), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

`Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx &= \frac{\int (b\cos(c+dx))^{7/2}(A+C\cos^2(c+dx)) dx}{b^4} \\ &= \frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} + \frac{(11A+9C)\int (b\cos(c+dx))^{7/2} dx}{11b^4} \\ &= \frac{2(11A+9C)(b\cos(c+dx))^{5/2}\sin(c+dx)}{77b^3d} + \frac{2C(b\cos(c+dx))^{9/2}\sin(c+dx)}{11b^5d} \\ &= \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd} + \frac{2(11A+9C)(b\cos(c+dx))^{5/2}}{77b^3d} \\ &= \frac{10(11A+9C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{231bd} + \frac{2(11A+9C)(b\cos(c+dx))^{5/2}}{77b^3d} \\ &= \frac{10(11A+9C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d\sqrt{b\cos(c+dx)}} + \frac{10(11A+9C)\sqrt{b\cos(c+dx)}}{231bd} \end{aligned}$$

Mathematica [A] time = 0.375043, size = 94, normalized size = 0.64

$$\frac{\sin(2(c+dx))(12(11A+16C)\cos(2(c+dx))+572A+21C\cos(4(c+dx))+531C)+80(11A+9C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{1848d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

`[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]`

`[Out] (80*(11*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (572*A + 531*C + 12*(11*A + 16*C)*Cos[2*(c + d*x)] + 21*C*Cos[4*(c + d*x)])*Sin[2*(c + d*x)]/(1848*d*Sqrt[b*Cos[c + d*x]])`

Maple [B] time = 3.71, size = 349, normalized size = 2.4

$$-\frac{2}{231d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(1344C\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^{12}-3360C\cos\right)$$

Verification of antiderivative is not currently implemented for this CAS.

`[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)`

`[Out] -2/231*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(1344*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12-3360*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(528*A+3792*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-792*A-2328*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(616*A+924*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-176*A-186*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+55*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+45*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*`

$\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^4}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^5 + A \cos(dx + c)^3) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^5 + A*cos(d*x + c)^3)*sqrt(b*cos(d*x + c))/b, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^4}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/sqrt(b*cos(d*x + c)), x)

$$3.62 \quad \int \frac{\cos^3(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=115

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^2d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))}{9b^4d}$$

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rubi [A] time = 0.0931476, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2640, 2639}

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^2d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))}{9b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3014

```
Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Ssin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Ssin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c+d*x]*(b*Ssin[c+d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Ssin[c+d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Ssin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx)) dx}{b^3} \\ &= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} + \frac{(9A+7C)\int (b\cos(c+dx))^{5/2} dx}{9b^3} \\ &= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} \\ &= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^4d} \\ &= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15bd\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^2d} \end{aligned}$$

Mathematica [A] time = 0.395381, size = 83, normalized size = 0.72

$$\frac{\sin(c+dx)\cos^2(c+dx)(18A+5C\cos(2(c+dx))+19C)+6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{45d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.267, size = 321, normalized size = 2.8

$$-\frac{2}{45d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-160C\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^{10}+320C\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^8\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c))+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^4 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

$$3.63 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=112

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b \cos(c+dx)}}{21bd} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{5/2}}{7b^3d}$$

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*C
os[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)

Rubi [A] time = 0.0888171, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2642, 2641}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b \cos(c+dx)}}{21bd} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{5/2}}{7b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*C
os[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d)
+ (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(
x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)] + (d_)*(x_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_)] + (d_)*(x_)], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c+dx) (A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx &= \frac{\int (b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx)) dx}{b^2} \\ &= \frac{2C(b \cos(c+dx))^{5/2} \sin(c+dx)}{7b^3 d} + \frac{(7A+5C) \int (b \cos(c+dx))^{3/2} dx}{7b^2} \\ &= \frac{2(7A+5C)\sqrt{b \cos(c+dx)} \sin(c+dx)}{21bd} + \frac{2C(b \cos(c+dx))^{5/2} \sin(c+dx)}{7b^3 d} \\ &= \frac{2(7A+5C)\sqrt{b \cos(c+dx)} \sin(c+dx)}{21bd} + \frac{2C(b \cos(c+dx))^{5/2} \sin(c+dx)}{7b^3 d} \\ &= \frac{2(7A+5C)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b \cos(c+dx)} \sin(c+dx)}{21bd} \end{aligned}$$

Mathematica [A] time = 0.23432, size = 77, normalized size = 0.69

$$\frac{\sin(2(c+dx))(14A+3C \cos(2(c+dx))+13C)+4(7A+5C)\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{42d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)]/(42*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.243, size = 293, normalized size = 2.6

$$-\frac{2}{21d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2 - 1\right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} \left(48C \left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^8 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right) - 72C \left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^6 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right) + (28A + 56C) \sin\left(\frac{1}{2} dx + \frac{c}{2}\right)^4 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right) + (-14A - 16C) \sin\left(\frac{1}{2} dx + \frac{c}{2}\right)^2 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right) + 7A \left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2 \right)^{(1/2)} \left(2 \sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^{(1/2)} \text{EllipticF}\left(\cos\left(\frac{1}{2} dx + \frac{c}{2}\right), 2^{(1/2)}\right) + 5C \left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^{(1/2)} \left(2 \sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^{(1/2)} \text{EllipticF}\left(\cos\left(\frac{1}{2} dx + \frac{c}{2}\right), 2^{(1/2)}\right) \right) / \left(-b \left(2 \sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^4 - \sin\left(\frac{1}{2} dx + \frac{c}{2}\right)^2\right)^{(1/2)} / \sin\left(\frac{1}{2} dx + \frac{c}{2}\right) / \left(b \left(2 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2 - 1\right)^{(1/2)} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)

[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^3 + A \cos(dx + c))\sqrt{b \cos(dx + c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)

$$3.64 \quad \int \frac{\cos(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=80

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^2d}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)

Rubi [A] time = 0.0630443, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3014, 2640, 2639}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx &= \frac{\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx)) dx}{b} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)} dx}{5b} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d} + \frac{((5A+3C)\sqrt{b\cos(c+dx)})\int \sqrt{\cos(c+dx)}}{5b\sqrt{\cos(c+dx)}} \\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5bd\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^2d}
\end{aligned}$$

Mathematica [A] time = 0.066358, size = 73, normalized size = 0.91

$$\frac{\sqrt{b\cos(c+dx)}\left(2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right) + C\sin(2(c+dx))\sqrt{\cos(c+dx)}\right)}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (Sqrt[b*Cos[c + d*x]]*(2*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[2*(c + d*x)]))/(5*b*d*Sqrt[Cos[c + d*x]])

Maple [B] time = 3.135, size = 260, normalized size = 3.3

$$\frac{2}{5d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(8C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^6\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 8C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)}{\sqrt{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/b, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)

$$3.65 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=75

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd}$$

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rubi [A] time = 0.0540866, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3014, 2642, 2641}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rule 3014

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx &= \frac{2C\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd} + \frac{1}{3}(3A+C) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx \\ &= \frac{2C\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd} + \frac{((3A+C)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c+dx)}} \\ &= \frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2C\sqrt{b \cos(c+dx)} \sin(c+dx)}{3bd} \end{aligned}$$

Mathematica [A] time = 0.119139, size = 58, normalized size = 0.77

$$\frac{2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right) + C\sin(2(c + dx))}{3d\sqrt{b\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.214, size = 236, normalized size = 3.2

$$-\frac{2}{3d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(4C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 3A\sqrt{\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C\cos(dx+c)^2 + A}{\sqrt{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)\sqrt{b\cos(dx+c)}}{b\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] `integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="giac")`

[Out] `integrate((C*cos(d*x + c)^2 + A)/sqrt(b*cos(d*x + c)), x)`

$$3.66 \quad \int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=71

$$\frac{2A \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0763436, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3012, 2640, 2639}

$$\frac{2A \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]],x]

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx &= b \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx \\
&= \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{(A - C) \int \sqrt{b \cos(c + dx)} dx}{b} \\
&= \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{((A - C)\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{b\sqrt{\cos(c + dx)}} \\
&= -\frac{2(A - C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2A \sin(c + dx)}{d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [C] time = 1.23607, size = 200, normalized size = 2.82

$$\csc(c) \left(3(A - C)(\cos(dx) - i \sin(dx)) \sqrt{i \sin(2(c + dx)) + \cos(2(c + dx)) + 1} {}_2F_1\left(-\frac{1}{4}, \frac{1}{2}; \frac{3}{4}; -e^{2idx}(\cos(c) + i \sin(c))^2\right) + (A - C) \sqrt{b \cos(c + dx)} \right) / \sqrt{b \cos(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]], x]

[Out] -(Csc[c]*(-6*A*Cos[d*x] + 3*C*Cos[d*x] + 3*C*Cos[2*c + d*x] + 3*(A - C)*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*(Cos[d*x] - I*Sin[d*x])*Sqrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]] + (A - C)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x))*(Cos[c] + I*Sin[c])^2])*(Cos[d*x] + I*Sin[d*x])*Sqrt[1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)]])/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.432, size = 213, normalized size = 3.

$$-2 \frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2 b} \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE}(\cos(1/2 dx + c/2), 2^{1/2}) - 2A \cos(1/2 dx + c/2) \right)}{\sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2), x)

[Out] -2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)} \sec(dx + c)}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/sqrt(b*cos(d*x + c)), x)

$$3.67 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=73

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0893228, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3012, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]], x]

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c+d*x]]/Sqrt[b*Sin[c+d*x]], Int[1/Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_)+(d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{1}{3}(A + 3C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\
&= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{((A + 3C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{2(A + 3C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.42459, size = 141, normalized size = 1.93

$$\frac{4b(A + C \cos^2(c + dx)) \left((A + 3C) \csc(c) \cos^2(c + dx) \sqrt{\cos^2(dx - \tan^{-1}(\cot(c)))} \sec(dx - \tan^{-1}(\cot(c))) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \sin(dx - \tan^{-1}(\cot(c)))\right) \right)}{3d\sqrt{\csc^2(c)(b \cos(c + dx))^{3/2}(2A + C \cos(2(c + dx)) + C)}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (-4*b*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3*d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])

Maple [B] time = 3.503, size = 291, normalized size = 4.

$$-\frac{2}{3d} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} (\sin(1/2 dx + c/2)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/sqrt(b*cos(d*x + c)), x)
```


$$3.68 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^3(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=112

$$\frac{2Ab^2 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2(3A+5C) \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.112419, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2Ab^2 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2(3A+5C) \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]],x]

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c+d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5}(b(3A + 5C)) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{(3A + 5C) \int \sqrt{b \cos(c + dx)}}{5b} \\ &= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{((3A + 5C)\sqrt{b \cos(c + dx)})}{5b\sqrt{\cos(c + dx)}} \\ &= -\frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [C] time = 6.28102, size = 522, normalized size = 4.66

$$b \left(\frac{\cos^4(c + dx) (A \sec^2(c + dx) + C) \left(\frac{4 \sec(c) \sec(c + dx) (3A \sin(dx) + 5C \sin(dx))}{5d} + \frac{4(3A + 5C) \csc(c) \sec(c)}{5d} + \frac{4A \sec(c) \sin(dx) \sec^3(c + dx)}{5d} + \frac{4}{5d} \right)}{(b \cos(c + dx))^{3/2} (2A + C \cos(2c + 2dx) + C)} \right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] b*((( -I/10)*(3*A + 5*C)*Cos[c + d*x]^(7/2)*Csc[c/2]*Sec[c/2]*(C + A*Sec[c +
d*x]^2)*((2*E^((2*I)*d*x))*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)
*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[
c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Si
n[c])))/(b*Cos[c + d*x]^(3/2)*(2*A + C + C*Cos[2*c + 2*d*x])) + (Cos[c +
d*x]^4*(C + A*Sec[c + d*x]^2)*((4*(3*A + 5*C)*Csc[c]*Sec[c])/(5*d) + (4*A*S
ec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (4*Sec[c]*Sec[c + d*x]*(3*A*Sin[d*x]
+ 5*C*Sin[d*x]))/(5*d) + (4*A*Sec[c + d*x]^2*Tan[c])/(5*d)))/(b*Cos[c + d
*x]^(3/2)*(2*A + C + C*Cos[2*c + 2*d*x])))
```

Maple [B] time = 9.168, size = 601, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2), x)
```

```
[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-10*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

$$3.69 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^4(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=110

$$\frac{2Ab^3 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2b(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d\sqrt{b \cos(c+dx)}}$$

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.122542, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2642, 2641}

$$\frac{2Ab^3 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2b(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c + d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^4(c + dx)}{\sqrt{b \cos(c + dx)}} dx &= b^4 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7} (b^2(5A + 7C)) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{1}{21} (5A + 7C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{((5A + 7C)\sqrt{\cos(c + dx)})}{21\sqrt{b \cos(c + dx)}} \\ &= \frac{2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.364563, size = 74, normalized size = 0.67

$$\frac{2 \left((5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + \tan(c + dx) (3A \sec^2(c + dx) + 5A + 7C) \right)}{21d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]
```

```
[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C +
3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 7.625, size = 412, normalized size = 3.8

$$-\frac{1}{d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(2C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b \left(2 (\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2} \right)}}{b \left((\cos(1/2 dx + c/2))^2 - 1/2 \right)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^4}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^4}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)

$$3.70 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^5(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=147

$$\frac{2b^2(7A+9C) \sin(c+dx)}{45d(b \cos(c+dx))^{5/2}} + \frac{2Ab^4 \sin(c+dx)}{9d(b \cos(c+dx))^{9/2}} + \frac{2(7A+9C) \sin(c+dx)}{15d\sqrt{b \cos(c+dx)}} - \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

[Out] (-2*(7*A + 9*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (2*b^2*(7*A + 9*C)*Sin[c + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (2*(7*A + 9*C)*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.150654, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2b^2(7A+9C) \sin(c+dx)}{45d(b \cos(c+dx))^{5/2}} + \frac{2Ab^4 \sin(c+dx)}{9d(b \cos(c+dx))^{9/2}} + \frac{2(7A+9C) \sin(c+dx)}{15d\sqrt{b \cos(c+dx)}} - \frac{2(7A+9C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]], x]

[Out] (-2*(7*A + 9*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(9*d*(b*Cos[c + d*x])^(9/2)) + (2*b^2*(7*A + 9*C)*Sin[c + d*x])/(45*d*(b*Cos[c + d*x])^(5/2)) + (2*(7*A + 9*C)*Sin[c + d*x])/(15*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c+d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^5(c + dx)}{\sqrt{b \cos(c + dx)}} dx &= b^5 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{11/2}} dx \\ &= \frac{2Ab^4 \sin(c + dx)}{9d(b \cos(c + dx))^{9/2}} + \frac{1}{9} (b^3(7A + 9C)) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2Ab^4 \sin(c + dx)}{9d(b \cos(c + dx))^{9/2}} + \frac{2b^2(7A + 9C) \sin(c + dx)}{45d(b \cos(c + dx))^{5/2}} + \frac{1}{15} (b(7A + 9C)) \int \frac{1}{b \cos(c + dx)} dx \\ &= \frac{2Ab^4 \sin(c + dx)}{9d(b \cos(c + dx))^{9/2}} + \frac{2b^2(7A + 9C) \sin(c + dx)}{45d(b \cos(c + dx))^{5/2}} + \frac{2(7A + 9C) \sin(c + dx)}{15d\sqrt{b \cos(c + dx)}} \\ &= \frac{2Ab^4 \sin(c + dx)}{9d(b \cos(c + dx))^{9/2}} + \frac{2b^2(7A + 9C) \sin(c + dx)}{45d(b \cos(c + dx))^{5/2}} + \frac{2(7A + 9C) \sin(c + dx)}{15d\sqrt{b \cos(c + dx)}} \\ &= -\frac{2(7A + 9C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15bd\sqrt{\cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{9d(b \cos(c + dx))^{9/2}} + \frac{2b^2}{4} \end{aligned}$$

Mathematica [A] time = 0.795612, size = 97, normalized size = 0.66

$$\frac{6(7A + 9C) \sin(c + dx) - 6(7A + 9C)\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2 \tan(c + dx) \sec(c + dx) (5A \sec^2(c + dx) + 7A)}{45d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^5)/Sqrt[b*Cos[c + d*x]],x]

[Out] (-6*(7*A + 9*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 6*(7*A + 9*C)*Sin[c + d*x] + 2*Sec[c + d*x]*(7*A + 9*C + 5*A*Sec[c + d*x]^2)*Tan[c + d*x])/(45*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 10.628, size = 729, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x)

[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/5*C/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+2*A*(-1/144*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/180

```
*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)} \sec(dx + c)^5}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**5/(b*cos(d*x+c))**(1/2),x)
```

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^5}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^5/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^5/sqrt(b*cos(d*x + c)), x)
```

$$3.71 \quad \int \frac{\cos^4(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(9A+7C)\sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^3d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{7/2}}{9b^5d}$$

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)

Rubi [A] time = 0.091136, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2640, 2639}

$$\frac{2(9A+7C)\sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^3d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{7/2}}{9b^5d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := -Simp[(b*Cos[c+d*x]*(b*Sin[c+d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c+d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(c + dx) (A + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx &= \frac{\int (b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx)) dx}{b^4} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^5d} + \frac{(9A + 7C) \int (b \cos(c + dx))^{5/2} dx}{9b^4} \\ &= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45b^3d} + \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^5d} \\ &= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45b^3d} + \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^5d} \\ &= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^2d\sqrt{\cos(c + dx)}} + \frac{2(9A + 7C)(b \cos(c + dx))^{3/2}}{45b^3d} \end{aligned}$$

Mathematica [A] time = 0.400068, size = 86, normalized size = 0.75

$$\frac{\sin(c + dx) \cos^2(c + dx) (18A + 5C \cos(2(c + dx)) + 19C) + 6(9A + 7C) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{45bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(3/2)), x]

[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.818, size = 324, normalized size = 2.8

$$-\frac{2}{45bd} \sqrt{b \left(2 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right)^2 - 1\right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} \left(-160C \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^{10} + 320C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c)^(3/2)), x)

[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^4 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)

$$3.72 \quad \int \frac{\cos^3(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}$$

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b *Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2 *d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)

Rubi [A] time = 0.0882239, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2642, 2641}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^2d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b *Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2 *d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx &= \frac{\int (b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx)) dx}{b^3} \\ &= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \frac{(7A+5C)\int (b\cos(c+dx))^{3/2} dx}{7b^3} \\ &= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \dots \\ &= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^4d} + \dots \\ &= \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^2d} \end{aligned}$$

Mathematica [A] time = 0.251986, size = 80, normalized size = 0.7

$$\frac{\sin(2(c+dx))(14A+3C\cos(2(c+dx))+13C)+4(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(3/2)), x]
```

```
[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.627, size = 296, normalized size = 2.6

$$-\frac{2}{21bd}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(48C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^8\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 72C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^6\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + (28A+56C)\sin^4\left(\frac{1}{2}dx + \frac{c}{2}\right)\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + (-14A-16C)\sin^2\left(\frac{1}{2}dx + \frac{c}{2}\right)\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 7A\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2\right)^{1/2} + (2\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 1)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right), 2^{1/2}\right) + 5C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2\right)^{1/2} + (2\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 1)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right), 2^{1/2}\right)\right)/(-b(2\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 1)^{1/2}}{21bd\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)
```

```
[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```


Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^3 + A \cos(dx + c))\sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

$$3.73 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=80

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^3d}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)

Rubi [A] time = 0.0714687, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3014, 2640, 2639}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(3/2), x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c+d*x]]/Sqrt[Sin[c+d*x]], Int[Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_)+(d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c-Pi/2+d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx &= \frac{\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx)) dx}{b^2} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)} dx}{5b^2} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d} + \frac{((5A+3C)\sqrt{b\cos(c+dx)})\int \sqrt{\cos(c+dx)} dx}{5b^2\sqrt{\cos(c+dx)}} \\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^3d}
\end{aligned}$$

Mathematica [A] time = 0.164004, size = 69, normalized size = 0.86

$$\frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + C\sin(2(c+dx))\cos(c+dx)}{5bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*x]*Sin[2*(c + d*x)])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.815, size = 263, normalized size = 3.3

$$\frac{2}{5bd}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(8C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^6\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) - 8C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 5A\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)^2}{(b\cos(dx+c))^{3/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/b^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

$$3.74 \quad \int \frac{\cos(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^2d}$$

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)

Rubi [A] time = 0.0676145, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3014, 2642, 2641}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx &= \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}} dx}{b} \\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}} dx}{3b} \\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d} + \frac{((3A+C)\sqrt{\cos(c+dx)})\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b\sqrt{b\cos(c+dx)}} \\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^2d}
\end{aligned}$$

Mathematica [A] time = 0.137771, size = 61, normalized size = 0.78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) + C\sin(2(c+dx))}{3bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.462, size = 239, normalized size = 3.1

$$-\frac{2}{3bd}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(4C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 3A\sqrt{\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)

$$3.75 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=74

$$\frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}}$$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0882502, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3012, 2640, 2639}

$$\frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)/(b*\text{Cos}[c + d*x])^{3/2}, x]$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)})^2, x_Symbol] :> \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /; \text{FreeQ}\{b, e, f, A, C\}, x] \&\& \text{LtQ}[m, -1]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx &= \frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{(A-C) \int \sqrt{b \cos(c+dx)} dx}{b^2} \\ &= \frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} - \frac{((A-C)\sqrt{b \cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{b^2 \sqrt{\cos(c+dx)}} \\ &= -\frac{2(A-C)\sqrt{b \cos(c+dx)}E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.112969, size = 57, normalized size = 0.77

$$\frac{2A \sin(c + dx) - 2(A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2), x]

[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.459, size = 216, normalized size = 2.9

$$-2 \frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \right)}{b \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - 1)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] -2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="fricas")

[Out] `integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)^2), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2), x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="giac")`

[Out] `integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(3/2), x)`

$$3.76 \quad \int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=75

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0844471, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3012, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx &= b \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{(A + 3C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b} \\
&= \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{((A + 3C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{3b\sqrt{b \cos(c + dx)}} \\
&= \frac{2(A + 3C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [C] time = 1.41602, size = 140, normalized size = 1.87

$$\frac{4(A + C \cos^2(c + dx)) \left((A + 3C) \csc(c) \cos^2(c + dx) \sqrt{\cos^2(dx - \tan^{-1}(\cot(c)))} \sec(dx - \tan^{-1}(\cot(c))) {}_2F_1\left(\frac{1}{4}, \frac{1}{2}; \frac{5}{4}; \dots\right) \right)}{3d\sqrt{\csc^2(c)}(b \cos(c + dx))^{3/2}(2A + C \cos(2(c + dx)) + C)}$$

Warning: Unable to verify antiderivative.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2), x]

[Out] (-4*(A + C*Cos[c + d*x]^2)*((A + 3*C)*Cos[c + d*x]^2*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcTan[Cot[c]]] - A*Sqrt[Csc[c]^2]*Sin[c + d*x]))/(3*d*(b*Cos[c + d*x])^(3/2)*(2*A + C + C*Cos[2*(c + d*x)])*Sqrt[Csc[c]^2])

Maple [B] time = 3.602, size = 294, normalized size = 3.9

$$-\frac{2}{3bd} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} (\sin(1/2 dx + c/2)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2), x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/b*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(3/2), x)

$$3.77 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=113

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.131559, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{5d(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2), x]

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c + d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5}(3A + 5C) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}} - \frac{(3A + 5C) \int \sqrt{b \cos(c + dx)}}{5b^2} \\ &= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}} - \frac{((3A + 5C)\sqrt{b \cos(c + dx)})}{5b^2\sqrt{\cos(c + dx)}} \\ &= -\frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2d\sqrt{\cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.272408, size = 81, normalized size = 0.72

$$\frac{2\left((3A + 5C) \sin(c + dx) - (3A + 5C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \sec(c + dx)\right)}{5bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 8.447, size = 601, normalized size = 5.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*si

$$\frac{n(1/2*d*x+1/2*c)^2+5*C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*EllipticE(\cos(1/2*d*x+1/2*c),2)^{(1/2)}-10*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)*(-2*b*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2*b)^{(1/2)}}{(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d}$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")


```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```

$$3.78 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=112

$$\frac{2Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21bd\sqrt{b \cos(c+dx)}}$$

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b *Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.147884, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2642, 2641}

$$\frac{2Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21bd\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b *Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c+d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c+d*x]]/Sqrt[b*Sin[c+d*x]], Int[1/Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{3/2}} dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\
 &= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7}(b(5A + 7C)) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
 &= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{(5A + 7C) \int \frac{1}{\sqrt{b \cos(c + dx)}}}{21b} \\
 &= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{((5A + 7C)\sqrt{\cos(c + dx)})}{21b\sqrt{b \cos(c + dx)}} \\
 &= \frac{2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21bd\sqrt{b \cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}}
 \end{aligned}$$

Mathematica [A] time = 0.360163, size = 77, normalized size = 0.69

$$\frac{2 \left((5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + \tan(c + dx)(3A \sec^2(c + dx) + 5A + 7C) \right)}{21bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C + 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 7.003, size = 413, normalized size = 3.7

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)}(\sin(1/2 dx + c/2))^2}{b \sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} d \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2), x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

$$3.79 \quad \int \frac{\cos^5(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^4d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{9b^6d}$$

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)

Rubi [A] time = 0.112227, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2640, 2639}

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^4d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{9b^6d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)),x]

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c + d*x]]) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_)), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c + d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^5(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx &= \frac{\int (b\cos(c+dx))^{5/2}(A+C\cos^2(c+dx)) dx}{b^5} \\ &= \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} + \frac{(9A+7C)\int (b\cos(c+dx))^{5/2} dx}{9b^5} \\ &= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} \\ &= \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} + \frac{2C(b\cos(c+dx))^{7/2}\sin(c+dx)}{9b^6d} \\ &= \frac{2(9A+7C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2(9A+7C)(b\cos(c+dx))^{3/2}\sin(c+dx)}{45b^4d} \end{aligned}$$

Mathematica [A] time = 0.393829, size = 86, normalized size = 0.75

$$\frac{\sin(c+dx)\cos^2(c+dx)(18A+5C\cos(2(c+dx))+19C)+6(9A+7C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{45b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^5*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2), x]
```

```
[Out] (6*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + Cos[c + d*x]^2*(18*A + 19*C + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x])/(45*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 3.361, size = 324, normalized size = 2.8

$$-\frac{2}{45b^2d}\sqrt{b(2(\cos(1/2dx+c/2))^2-1)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(-160C\cos(1/2dx+c/2)(\sin(1/2dx+c/2))^{10}+320C(\sin(1/2dx+c/2))^{10}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)
```

```
[Out] -2/45*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-160*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+320*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-72*A-296*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(72*A+136*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-18*A-24*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-27*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-21*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^5}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^4 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**5*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^5}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)

$$3.80 \quad \int \frac{\cos^4(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}$$

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)

Rubi [A] time = 0.102995, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3014, 2635, 2642, 2641}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{21b^3d} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{5/2}}{7b^5d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^(n_), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c+d*x]*(b*Sin[c+d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c+d*x])^(n-2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c+d*x]]/Sqrt[b*Sin[c+d*x]], Int[1/Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx &= \frac{\int (b\cos(c+dx))^{3/2}(A+C\cos^2(c+dx)) dx}{b^4} \\ &= \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} + \frac{(7A+5C)\int (b\cos(c+dx))^{3/2} dx}{7b^4} \\ &= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} \\ &= \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} + \frac{2C(b\cos(c+dx))^{5/2}\sin(c+dx)}{7b^5d} \\ &= \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(7A+5C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{21b^3d} \end{aligned}$$

Mathematica [A] time = 0.239203, size = 80, normalized size = 0.7

$$\frac{\sin(2(c+dx))(14A+3C\cos(2(c+dx))+13C)+4(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{42b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (4*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (14*A + 13*C + 3*C*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(42*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.497, size = 296, normalized size = 2.6

$$-\frac{2}{21b^2d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(48C\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^8\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)-72C\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^6\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)+28A+56C\right)\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)+7A\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2-1\right)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{c}{2}\right),2^{1/2}\right)+5C\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2\left(\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2-1\right)^{1/2}\text{EllipticF}\left(\cos\left(\frac{1}{2}dx+\frac{c}{2}\right),2^{1/2}\right)\right)/\left(-b\left(2\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^4-\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)^2\right)^{1/2}/\sin\left(\frac{1}{2}dx+\frac{c}{2}\right)/(b\left(2\cos\left(\frac{1}{2}dx+\frac{c}{2}\right)\right)^2-1)^{1/2}/d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c)^(5/2)), x)

[Out] -2/21*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(48*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)-72*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(28*A+56*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-14*A-16*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+7*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^3 + A \cos(dx + c))\sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

$$3.81 \quad \int \frac{\cos^3(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=80

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5b^3 d \sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5b^4 d}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)

Rubi [A] time = 0.0703028, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3014, 2640, 2639}

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5b^3 d \sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^{3/2}}{5b^4 d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)),x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx &= \frac{\int \sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx)) dx}{b^3} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{(5A+3C)\int \sqrt{b\cos(c+dx)} dx}{5b^3} \\
&= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{((5A+3C)\sqrt{b\cos(c+dx)})\int \sqrt{\cos(c+dx)} dx}{5b^3\sqrt{\cos(c+dx)}} \\
&= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d}
\end{aligned}$$

Mathematica [A] time = 0.156841, size = 69, normalized size = 0.86

$$\frac{2(5A+3C)\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)+C\sin(2(c+dx))\cos(c+dx)}{5b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)),x]

[Out] (2*(5*A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + C*Cos[c + d*x]*Sin[2*(c + d*x)])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.615, size = 263, normalized size = 3.3

$$\frac{2}{5b^2d}\sqrt{b\left(2(\cos(1/2dx+c/2))^2-1\right)\left(\sin\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}\left(8C(\sin(1/2dx+c/2))^6\cos(1/2dx+c/2)-8C(\sin(1/2dx+c/2))\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(8*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-8*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2+A)\cos(dx+c)^3}{(b\cos(dx+c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/b^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)

$$3.82 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^3d}$$

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)

Rubi [A] time = 0.0659925, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {16, 3014, 2642, 2641}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)),x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^(m+1)), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_)+(d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c+d*x]]/Sqrt[b*Sin[c+d*x]], Int[1/Sqrt[Sin[c+d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_)+(d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx &= \int \frac{\frac{A+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}} dx}{b^2} \\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{(3A+C)\int \frac{1}{\sqrt{b\cos(c+dx)}} dx}{3b^2} \\
&= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{\left((3A+C)\sqrt{\cos(c+dx)}\right)\int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^2\sqrt{b\cos(c+dx)}} \\
&= \frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d}
\end{aligned}$$

Mathematica [A] time = 0.125944, size = 61, normalized size = 0.78

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) + C\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.711, size = 239, normalized size = 3.1

$$-\frac{2}{3b^2d}\sqrt{b\left(2\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)^2 - 1\right)}\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2\left(4C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 3A\sqrt{\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-2*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)^2}{(b\cos(dx+c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)

$$3.83 \quad \int \frac{\cos(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=74

$$\frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{b^3 d \sqrt{\cos(c+dx)}}$$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0706919, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3012, 2640, 2639}

$$\frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} - \frac{2(A-C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{b^3 d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]*(A + C*\text{Cos}[c + d*x]^2))/(b*\text{Cos}[c + d*x])^{5/2}, x]$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_.)*(v_)^{(m_.)*((b_.)*(v_))^{(n_.)}}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_.)*((A_.) + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2)}, x_Symbol] \rightarrow \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /;$ FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2640

$\text{Int}[\text{Sqrt}[(b_.)*\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \rightarrow \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx &= \frac{\int \frac{A+C\cos^2(c+dx)}{(b\cos(c+dx))^{3/2}} dx}{b} \\
&= \frac{2A\sin(c+dx)}{b^2 d \sqrt{b\cos(c+dx)}} - \frac{(A-C) \int \sqrt{b\cos(c+dx)} dx}{b^3} \\
&= \frac{2A\sin(c+dx)}{b^2 d \sqrt{b\cos(c+dx)}} - \frac{((A-C)\sqrt{b\cos(c+dx)}) \int \sqrt{\cos(c+dx)} dx}{b^3 \sqrt{\cos(c+dx)}} \\
&= -\frac{2(A-C)\sqrt{b\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{b^2 d \sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.115142, size = 57, normalized size = 0.77

$$\frac{2A\sin(c+dx) - 2(A-C)\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{b^2 d \sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (-2*(A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.707, size = 216, normalized size = 2.9

$$\frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE}\left(\cos\left(\frac{1}{2} dx + \frac{c}{2}\right), 2\right) - 2A \sin\left(\frac{1}{2} dx + \frac{c}{2}\right) \right)}{b^2 \sqrt{-b \left(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2 \right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] -2/b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(5/2), x)

$$3.84 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=78

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}}$$

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.0660394, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.12$, Rules used = {3012, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2))

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx &= \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{(A+3C) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{3b^2} \\ &= \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{((A+3C)\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3b^2\sqrt{b \cos(c+dx)}} \\ &= \frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.185803, size = 58, normalized size = 0.74

$$\frac{2 \left((A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \right)}{3b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*((A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Tan[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 3.767, size = 294, normalized size = 3.8

$$-\frac{2}{3b^2d} \left(-2A \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^2 - 2 \operatorname{EllipticF}\left(\cos(1/2 dx + c/2), \sqrt{2}\right) \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] -2/3*(-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A+3*C)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/b^2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(5/2), x)

$$3.85 \quad \int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=112

$$\frac{2(3A+5C) \sin(c+dx)}{5b^2 d \sqrt{b \cos(c+dx)}} - \frac{2(3A+5C) E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}}$$

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.107227, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {16, 3012, 2636, 2640, 2639}

$$\frac{2(3A+5C) \sin(c+dx)}{5b^2 d \sqrt{b \cos(c+dx)}} - \frac{2(3A+5C) E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2), x]

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c + d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_.)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx &= b \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{(3A + 5C) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx}{5b} \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}} - \frac{(3A + 5C) \int \sqrt{b \cos(c + dx)} dx}{5b^3} \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}} - \frac{((3A + 5C) \sqrt{b \cos(c + dx)}) \int dx}{5b^3 \sqrt{\cos(c + dx)}} \\ &= -\frac{2(3A + 5C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3 d \sqrt{\cos(c + dx)}} + \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^2 d \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0917273, size = 81, normalized size = 0.72

$$\frac{2 \left((3A + 5C) \sin(c + dx) - (3A + 5C) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \tan(c + dx) \sec(c + dx) \right)}{5b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + (3*A + 5*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x]))/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 8.561, size = 601, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*si

$$n(1/2*d*x+1/2*c)^2+5*C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-10*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)*(-2*b*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2*b)^{(1/2)}/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b^3 \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)/(b^3*cos(d*x + c)^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(5/2), x)
```

$$3.86 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=113

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.122383, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {16, 3012, 2636, 2642, 2641}

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b\cos(c+dx)}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b\cos(c+dx))^{3/2}} + \frac{2Ab\sin(c+dx)}{7d(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2))

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), Int[(b*Sin[c + d*x])^(n+2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_.)], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\ &= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7}(5A + 7C) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21bd(b \cos(c + dx))^{3/2}} + \frac{(5A + 7C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{21b^2} \\ &= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21bd(b \cos(c + dx))^{3/2}} + \frac{((5A + 7C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{21b^2 \sqrt{b \cos(c + dx)}} \\ &= \frac{2(5A + 7C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21b^2 d \sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21bd(b \cos(c + dx))^{3/2}} \end{aligned}$$

Mathematica [A] time = 0.294516, size = 77, normalized size = 0.68

$$\frac{2 \left((5A + 7C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + \tan(c + dx) (3A \sec^2(c + dx) + 5A + 7C) \right)}{21b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x]^(5/2)), x]

[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C + 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 7.829, size = 413, normalized size = 3.7

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)} (\sin(1/2 dx + c/2))^2}{b^2 \sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)}} \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2), x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^3 \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^3*cos(d*x + c)^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)

$$3.87 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{7/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

[Out] $(-2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*b^4*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(5*b*d*(b*\text{Cos}[c + d*x])^{(5/2)}) + (2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*b^3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0917528, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3012, 2636, 2640, 2639}

$$\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + C*\text{Cos}[c + d*x]^2)/(b*\text{Cos}[c + d*x])^{(7/2)}, x]$

[Out] $(-2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*b^4*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(5*b*d*(b*\text{Cos}[c + d*x])^{(5/2)}) + (2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*b^3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 3012

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^{(n_*)})^2, x_Symbol] := \text{Simp}[(A*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+1)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(b^2*(m+1)), \text{Int}[(b*\text{Sin}[e + f*x])^{(m+2)}, x], x] /;$ $\text{FreeQ}\{b, e, f, A, C\}, x \ \&\& \ \text{LtQ}[m, -1]$

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] := \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)})/(b*d*(n+1)), x] + \text{Dist}[(n+2)/(b^2*(n+1)), \text{Int}[(b*\text{Sin}[c + d*x])^{(n+2)}, x], x] /;$ $\text{FreeQ}\{b, c, d\}, x \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] := \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ $\text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] := \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /;$ $\text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx &= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{(3A + 5C) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx}{5b^2} \\
&= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}} - \frac{(3A + 5C) \int \sqrt{b \cos(c + dx)} dx}{5b^4} \\
&= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}} - \frac{((3A + 5C) \sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{5b^4 \sqrt{\cos(c + dx)}} \\
&= -\frac{2(3A + 5C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^4 d \sqrt{\cos(c + dx)}} + \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0850205, size = 81, normalized size = 0.7

$$\frac{2 \left((3A + 5C) \sin(c + dx) - (3A + 5C) \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right) + A \tan(c + dx) \sec(c + dx)}{5b^3 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2), x]

[Out] (2*(-((3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])) + (3*A + 5*C)*Sin[c + d*x] + A*Sec[c + d*x]*Tan[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 9.02, size = 601, normalized size = 5.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2), x)

[Out] 2/5*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-40*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+40*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-10*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^4*cos(d*x + c)^4), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(7/2), x)

$$3.88 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{9/2}} dx$$

Optimal. Leaf size=115

$$\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))

Rubi [A] time = 0.116477, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3012, 2636, 2642, 2641}

$$\frac{2(5A+7C)\sin(c+dx)}{21b^3d(b\cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^4d\sqrt{b\cos(c+dx)}} + \frac{2A\sin(c+dx)}{7bd(b\cos(c+dx))^{7/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2), x]

[Out] (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^4*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(7/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b^3*d*(b*Cos[c + d*x])^(3/2))

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx &= \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{(5A + 7C) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx}{7b^2} \\
&= \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21b^3d(b \cos(c + dx))^{3/2}} + \frac{(5A + 7C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{21b^4} \\
&= \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21b^3d(b \cos(c + dx))^{3/2}} + \frac{((5A + 7C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{21b^4\sqrt{b \cos(c + dx)}} \\
&= \frac{2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21b^4d\sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{7bd(b \cos(c + dx))^{7/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21b^3d(b \cos(c + dx))^{3/2}}
\end{aligned}$$

Mathematica [A] time = 0.0920302, size = 77, normalized size = 0.67

$$\frac{2\left((5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + \tan(c + dx)(3A \sec^2(c + dx) + 5A + 7C)\right)}{21b^4d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(9/2), x]

[Out] (2*((5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*A + 7*C + 3*A*Sec[c + d*x]^2)*Tan[c + d*x]))/(21*b^4*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 7.634, size = 413, normalized size = 3.6

$$-2 \frac{\sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)(\sin(1/2 dx + c/2))^2}}{b^4 \sin(1/2 dx + c/2) \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1)}} d \left(C \left(-1/6 \frac{\cos(1/2 dx + c/2) \sqrt{-b(2(\sin(1/2 dx + c/2))^4 - (\sin(1/2 dx + c/2))^2)}}{b((\cos(1/2 dx + c/2))^2 - 1/2)^2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2), x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^4*(C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2))/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))+A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)\sqrt{b \cos(dx + c)}}{b^5 \cos(dx + c)^5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/(b^5*cos(d*x + c)^5), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(9/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(9/2), x)

$$3.89 \quad \int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} \left(A + C \cos^2(c+dx) \right) dx$$

Optimal. Leaf size=116

$$-\frac{(A+2C) \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{(A+C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{C \sin^5(c+dx) \sqrt{b \cos(c+dx)}}{5d \sqrt{\cos(c+dx)}}$$

[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0589568, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3013, 373}

$$-\frac{(A+2C) \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{(A+C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{C \sin^5(c+dx) \sqrt{b \cos(c+dx)}}{5d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - ((A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A+C \cos^2(c+dx)) dx &= \frac{\sqrt{b \cos(c+dx)} \int \cos^3(c+dx) (A+C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{\sqrt{b \cos(c+dx)} \operatorname{Subst}\left(\int (1-x^2) (A+C-Cx^2) dx, x, -\sqrt{\cos(c+dx)}\right)}{d \sqrt{\cos(c+dx)}} \\
&= -\frac{\sqrt{b \cos(c+dx)} \operatorname{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right) - (A+2C)x^2 + Cx^4\right) dx, x, -\sqrt{\cos(c+dx)}\right)}{d \sqrt{\cos(c+dx)}} \\
&= \frac{(A+C) \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{(A+2C) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.22274, size = 70, normalized size = 0.6

$$\frac{\sin(c+dx) \sqrt{b \cos(c+dx)} (4(5A+7C) \cos(2(c+dx)) + 100A + 3C \cos(4(c+dx)) + 89C)}{120d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.461, size = 70, normalized size = 0.6

$$\frac{(3C(\cos(dx+c))^4 + 5A(\cos(dx+c))^2 + 4C(\cos(dx+c))^2 + 10A + 8C)\sin(dx+c)}{15d} \sqrt{b \cos(dx+c)} \frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x)

[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*(b*cos(d*x+c))^(1/2)*sin(d*x+c)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.10699, size = 150, normalized size = 1.29

$$\frac{C \sqrt{b} \left(3 \sin(5dx + 5c) + 25 \sin\left(\frac{3}{5} \arctan(\sin(5dx + 5c), \cos(5dx + 5c))\right) + 150 \sin\left(\frac{1}{5} \arctan(\sin(5dx + 5c), \cos(5dx + 5c))\right) \right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/240*(C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 20*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d

Fricas [A] time = 1.4845, size = 170, normalized size = 1.47

$$\frac{(3C \cos(dx+c)^4 + (5A+4C) \cos(dx+c)^2 + 10A+8C) \sqrt{b \cos(dx+c)} \sin(dx+c)}{15d \sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*C*cos(d*x + c)^4 + (5*A + 4*C)*cos(d*x + c)^2 + 10*A + 8*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Timed out

3.90 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}(A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=113

$$\frac{x(4A + 3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A + 3C) \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d} + \frac{C \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}}{4d}$$

[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rubi [A] time = 0.072712, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3014, 2635, 8}

$$\frac{x(4A + 3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A + 3C) \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d} + \frac{C \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx &= \frac{\sqrt{b \cos(c+dx)} \int \cos^2(c+dx) (A + C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{C \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d} + \frac{((4A+3C)\sqrt{b \cos(c+dx)}) \sin(c+dx)}{4d} \\
&= \frac{(4A+3C)\sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{8d} + \frac{C \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d} \\
&= \frac{(4A+3C)x \sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A+3C)\sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{8d}
\end{aligned}$$

Mathematica [A] time = 0.168615, size = 67, normalized size = 0.59

$$\frac{\sqrt{b \cos(c+dx)}(4(4A+3C)(c+dx) + 8(A+C)\sin(2(c+dx)) + C\sin(4(c+dx)))}{32d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)]))/(32*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.489, size = 88, normalized size = 0.8

$$\frac{2C(\cos(dx+c))^3 \sin(dx+c) + 4A \cos(dx+c) \sin(dx+c) + 3C \cos(dx+c) \sin(dx+c) + 4A(dx+c) + 3C(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x)

[Out] 1/8/d*(b*cos(d*x+c))^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(1/2)

Maxima [A] time = 2.05079, size = 101, normalized size = 0.89

$$\frac{8(2dx+2c+\sin(2dx+2c))A\sqrt{b} + (12dx+12c+\sin(4dx+4c) + 8\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c), \cos(4dx+4c))\right))C\sqrt{b}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d

Fricas [A] time = 1.74705, size = 549, normalized size = 4.86

$$\frac{2 \left(2 C \cos(dx + c)^2 + 4 A + 3 C \right) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) + (4 A + 3 C) \sqrt{-b} \log \left(2 b \cos(dx + c)^2 - \right)}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b*cos(d*x + c)^(3/2)))))/d]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

3.91 $\int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))dx$

Optimal. Leaf size=74

$$\frac{(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{C\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0375622, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3013}

$$\frac{(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} - \frac{C\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2), x]

[Out] ((A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^(m - 1/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}(A+C\cos^2(c+dx))dx &= \frac{\sqrt{b\cos(c+dx)} \int \cos(c+dx)(A+C\cos^2(c+dx))dx}{\sqrt{\cos(c+dx)}} \\ &= -\frac{\sqrt{b\cos(c+dx)} \text{Subst}\left(\int (A+C-Cx^2)dx, x, -\sin(c+dx)\right)}{d\sqrt{\cos(c+dx)}} \\ &= \frac{(A+C)\sqrt{b\cos(c+dx)}\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{C\sqrt{b\cos(c+dx)}\sin^3(c+dx)}{3d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0856498, size = 52, normalized size = 0.7

$$\frac{\sin(c+dx)\sqrt{b\cos(c+dx)}(6A+C\cos(2(c+dx))+5C)}{6d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2),x]

[Out] (Sqrt[b*Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.379, size = 47, normalized size = 0.6

$$\frac{(C(\cos(dx+c))^2 + 3A + 2C)\sin(dx+c)}{3d} \sqrt{b\cos(dx+c)} \frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x)

[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.03668, size = 77, normalized size = 1.04

$$\frac{C\sqrt{b}\left(\sin(3dx+3c) + 9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right) + 12A\sqrt{b}\sin(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/12*(C*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 12*A*sqrt(b)*sin(d*x + c))/d

Fricas [A] time = 1.39179, size = 126, normalized size = 1.7

$$\frac{(C\cos(dx+c)^2 + 3A + 2C)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algo  
ithm="giac")
```

```
[Out] Timed out
```

$$3.92 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=90

$$\frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0231953, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2635, 8}

$$\frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b \cos(c+dx)}(A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx &= \frac{\sqrt{b \cos(c+dx)} \int (A+C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{(C\sqrt{b \cos(c+dx)}) \int \cos^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)} \sin(c+dx)}{2d} + \frac{(C\sqrt{b \cos(c+dx)}) \int \cos^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d} \end{aligned}$$

Mathematica [A] time = 0.0734354, size = 52, normalized size = 0.58

$$\frac{\sqrt{b \cos(c + dx)}(2(2A + C)(c + dx) + C \sin(2(c + dx)))}{4d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]

[Out] (Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.428, size = 54, normalized size = 0.6

$$\frac{C \cos(dx + c) \sin(dx + c) + 2A(dx + c) + C(dx + c)}{2d} \sqrt{b \cos(dx + c)} \frac{1}{\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2), x)

[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/cos(d*x+c)^(1/2)

Maxima [A] time = 1.89713, size = 70, normalized size = 0.78

$$\frac{(2dx + 2c + \sin(2dx + 2c))C\sqrt{b} + 8A\sqrt{b} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)))/d

Fricas [A] time = 1.71854, size = 459, normalized size = 5.1

$$\left[\frac{2\sqrt{b \cos(dx + c)}C\sqrt{\cos(dx + c)} \sin(dx + c) + (2A + C)\sqrt{-b} \log\left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\sqrt{-b}\sqrt{\cos(dx + c)}\right)}{4d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2), x, algorithm="fricas")

```
[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*
sqrt(-b)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(
d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x +
c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x
)
```

$$3.93 \quad \int \frac{\sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=68

$$\frac{A\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0389626, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3014, 3770}

$$\frac{A\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx &= \frac{\sqrt{b \cos(c+dx)} \int (A + C \cos^2(c+dx)) \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{C\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{(A\sqrt{b \cos(c+dx)}) \int \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{A \tanh^{-1}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{C\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0458876, size = 44, normalized size = 0.65

$$\frac{\sqrt{b \cos(c + dx)} \left(A \tanh^{-1}(\sin(c + dx)) + C \sin(c + dx) \right)}{d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.415, size = 55, normalized size = 0.8

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - \sin(dx + c) C \right) \sqrt{b \cos(dx + c)} \frac{1}{\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.06666, size = 108, normalized size = 1.59

$$\frac{A\sqrt{b}(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2\sin(dx+c) + 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 2*C*sqrt(b)*sin(d*x + c))/d

Fricas [A] time = 1.74861, size = 554, normalized size = 8.15

$$\left[\frac{A\sqrt{b} \cos(dx + c) \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b \cos(dx+c)} C \sqrt{\cos(dx+c)} \sin(dx+c)}{2d \cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="fricas")

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x +
c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt
(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*
sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate(((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c)))/cos(d*x + c)^(3/2), x
)
```

$$3.94 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=59

$$\frac{A \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{Cx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0311298, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 8}

$$\frac{A \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{Cx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]

[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]))^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b \cos(c+dx)}(A+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx &= \frac{\sqrt{b \cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{A\sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{(C\sqrt{b \cos(c+dx)}) \int 1 dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Cx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.0656081, size = 45, normalized size = 0.76

$$\frac{\sqrt{b \cos(c + dx)}(A \sin(c + dx) + C dx \cos(c + dx))}{d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.377, size = 45, normalized size = 0.8

$$\frac{C \cos(dx + c)(dx + c) + A \sin(dx + c)}{d} \sqrt{b \cos(dx + c)} (\cos(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2), x)

[Out] 1/d*(b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)

Maxima [A] time = 1.89028, size = 108, normalized size = 1.83

$$\frac{2 \left(C \sqrt{b} \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) + \frac{A \sqrt{b} \sin(2dx+2c)}{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c)+1} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] 2*(C*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d

Fricas [A] time = 1.68782, size = 518, normalized size = 8.78

$$\left[\frac{C \sqrt{-b} \cos(dx + c)^2 \log \left(2b \cos(dx + c)^2 - 2 \sqrt{b \cos(dx + c)} \sqrt{-b \cos(dx + c)} \sin(dx + c) - b \right) + 2 \sqrt{b \cos(dx + c)} A \sqrt{\cos(dx + c)}}{2 d \cos(dx + c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2), x, algorithm="fricas")

```
[Out] [1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(5/2), x)
```

$$3.95 \quad \int \frac{\sqrt{b \cos(c+dx)} (A+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=78

$$\frac{(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))

Rubi [A] time = 0.043072, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 3770}

$$\frac{(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{b \cos(c+dx)} (A+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx &= \frac{\sqrt{b \cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec^3(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{A\sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{((A+2C)\sqrt{b \cos(c+dx)}) \int \sec(c+dx) dx}{2\sqrt{\cos(c+dx)}} \\ &= \frac{(A+2C) \tanh^{-1}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{A\sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.0983213, size = 59, normalized size = 0.76

$$\frac{\sqrt{b \cos(c + dx)} \left((A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)) + A \sin(c + dx) \right)}{2d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))

Maple [B] time = 0.411, size = 134, normalized size = 1.7

$$\frac{1}{2d} \left(-A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + A (\cos(dx + c))^2 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x)

[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)

Maxima [B] time = 2.14279, size = 983, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x, algorithm="maxima")

[Out] 1/4*(2*C*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*

$d*x + 2*c))))*A*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1))/d$

Fricas [A] time = 1.72237, size = 591, normalized size = 7.58

$$\left[\frac{(A + 2C)\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b} \cos(dx+c) A \sqrt{\cos(dx+c)}}{4d \cos(dx+c)^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)

$$3.96 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=79

$$\frac{(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)}$$

[Out] (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + ((2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0442948, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3012, 3767, 8}

$$\frac{(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]

[Out] (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + ((2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_.) + (d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{\sqrt{b \cos(c+dx)} \int (A + C \cos^2(c+dx)) \sec^4(c+dx) dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{((2A+3C)\sqrt{b \cos(c+dx)}) \int \sec^2(c+dx) dx}{3d \sqrt{\cos(c+dx)}} \\
&= \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{7}{2}}(c+dx)} - \frac{((2A+3C)\sqrt{b \cos(c+dx)}) \text{Subst}(\int 1 dx, x)}{3d \sqrt{\cos(c+dx)}} \\
&= \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{(2A+3C)\sqrt{b \cos(c+dx)} \sin(c+dx)}{3d \cos^{\frac{3}{2}}(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.17822, size = 51, normalized size = 0.65

$$\frac{\sin(c+dx)\sqrt{b \cos(c+dx)} (A \tan^2(c+dx) + 3(A+C))}{3d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.397, size = 54, normalized size = 0.7

$$\frac{(2A(\cos(dx+c))^2 + 3C(\cos(dx+c))^2 + A)\sin(dx+c)\sqrt{b \cos(dx+c)}(\cos(dx+c))^{-\frac{7}{2}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2), x)

[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2)

Maxima [B] time = 2.12433, size = 473, normalized size = 5.99

$$2 \left(\frac{2((3 \cos(2dx+2c)+1)\sin(6dx+6c)+3(3 \cos(2dx+2c)+1)\sin(4dx+4c)-3 \cos(2dx+2c)\sin(6dx+6c)+3 \cos(2dx+2c)\sin(4dx+4c)-3 \cos(6dx+6c)\sin(2dx+2c)-9 \cos(4dx+4c)\sin(2dx+2c))A\sqrt{b}}{2(3 \cos(4dx+4c)+3 \cos(2dx+2c)+1)\cos(6dx+6c)+\cos(6dx+6c)^2+6(3 \cos(2dx+2c)+1)\cos(4dx+4c)+9 \cos(4dx+4c)^2+9 \cos(2dx+2c)^2+6(\sin(4dx+4c)+\sin(2dx+2c))} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2), x, algorith="maxima")

[Out] 2/3*(2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c)

) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 3*C*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d

Fricas [A] time = 1.43556, size = 128, normalized size = 1.62

$$\frac{\left((2A + 3C) \cos(dx + c)^2 + A \right) \sqrt{b \cos(dx + c)} \sin(dx + c)}{3d \cos(dx + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")

[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^(7/2))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)

$$3.97 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=122

$$\frac{(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + ((3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))

Rubi [A] time = 0.0647402, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3012, 3768, 3770}

$$\frac{(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + ((3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\sqrt{b \cos(c+dx)} (A + C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx = \frac{\sqrt{b \cos(c+dx)} \int (A + C \cos^2(c+dx)) \sec^5(c+dx) dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d \cos^{\frac{9}{2}}(c+dx)} + \frac{((3A+4C)\sqrt{b \cos(c+dx)}) \int \sec^3(c+dx) dx}{4\sqrt{\cos(c+dx)}}$$

$$= \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d \cos^{\frac{9}{2}}(c+dx)} + \frac{(3A+4C)\sqrt{b \cos(c+dx)} \sin(c+dx)}{8d \cos^{\frac{5}{2}}(c+dx)} + \dots$$

$$= \frac{(3A+4C) \tanh^{-1}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{8d \sqrt{\cos(c+dx)}} + \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Mathematica [A] time = 0.261821, size = 80, normalized size = 0.66

$$\frac{\sqrt{b \cos(c+dx)} (\sin(c+dx) ((3A+4C) \cos^2(c+dx) + 2A) + (3A+4C) \cos^4(c+dx) \tanh^{-1}(\sin(c+dx)))}{8d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(9/2))

Maple [B] time = 0.334, size = 214, normalized size = 1.8

$$\frac{1}{8d} \left(-3A (\cos(dx+c))^4 \ln \left(-\frac{-1 + \cos(dx+c) + \sin(dx+c)}{\sin(dx+c)} \right) + 3A (\cos(dx+c))^4 \ln \left(\frac{1 - \cos(dx+c) + \sin(dx+c)}{\sin(dx+c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2), x)

[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*sin(d*x+c)*cos(d*x+c)^2+2*A*sin(d*x+c))*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2)

Maxima [B] time = 2.50744, size = 3129, normalized size = 25.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2), x, algo rithm="maxima")

[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44

$$\begin{aligned}
& *(\sin(8*d*x + 8*c) + 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x \\
& + 2*c))*\cos(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\sin(8*d*x \\
& + 8*c) + 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 12*(\sin(8*d*x + 8*c) + \\
& 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 3*(2*(4*\cos(6*d*x + 6*c) + 6*\cos \\
& (4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c) \\
& ^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16* \\
& \cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(\\
& 4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d* \\
& x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(\\
& 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + \\
& 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16* \\
& \sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x \\
& + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\
& + 2*c))))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + \\
& 3*(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos \\
& (8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + \\
& 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) \\
& + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4* \\
& (2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + \\
& 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin \\
& (6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d* \\
& *x + 4*c)*\sin(2*d*x + 2*c) + 16*\sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1 \\
&)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 12*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(7/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 44*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 12*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*A*\sqrt{b}/(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*\sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1) + 4*(4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*C*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin
\end{aligned}$$

$$\frac{(4dx + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1}{d}$$

Fricas [A] time = 1.80536, size = 687, normalized size = 5.63

$$\left[\frac{(3A + 4C)\sqrt{b}\cos(dx + c)^5 \log\left(-\frac{b\cos(dx+c)^3 - 2\sqrt{b}\cos(dx+c)\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2((3A + 4C)\cos(dx + c) + 2A)\sqrt{b}\cos(dx + c)^5}{16d\cos(dx + c)^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="fricas")

[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt(b)*cos(d*x + c)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx + c)^2 + A)\sqrt{b\cos(dx + c)}}{\cos(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2),x)

$$3.98 \quad \int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2} \left(A + C \cos^2(c + dx) \right) dx$$

Optimal. Leaf size=119

$$-\frac{b(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*(A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0575295, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3013, 373}

$$-\frac{b(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*(A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rule 373

Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]

Rubi steps

$$\begin{aligned}
\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx)) dx &= \frac{(b\sqrt{b \cos(c+dx)}) \int \cos^3(c+dx) (A+C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\
&= -\frac{(b\sqrt{b \cos(c+dx)}) \operatorname{Subst}\left(\int (1-x^2) (A+C-Cx^2) dx\right)}{d\sqrt{\cos(c+dx)}} \\
&= -\frac{(b\sqrt{b \cos(c+dx)}) \operatorname{Subst}\left(\int \left(A\left(1+\frac{C}{A}\right) - (A+2C)x^2 - Cx^4\right) dx\right)}{d\sqrt{\cos(c+dx)}} \\
&= \frac{b(A+C)\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{b(A+2C)\sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.243569, size = 70, normalized size = 0.59

$$\frac{\sin(c+dx)(b \cos(c+dx))^{3/2}(4(5A+7C) \cos(2(c+dx)) + 100A + 3C \cos(4(c+dx)) + 89C)}{120d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.303, size = 70, normalized size = 0.6

$$\frac{(3C(\cos(dx+c))^4 + 5A(\cos(dx+c))^2 + 4C(\cos(dx+c))^2 + 10A + 8C) \sin(dx+c)}{15d} (b \cos(dx+c))^{\frac{3}{2}} (\cos(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2), x)

[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(3/2)

Maxima [A] time = 2.11192, size = 158, normalized size = 1.33

$$\frac{20 \left(b \sin(3dx+3c) + 9b \sin\left(\frac{1}{3} \arctan(\sin(3dx+3c), \cos(3dx+3c))\right) \right) A\sqrt{b} + \left(3b \sin(5dx+5c) + 25b \sin\left(\frac{3}{5} \arctan(\sin(5dx+5c), \cos(5dx+5c))\right) \right) b}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2), x, algor
ithm="maxima")

[Out] 1/240*(20*(b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*A*sqrt(b) + (3*b*sin(5*d*x + 5*c) + 25*b*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 150*b*sin(1/5*arctan2(sin(5*d*x + 5*c),

$\cos(5dx + 5c))) * C * \sqrt{b}) / d$

Fricas [A] time = 1.52038, size = 182, normalized size = 1.53

$$\frac{(3Cb \cos(dx + c)^4 + (5A + 4C)b \cos(dx + c)^2 + 2(5A + 4C)b) \sqrt{b \cos(dx + c)} \sin(dx + c)}{15d \sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] 1/15*(3*C*b*cos(d*x + c)^4 + (5*A + 4*C)*b*cos(d*x + c)^2 + 2*(5*A + 4*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] Timed out

3.99 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=116

$$\frac{bx(4A+3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d} + \frac{bC \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}}{4d}$$

[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rubi [A] time = 0.0566941, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3014, 2635, 8}

$$\frac{bx(4A+3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b(4A+3C)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d} + \frac{bC \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_)*sin[(c_)] + (d_)*(x_))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx)) dx &= \frac{(b\sqrt{b \cos(c+dx)}) \int \cos^2(c+dx) (A+C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\
&= \frac{bC \cos^2(c+dx) \sqrt{b \cos(c+dx)} \sin(c+dx)}{4d} + \frac{(b(4A+3C))}{8d} \\
&= \frac{b(4A+3C) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{8d} + \frac{bC \cos^2(c+dx)}{8d} \\
&= \frac{b(4A+3C) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} \sin(c+dx)}{8\sqrt{\cos(c+dx)}} + \frac{b(4A+3C) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{8d}
\end{aligned}$$

Mathematica [A] time = 0.184001, size = 67, normalized size = 0.58

$$\frac{(b \cos(c+dx))^{3/2} (4(4A+3C)(c+dx) + 8(A+C) \sin(2(c+dx)) + C \sin(4(c+dx)))}{32d \cos^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Ssin[4*(c + d*x)]))/(32*d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.515, size = 88, normalized size = 0.8

$$\frac{2C(\cos(dx+c))^3 \sin(dx+c) + 4A \cos(dx+c) \sin(dx+c) + 3C \cos(dx+c) \sin(dx+c) + 4A(dx+c) + 3C(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

[Out] 1/8/d*(b*cos(d*x+c))^(3/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(3/2)

Maxima [A] time = 2.06465, size = 111, normalized size = 0.96

$$\frac{8(2(dx+c)b + b \sin(2dx+2c))A\sqrt{b} + \left(12(dx+c)b + b \sin(4dx+4c) + 8b \sin\left(\frac{1}{2} \arctan(\sin(4dx+4c), \cos(4dx+4c))\right)\right)C\sqrt{b}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] 1/32*(8*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d

Fricas [A] time = 1.67886, size = 568, normalized size = 4.9

$$\frac{(4A + 3C)\sqrt{-b} \log(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b) + 2(2Cb \cos(dx + c)^2)}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*((4*A + 3*C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x + c)^2 + (4*A + 3*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1/8*((4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))) + (2*C*b*cos(d*x + c)^2 + (4*A + 3*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.100 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=76

$$\frac{b(A+C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} - \frac{bC \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}$$

[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0326561, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3013}

$$\frac{b(A+C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} - \frac{bC \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (b*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^(m - 1/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx &= \frac{(b \sqrt{b \cos(c+dx)}) \int \cos(c+dx) (A+C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\ &= -\frac{(b \sqrt{b \cos(c+dx)}) \text{Subst}\left(\int (A+C-Cx^2) dx, x, -\sin(c+dx)\right)}{d \sqrt{\cos(c+dx)}} \\ &= \frac{b(A+C) \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{bC \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0307472, size = 53, normalized size = 0.7

$$\frac{b \sin(c+dx) \sqrt{b \cos(c+dx)} (6A + C \cos(2(c+dx)) + 5C)}{6d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(3/2)*(A + C*cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (b*Sqrt[b*cos[c + d*x]]*(6*A + 5*C + C*cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.356, size = 47, normalized size = 0.6

$$\frac{(C(\cos(dx+c))^2 + 3A + 2C)\sin(dx+c)}{3d} (b\cos(dx+c))^{\frac{3}{2}} (\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)

[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*(b*cos(d*x+c))^(3/2)*sin(d*x+c)/cos(d*x+c)^(3/2)

Maxima [A] time = 2.07042, size = 81, normalized size = 1.07

$$\frac{12Ab^{\frac{3}{2}}\sin(dx+c) + \left(b\sin(3dx+3c) + 9b\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)C\sqrt{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/12*(12*A*b^(3/2)*sin(d*x + c) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d

Fricas [A] time = 1.47297, size = 134, normalized size = 1.76

$$\frac{(Cb\cos(dx+c)^2 + (3A + 2C)b)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/3*(C*b*cos(d*x + c)^2 + (3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{3}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x)

$$3.101 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=93

$$\frac{Abx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0304866, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2635, 8}

$$\frac{Abx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Abx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{(bC\sqrt{b \cos(c + dx)}) \int \cos^2(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Abx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{bC\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)} \sin(c + dx)}{2d} + \frac{(bC}{2d} \\
&= \frac{Abx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{bCx\sqrt{b \cos(c + dx)}}{2\sqrt{\cos(c + dx)}} + \frac{bC\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)}}{2d}
\end{aligned}$$

Mathematica [A] time = 0.095183, size = 52, normalized size = 0.56

$$\frac{(b \cos(c + dx))^{3/2} (2(2A + C)(c + dx) + C \sin(2(c + dx)))}{4d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.25, size = 54, normalized size = 0.6

$$\frac{C \cos(dx + c) \sin(dx + c) + 2A(dx + c) + C(dx + c)}{2d} (b \cos(dx + c))^{\frac{3}{2}} (\cos(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)

[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/cos(d*x+c)^(3/2)

Maxima [A] time = 1.8626, size = 74, normalized size = 0.8

$$\frac{8Ab^{\frac{3}{2}} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + (2(dx+c)b + b \sin(2dx+2c))C\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*C*sqrt(b))/d

Fricas [A] time = 1.77867, size = 467, normalized size = 5.02

$$\left[\frac{2 \sqrt{b \cos(dx+c)} C b \sqrt{\cos(dx+c)} \sin(dx+c) + (2A+C) \sqrt{-bb} \log(2b \cos(dx+c)^2 - 2 \sqrt{b \cos(dx+c)} \sqrt{-b} \sqrt{\cos(dx+c)})}{4d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(-b)*b*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A)(b \cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)

$$3.102 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{5/2}(c+dx)} dx$$

Optimal. Leaf size=70

$$\frac{Ab\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0328485, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3014, 3770}

$$\frac{Ab\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_)] + (d_)*(x_), x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{5/2}(c+dx)} dx &= \frac{(b\sqrt{b \cos(c+dx)}) \int (A+C \cos^2(c+dx)) \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{bC\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{(Ab\sqrt{b \cos(c+dx)}) \int \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ab \tanh^{-1}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bC\sqrt{b \cos(c+dx)} \sin(c+dx)}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0555066, size = 44, normalized size = 0.63

$$\frac{(b \cos(c + dx))^{3/2} (A \tanh^{-1}(\sin(c + dx)) + C \sin(c + dx))}{d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(3/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]

[Out] ((b*cos[c + d*x])^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*sin[c + d*x]))/(d*cos[c + d*x]^(3/2))

Maple [A] time = 0.226, size = 55, normalized size = 0.8

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - \sin(dx + c) C \right) (b \cos(dx + c))^{\frac{3}{2}} (\cos(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*C)*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)

Maxima [A] time = 2.01744, size = 112, normalized size = 1.6

$$\frac{2 C b^{\frac{3}{2}} \sin(dx + c) + (b \log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - b \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2 \sin(dx + c) + 1)) A \sqrt{b}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorith="maxima")

[Out] 1/2*(2*C*b^(3/2)*sin(d*x + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d

Fricas [A] time = 1.76037, size = 562, normalized size = 8.03

$$\frac{A b^{\frac{3}{2}} \cos(dx + c) \log \left(-\frac{b \cos(dx + c)^3 - 2 \sqrt{b \cos(dx + c)} \sqrt{b \cos(dx + c)} \sin(dx + c) - 2 b \cos(dx + c)}{\cos(dx + c)^3} \right) + 2 \sqrt{b \cos(dx + c)} C b \sqrt{\cos(dx + c)}}{2 d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorith="fricas")

```
[Out] [1/2*(A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)), -(A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x
+ c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2),
x)
```

$$3.103 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{7/2}(c+dx)} dx$$

Optimal. Leaf size=61

$$\frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{3/2}(c+dx)} + \frac{bCx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0322901, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 8}

$$\frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{3/2}(c+dx)} + \frac{bCx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{7/2}(c+dx)} dx &= \frac{(b \sqrt{b \cos(c+dx)}) \int (A+C \cos^2(c+dx)) \sec^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ab \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{3/2}(c+dx)} + \frac{(bC \sqrt{b \cos(c+dx)}) \int 1 dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{bCx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{3/2}(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.0595929, size = 45, normalized size = 0.74

$$\frac{(b \cos(c + dx))^{3/2} (A \sin(c + dx) + C dx \cos(c + dx))}{d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.228, size = 45, normalized size = 0.7

$$\frac{C \cos(dx + c)(dx + c) + A \sin(dx + c)}{d} (b \cos(dx + c))^{\frac{3}{2}} (\cos(dx + c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

[Out] 1/d*(b*cos(d*x+c))^(3/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(5/2)

Maxima [A] time = 1.82936, size = 108, normalized size = 1.77

$$\frac{2 \left(C b^{\frac{3}{2}} \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) + \frac{A b^{\frac{3}{2}} \sin(2dx+2c)}{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c)+1} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 2*(C*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d

Fricas [A] time = 1.61463, size = 527, normalized size = 8.64

$$\left[\frac{C \sqrt{-bb} \cos(dx + c)^2 \log \left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)} \sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b \right) + 2\sqrt{b \cos(dx + c)} Ab}{2d \cos(dx + c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")


```
[Out] [1/2*(C*sqrt(-b)*b*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*b*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2),x)
```

$$3.104 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=80

$$\frac{b(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)}$$

[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))

Rubi [A] time = 0.0401435, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 3770}

$$\frac{b(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2))

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{(b\sqrt{b \cos(c+dx)}) \int (A+C \cos^2(c+dx)) \sec^3(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ab\sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{(b(A+2C)\sqrt{b \cos(c+dx)}) \int \sec(c+dx) dx}{2\sqrt{\cos(c+dx)}} \\ &= \frac{b(A+2C) \tanh^{-1}(\sin(c+dx))\sqrt{b \cos(c+dx)}}{2d\sqrt{\cos(c+dx)}} + \frac{Ab\sqrt{b \cos(c+dx)} \sin(c+dx)}{2d \cos^{\frac{5}{2}}(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.110523, size = 59, normalized size = 0.74

$$\frac{(b \cos(c + dx))^{3/2} \left((A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)) + A \sin(c + dx) \right)}{2d \cos^{7/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))

Maple [A] time = 0.27, size = 134, normalized size = 1.7

$$\frac{1}{2d} \left(-A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + A (\cos(dx + c))^2 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)

[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(7/2)

Maxima [B] time = 2.122, size = 1027, normalized size = 12.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/4*(2*(b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c)

$c) + b) \cdot \sin\left(\frac{1}{2} \arctan\left(\frac{\sin(2dx + 2c)}{\cos(2dx + 2c)}\right)\right) \cdot A \sqrt{b} / (2 \cdot (2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4 \cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c)^2 + 4 \cos(2dx + 2c) + 1) / d$

Fricas [A] time = 1.66939, size = 599, normalized size = 7.49

$$\frac{(A + 2C)b^{\frac{3}{2}} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b \cos(dx+c)}Ab\sqrt{\cos(dx+c)}}{4d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(3/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(9/2),x, algorith="fricas")

[Out] [1/4*((A + 2*C)*b^(3/2)*cos(dx + c)^3*log(-(b*cos(dx + c))^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3 + 2*sqrt(b*cos(dx + c))*A*b*sqrt(cos(dx + c))*sin(dx + c))/(d*cos(dx + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c)^3 - sqrt(b*cos(dx + c))*A*b*sqrt(cos(dx + c))*sin(dx + c))/(d*cos(dx + c)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))**(3/2)*(A+C*cos(dx+c)**2)/cos(dx+c)**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(3/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(9/2),x, algorith="giac")

[Out] integrate(((C*cos(dx + c)^2 + A)*(b*cos(dx + c))^(3/2)/cos(dx + c)^(9/2), x)

$$3.105 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=81

$$\frac{b(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)}$$

[Out] (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0476186, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3012, 3767, 8}

$$\frac{b(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]

[Out] (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_.) + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx))}{\cos^{11/2}(c + dx)} dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{(b(2A + 3C)\sqrt{b \cos(c + dx)}) \int \sec^2(c + dx) dx}{3d\sqrt{\cos(c + dx)}} \\
&= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} - \frac{(b(2A + 3C)\sqrt{b \cos(c + dx)}) \text{Subst}(\int 1/\sqrt{u} du, u = \cos(c + dx))}{3d\sqrt{\cos(c + dx)}} \\
&= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{b(2A + 3C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{3/2}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.142715, size = 52, normalized size = 0.64

$$\frac{b \sin(c + dx) \sqrt{b \cos(c + dx)} (A \tan^2(c + dx) + 3(A + C))}{3d \cos^{3/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(3/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]

[Out] (b*Sqrt[b*cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*cos[c + d*x]^(3/2))

Maple [A] time = 0.262, size = 54, normalized size = 0.7

$$\frac{(2A(\cos(dx + c))^2 + 3C(\cos(dx + c))^2 + A)\sin(dx + c)}{3d} (b \cos(dx + c))^{3/2} (\cos(dx + c))^{-9/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2), x)

[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(9/2)

Maxima [B] time = 2.08734, size = 479, normalized size = 5.91

$$2 \left(\frac{3Cb^{3/2} \sin(2dx+2c)}{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c) + 1} - \frac{2(3b \cos(6dx+6c) \sin(2dx+2c))}{2(3 \cos(4dx+4c) + 3 \cos(2dx+2c) + 1) \cos(6dx+6c) + \cos(6dx+6c)^2 + 6(3 \cos(2dx+2c) + 1) \cos(4dx+2c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2), x, algorithm="maxima")

[Out] 2/3*(3*C*b^(3/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - 2*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*

$$\cos(4dx + 4c)\sin(2dx + 2c) - (3b\cos(2dx + 2c) + b)\sin(6dx + 6c) - 3(3b\cos(2dx + 2c) + b)\sin(4dx + 4c) \cdot A\sqrt{b} / (2(3\cos(4dx + 4c) + 3\cos(2dx + 2c) + 1)\cos(6dx + 6c) + \cos(6dx + 6c)^2 + 6(3\cos(2dx + 2c) + 1)\cos(4dx + 4c) + 9\cos(4dx + 4c)^2 + 9\cos(2dx + 2c)^2 + 6(\sin(4dx + 4c) + \sin(2dx + 2c))\sin(6dx + 6c) + \sin(6dx + 6c)^2 + 9\sin(4dx + 4c)^2 + 18\sin(4dx + 4c)\sin(2dx + 2c) + 9\sin(2dx + 2c)^2 + 6\cos(2dx + 2c) + 1) / d$$

Fricas [A] time = 1.35158, size = 134, normalized size = 1.65

$$\frac{((2A + 3C)b\cos(dx + c)^2 + Ab)\sqrt{b\cos(dx + c)}\sin(dx + c)}{3d\cos(dx + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(3/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(11/2),x, algorithm="fricas")

[Out] 1/3*((2*A + 3*C)*b*cos(dx + c)^2 + A*b)*sqrt(b*cos(dx + c))*sin(dx + c)/(d*cos(dx + c)^(7/2))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))**(3/2)*(A+C*cos(dx+c)**2)/cos(dx+c)**(11/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx + c)^2 + A)(b\cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(3/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(11/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + A)*(b*cos(dx + c))^(3/2)/cos(dx + c)^(11/2), x)

$$3.106 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+C \cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx$$

Optimal. Leaf size=125

$$\frac{b(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rubi [A] time = 0.0623372, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3012, 3768, 3770}

$$\frac{b(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(3/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2), x]
```

```
[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] := -Simp[(b*Csc[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{3/2} (A + C \cos^2(c + dx))}{\cos^{13}(c + dx)} dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{(b(3A + 4C)\sqrt{b \cos(c + dx)}) \int \sec^3(c + dx) dx}{4\sqrt{\cos(c + dx)}} \\
&= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{b(3A + 4C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{8d \cos^{5/2}(c + dx)} \\
&= \frac{b(3A + 4C) \tanh^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{8d\sqrt{\cos(c + dx)}} + \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.20471, size = 81, normalized size = 0.65

$$\frac{b\sqrt{b \cos(c + dx)} (\sin(c + dx) ((3A + 4C) \cos^2(c + dx) + 2A) + (3A + 4C) \cos^4(c + dx) \tanh^{-1}(\sin(c + dx)))}{8d \cos^{9/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(3/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]

[Out] (b*Sqrt[b*cos[c + d*x]]*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*cos[c + d*x]^(9/2))

Maple [A] time = 0.266, size = 214, normalized size = 1.7

$$\frac{1}{8d} \left(-3A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + 3A (\cos(dx + c))^4 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x)

[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*sin(d*x+c)*cos(d*x+c)^2+2*A*sin(d*x+c))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2)

Maxima [B] time = 2.55827, size = 3286, normalized size = 26.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="maxima")

[Out] -1/16*((12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)

$$\begin{aligned}
&)) + 44*(b*\sin(8*d*x + 8*c) + 4*b*\sin(6*d*x + 6*c) + 6*b*\sin(4*d*x + 4*c) \\
& + 4*b*\sin(2*d*x + 2*c))*\cos(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)) \\
&) - 44*(b*\sin(8*d*x + 8*c) + 4*b*\sin(6*d*x + 6*c) + 6*b*\sin(4*d*x + 4*c) + \\
& 4*b*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) \\
& - 12*(b*\sin(8*d*x + 8*c) + 4*b*\sin(6*d*x + 6*c) + 6*b*\sin(4*d*x + 4*c) + 4* \\
& b*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - \\
& 3*(b*\cos(8*d*x + 8*c)^2 + 16*b*\cos(6*d*x + 6*c)^2 + 36*b*\cos(4*d*x + 4*c)^2 \\
& + 16*b*\cos(2*d*x + 2*c)^2 + b*\sin(8*d*x + 8*c)^2 + 16*b*\sin(6*d*x + 6*c)^2 \\
& + 36*b*\sin(4*d*x + 4*c)^2 + 48*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*b* \\
& \sin(2*d*x + 2*c)^2 + 2*(4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) + 4*b*c \\
& \cos(2*d*x + 2*c) + b)*\cos(8*d*x + 8*c) + 8*(6*b*\cos(4*d*x + 4*c) + 4*b*\cos(2 \\
& *d*x + 2*c) + b)*\cos(6*d*x + 6*c) + 12*(4*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x \\
& + 4*c) + 8*b*\cos(2*d*x + 2*c) + 4*(2*b*\sin(6*d*x + 6*c) + 3*b*\sin(4*d*x + \\
& 4*c) + 2*b*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*b*\sin(4*d*x + 4*c) + \\
& 2*b*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + b)*\log(\cos(1/2*\arctan2(\sin(2*d*x + \\
& 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
& 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) + 3 \\
& *(b*\cos(8*d*x + 8*c)^2 + 16*b*\cos(6*d*x + 6*c)^2 + 36*b*\cos(4*d*x + 4*c)^2 \\
& + 16*b*\cos(2*d*x + 2*c)^2 + b*\sin(8*d*x + 8*c)^2 + 16*b*\sin(6*d*x + 6*c)^2 \\
& + 36*b*\sin(4*d*x + 4*c)^2 + 48*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*b*s \\
& \sin(2*d*x + 2*c)^2 + 2*(4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) + 4*b*co \\
& s(2*d*x + 2*c) + b)*\cos(8*d*x + 8*c) + 8*(6*b*\cos(4*d*x + 4*c) + 4*b*\cos(2 \\
& *d*x + 2*c) + b)*\cos(6*d*x + 6*c) + 12*(4*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x \\
& + 4*c) + 8*b*\cos(2*d*x + 2*c) + 4*(2*b*\sin(6*d*x + 6*c) + 3*b*\sin(4*d*x + 4 \\
& *c) + 2*b*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*b*\sin(4*d*x + 4*c) + 2 \\
& *b*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + b)*\log(\cos(1/2*\arctan2(\sin(2*d*x + \\
& 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
& 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - 12 \\
& *(b*\cos(8*d*x + 8*c) + 4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) + 4*b*co \\
& s(2*d*x + 2*c) + b)*\sin(7/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - \\
& 44*(b*\cos(8*d*x + 8*c) + 4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) + 4*b* \\
& \cos(2*d*x + 2*c) + b)*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) \\
& + 44*(b*\cos(8*d*x + 8*c) + 4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) + 4* \\
& b*\cos(2*d*x + 2*c) + b)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) \\
&) + 12*(b*\cos(8*d*x + 8*c) + 4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) + \\
& 4*b*\cos(2*d*x + 2*c) + b)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c \\
&))) *A*\sqrt{b}/(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + \\
& 2*c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4 \\
& *\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos \\
& (2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x \\
& + 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c)) \\
& *\sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d \\
& *x + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 \\
& + 48*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16*\sin(2*d*x + 2*c)^2 + 8*\cos(2*d \\
& *x + 2*c) + 1) + 4*(4*(b*\sin(4*d*x + 4*c) + 2*b*\sin(2*d*x + 2*c))*\cos(3/2*a \\
& rctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(b*\sin(4*d*x + 4*c) + 2*b*s \\
& \sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (b* \\
& \cos(4*d*x + 4*c)^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*d*x + 4*c)^2 + 4*b*si \\
& n(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + 2*(2*b*\cos(2*d*x \\
& + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*\log(\cos(1/2*arcta \\
& n2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c \\
&), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\
& *c))) + 1) + (b*\cos(4*d*x + 4*c)^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*d*x + \\
& 4*c)^2 + 4*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + \\
& 2*(2*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*l \\
& \log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2 \\
& (\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c \\
&), \cos(2*d*x + 2*c))) + 1) - 4*(b*\cos(4*d*x + 4*c) + 2*b*\cos(2*d*x + 2*c) + \\
& b)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(b*\cos(4*d*x +
\end{aligned}$$

$$4*c) + 2*b*\cos(2*d*x + 2*c) + b)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*C*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1))/d$$

Fricas [A] time = 1.64118, size = 701, normalized size = 5.61

$$\frac{(3A + 4C)b^{\frac{3}{2}} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2((3A + 4C)b \cos(dx + c) + \dots)}{16 d \cos(dx + c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorith="fricas")

[Out] [1/16*((3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b*cos(d*x + c)^2 + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorith="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2), x)

3.107 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=125

$$-\frac{b^2(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b^2(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*(A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.0620607, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3013, 373}

$$-\frac{b^2(A+2C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{b^2(A+C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2C\sin^5(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2), x]
```

```
[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*(A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^5)/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)], x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol] := Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx)) dx &= \frac{(b^2 \sqrt{b \cos(c+dx)}) \int \cos^3(c+dx) (A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} \\
&= -\frac{(b^2 \sqrt{b \cos(c+dx)}) \text{Subst} \left(\int (1-x^2) (A+C-Cx^2) dx \right)}{d \sqrt{\cos(c+dx)}} \\
&= -\frac{(b^2 \sqrt{b \cos(c+dx)}) \text{Subst} \left(\int \left(A \left(1 + \frac{C}{A} \right) - (A+2C)x^2 \right) dx \right)}{d \sqrt{\cos(c+dx)}} \\
&= \frac{b^2(A+C) \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{b^2(A+2C) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.276887, size = 70, normalized size = 0.56

$$\frac{\sin(c+dx)(b \cos(c+dx))^{5/2}(4(5A+7C) \cos(2(c+dx)) + 100A + 3C \cos(4(c+dx)) + 89C)}{120d \cos^2(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(5/2)*(100*A + 89*C + 4*(5*A + 7*C)*Cos[2*(c + d*x)] + 3*C*Cos[4*(c + d*x)])*Sin[c + d*x])/(120*d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.408, size = 70, normalized size = 0.6

$$\frac{(3C(\cos(dx+c))^4 + 5A(\cos(dx+c))^2 + 4C(\cos(dx+c))^2 + 10A + 8C) \sin(dx+c)}{15d} (b \cos(dx+c))^{5/2} (\cos(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

[Out] 1/15/d*(3*C*cos(d*x+c)^4+5*A*cos(d*x+c)^2+4*C*cos(d*x+c)^2+10*A+8*C)*(b*cos(d*x+c))^(5/2)*sin(d*x+c)/cos(d*x+c)^(5/2)

Maxima [A] time = 2.11695, size = 171, normalized size = 1.37

$$\frac{20 \left(b^2 \sin(3dx+3c) + 9b^2 \sin\left(\frac{1}{3} \arctan(\sin(3dx+3c), \cos(3dx+3c))\right) \right) A \sqrt{b} + \left(3b^2 \sin(5dx+5c) + 25b^2 \sin\left(\frac{1}{5} \arctan(\sin(5dx+5c), \cos(5dx+5c))\right) \right) A \sqrt{b}}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x, algor
ithm="maxima")

[Out] 1/240*(20*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*A*sqrt(b) + (3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*A*sqrt(b)

$*x + 5*c), \cos(5*d*x + 5*c))))*C*\sqrt{b})/d$

Fricas [A] time = 1.42269, size = 190, normalized size = 1.52

$$\frac{(3Cb^2 \cos(dx+c)^4 + (5A+4C)b^2 \cos(dx+c)^2 + 2(5A+4C)b^2)\sqrt{b \cos(dx+c)} \sin(dx+c)}{15d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] 1/15*(3*C*b^2*cos(d*x + c)^4 + (5*A + 4*C)*b^2*cos(d*x + c)^2 + 2*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] Timed out

$$3.108 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=122

$$\frac{b^2 x (4A + 3C) \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{b^2 (4A + 3C) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{8d} + \frac{b^2 C \sin(c + dx) \cos^2(c + dx)^{5/2}}{4d}$$

[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rubi [A] time = 0.0537611, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3014, 2635, 8}

$$\frac{b^2 x (4A + 3C) \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{b^2 (4A + 3C) \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{8d} + \frac{b^2 C \sin(c + dx) \cos^2(c + dx)^{5/2}}{4d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d)

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int \cos^2(c + dx) (A + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{b^2 C \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{(b^2(4A + 3C) \sqrt{b \cos(c + dx)})}{4 \sqrt{\cos(c + dx)}} \\
&= \frac{b^2(4A + 3C) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} + \frac{b^2 C \cos^{\frac{5}{2}}(c + dx)}{4 \sqrt{\cos(c + dx)}} \\
&= \frac{b^2(4A + 3C)x \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{b^2(4A + 3C) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{8d}
\end{aligned}$$

Mathematica [A] time = 0.179927, size = 67, normalized size = 0.55

$$\frac{(b \cos(c + dx))^{5/2} (4(4A + 3C)(c + dx) + 8(A + C) \sin(2(c + dx)) + C \sin(4(c + dx)))}{32d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]

[Out] ((b*cos[c + d*x])^(5/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Ssin[4*(c + d*x)]))/(32*d*cos[c + d*x]^(5/2))

Maple [A] time = 0.431, size = 88, normalized size = 0.7

$$\frac{2C(\cos(dx + c))^3 \sin(dx + c) + 4A \cos(dx + c) \sin(dx + c) + 3C \cos(dx + c) \sin(dx + c) + 4A(dx + c) + 3C(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x)

[Out] 1/8/d*(b*cos(d*x+c))^(5/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/cos(d*x+c)^(5/2)

Maxima [A] time = 2.13532, size = 124, normalized size = 1.02

$$\frac{8(2(dx + c)b^2 + b^2 \sin(2dx + 2c))A\sqrt{b} + (12(dx + c)b^2 + b^2 \sin(4dx + 4c) + 8b^2 \sin\left(\frac{1}{2} \arctan(\sin(4dx + 4c), \cos(4dx + 4c))\right))C\sqrt{b}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] 1/32*(8*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + (12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d

Fricas [A] time = 1.72187, size = 582, normalized size = 4.77

$$\left[\frac{(4A + 3C)\sqrt{-bb^2} \log(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b) + 2(2Cb^2 \cos(dx + c) + C^2 b^2)}{16d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/16*((4*A + 3*C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*x + c)^2 + (4*A + 3*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d, 1/8*((4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b*cos(d*x + c))^(3/2))) + (2*C*b^2*cos(d*x + c)^2 + (4*A + 3*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/d]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{5}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x)

$$3.109 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=80

$$\frac{b^2(A+C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} - \frac{b^2 C \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}$$

[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0340368, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3013}

$$\frac{b^2(A+C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} - \frac{b^2 C \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (b^2*(A + C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) - (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^(m - 1/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx &= \frac{(b^2 \sqrt{b \cos(c+dx)}) \int \cos(c+dx) (A+C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}} \\ &= -\frac{(b^2 \sqrt{b \cos(c+dx)}) \text{Subst}\left(\int (A+C-Cx^2) dx, x, -\sin(c+dx)\right)}{d \sqrt{\cos(c+dx)}} \\ &= \frac{b^2(A+C) \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} - \frac{b^2 C \sqrt{b \cos(c+dx)} \sin^3(c+dx)}{3d \sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.133093, size = 52, normalized size = 0.65

$$\frac{\sin(c+dx)(b \cos(c+dx))^{5/2}(6A+C \cos(2(c+dx))+5C)}{6d \cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/cos[c + d*x]^(3/2), x]

[Out] ((b*cos[c + d*x])^(5/2)*(6*A + 5*C + C*cos[2*(c + d*x)])*sin[c + d*x])/(6*d*cos[c + d*x]^(5/2))

Maple [A] time = 0.261, size = 47, normalized size = 0.6

$$\frac{(C(\cos(dx+c))^2 + 3A + 2C)\sin(dx+c)}{3d} (b\cos(dx+c))^{\frac{5}{2}} (\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)

[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)

Maxima [A] time = 2.14171, size = 86, normalized size = 1.08

$$\frac{12Ab^{\frac{5}{2}}\sin(dx+c) + \left(b^2\sin(3dx+3c) + 9b^2\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c), \cos(3dx+3c))\right)\right)C\sqrt{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/12*(12*A*b^(5/2)*sin(d*x + c) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d

Fricas [A] time = 1.35093, size = 139, normalized size = 1.74

$$\frac{(Cb^2\cos(dx+c)^2 + (3A + 2C)b^2)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] 1/3*(C*b^2*cos(d*x + c)^2 + (3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2),x)

$$3.110 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=99

$$\frac{Ab^2x\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0262921, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2635, 8}

$$\frac{Ab^2x\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx))}{\cos^2(c + dx)} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{(b^2 C \sqrt{b \cos(c + dx)}) \int \cos^2(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 C \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d} + \frac{(b^2 C \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)})}{2d} \\
&= \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 C x \sqrt{b \cos(c + dx)}}{2\sqrt{\cos(c + dx)}} + \frac{b^2 C \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{2d}
\end{aligned}$$

Mathematica [A] time = 0.108027, size = 52, normalized size = 0.53

$$\frac{(b \cos(c + dx))^{5/2} (2(2A + C)(c + dx) + C \sin(2(c + dx)))}{4d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] ((b*cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + C*sin[2*(c + d*x)]))/(4*d*cos[c + d*x]^(5/2))

Maple [A] time = 0.234, size = 54, normalized size = 0.6

$$\frac{C \cos(dx + c) \sin(dx + c) + 2A(dx + c) + C(dx + c)}{2d} (b \cos(dx + c))^{5/2} (\cos(dx + c))^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)

[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/cos(d*x+c)^(5/2)

Maxima [A] time = 1.97368, size = 80, normalized size = 0.81

$$\frac{8Ab^2 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + (2(dx+c)b^2 + b^2 \sin(2dx+2c))C\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] 1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d

Fricas [A] time = 1.67028, size = 475, normalized size = 4.8

$$\left[\frac{2 \sqrt{b \cos(dx + c)} C b^2 \sqrt{\cos(dx + c)} \sin(dx + c) + (2A + C) \sqrt{-bb^2} \log\left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)} \sqrt{-b} \sqrt{\cos(dx + c)}\right)}{4d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(-b)*b^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/d, 1/2*(sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/d]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x)

$$3.111 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{7/2}(c+dx)} dx$$

Optimal. Leaf size=74

$$\frac{Ab^2 \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{b^2 C \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0369129, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3014, 3770}

$$\frac{Ab^2 \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{b^2 C \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_)] + (d_)*(x_), x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{7/2}(c+dx)} dx &= \frac{(b^2 \sqrt{b \cos(c+dx)}) \int (A+C \cos^2(c+dx)) \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{b^2 C \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{(Ab^2 \sqrt{b \cos(c+dx)}) \int \sec(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ab^2 \tanh^{-1}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} + \frac{b^2 C \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0802229, size = 44, normalized size = 0.59

$$\frac{(b \cos(c + dx))^{5/2} (A \tanh^{-1}(\sin(c + dx)) + C \sin(c + dx))}{d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] ((b*cos[c + d*x])^(5/2)*(A*ArcTanh[Sin[c + d*x]] + C*sin[c + d*x]))/(d*cos[c + d*x]^(5/2))

Maple [A] time = 0.221, size = 55, normalized size = 0.7

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - \sin(dx + c) C \right) (b \cos(dx + c))^{\frac{5}{2}} (\cos(dx + c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*C)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)

Maxima [A] time = 2.08183, size = 117, normalized size = 1.58

$$\frac{2 C b^{\frac{5}{2}} \sin(dx + c) + (b^2 \log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2 \sin(dx + c) + 1) - b^2 \log(\cos(dx + c)^2 + \sin(dx + c)^2))}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/2*(2*C*b^(5/2)*sin(d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d

Fricas [A] time = 1.75235, size = 570, normalized size = 7.7

$$\left[\frac{A b^{\frac{5}{2}} \cos(dx + c) \log \left(-\frac{b \cos(dx + c)^3 - 2 \sqrt{b \cos(dx + c)} \sqrt{b \cos(dx + c)} \sin(dx + c) - 2 b \cos(dx + c)}{\cos(dx + c)^3} \right) + 2 \sqrt{b \cos(dx + c)} C b^2 \sqrt{\cos(dx + c)}}{2 d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

```
[Out] [1/2*(A*b^(5/2)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c)), -(A*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c
))/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*b^2*sqrt(co
s(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(7/2),
x)
```

$$3.112 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=65

$$\frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0334339, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 8}

$$\frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]

[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)])^(2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx &= \frac{(b^2 \sqrt{b \cos(c+dx)}) \int (A+C \cos^2(c+dx)) \sec^2(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{(b^2 C \sqrt{b \cos(c+dx)}) \int 1 dx}{\sqrt{\cos(c+dx)}} \\ &= \frac{b^2 Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.0760676, size = 45, normalized size = 0.69

$$\frac{(b \cos(c + dx))^{5/2} (A \sin(c + dx) + C dx \cos(c + dx))}{d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]

[Out] ((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(7/2))

Maple [A] time = 0.21, size = 45, normalized size = 0.7

$$\frac{C \cos(dx + c)(dx + c) + A \sin(dx + c)}{d} (b \cos(dx + c))^{\frac{5}{2}} (\cos(dx + c))^{-\frac{7}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)

[Out] 1/d*(b*cos(d*x+c))^(5/2)*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(7/2)

Maxima [A] time = 1.8294, size = 108, normalized size = 1.66

$$\frac{2 \left(C b^{\frac{5}{2}} \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) + \frac{A b^{\frac{5}{2}} \sin(2dx+2c)}{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c)+1} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 2*(C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + A*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d

Fricas [A] time = 1.6215, size = 535, normalized size = 8.23

$$\left[\frac{C \sqrt{-bb^2} \cos(dx + c)^2 \log \left(2b \cos(dx + c)^2 - 2 \sqrt{b \cos(dx + c)} \sqrt{-b \cos(dx + c)} \sin(dx + c) - b \right) + 2 \sqrt{b \cos(dx + c)} A}{2d \cos(dx + c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")

```
[Out] [1/2*(C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2), (C*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(9/2),x)
```

$$3.113 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{11/2}(c+dx)} dx$$

Optimal. Leaf size=84

$$\frac{b^2(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{5/2}(c+dx)}$$

[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)))

Rubi [A] time = 0.04534, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 3770}

$$\frac{b^2(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{5/2}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]

[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^(2)), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx))}{\cos^{11/2}(c + dx)} dx = \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}}$$

$$= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} + \frac{(b^2(A + 2C) \sqrt{b \cos(c + dx)}) \int \sec(c + dx) dx}{2\sqrt{\cos(c + dx)}}$$

$$= \frac{b^2(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)}}{2d \cos^{5/2}(c + dx)}$$

Mathematica [A] time = 0.100727, size = 59, normalized size = 0.7

$$\frac{(b \cos(c + dx))^{5/2} ((A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)) + A \sin(c + dx))}{2d \cos^{9/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]

[Out] ((b*cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x]))/(2*d*cos[c + d*x]^(9/2))

Maple [A] time = 0.27, size = 134, normalized size = 1.6

$$\frac{1}{2d} \left(-A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + A (\cos(dx + c))^2 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x)

[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(9/2)

Maxima [B] time = 2.27737, size = 1108, normalized size = 13.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="maxima")

[Out] 1/4*(2*(b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*C*sqrt(b) - (4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (b^2*cos(4*d*x +

$$4c)^2 + 4b^2 \cos(2dx + 2c)^2 + b^2 \sin(4dx + 4c)^2 + 4b^2 \sin(4dx + 4c) \sin(2dx + 2c) + 4b^2 \sin(2dx + 2c)^2 + 4b^2 \cos(2dx + 2c) + b^2 + 2(2b^2 \cos(2dx + 2c) + b^2) \cos(4dx + 4c) \log(\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))^2 + \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2 \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) + (b^2 \cos(4dx + 4c)^2 + 4b^2 \cos(2dx + 2c)^2 + b^2 \sin(4dx + 4c)^2 + 4b^2 \sin(4dx + 4c) \sin(2dx + 2c) + 4b^2 \sin(2dx + 2c)^2 + 4b^2 \cos(2dx + 2c) + b^2 + 2(2b^2 \cos(2dx + 2c) + b^2) \cos(4dx + 4c) \log(\cos(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))))^2 + \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 - 2 \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) - 4(b^2 \cos(4dx + 4c) + 2b^2 \cos(2dx + 2c) + b^2) \sin(3/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4(b^2 \cos(4dx + 4c) + 2b^2 \cos(2dx + 2c) + b^2) \sin(1/2 \arctan2(\sin(2dx + 2c), \cos(2dx + 2c))) A \sqrt{b} / (2(2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4 \cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c)^2 + 4 \cos(2dx + 2c) + 1) / d$$

Fricas [A] time = 1.66846, size = 608, normalized size = 7.24

$$\left[\frac{(A + 2C)b^{\frac{5}{2}} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b \cos(dx+c)}Ab^2\sqrt{\cos(dx+c)}}{4d \cos(dx+c)^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(11/2),x, algorithm="fricas")

[Out] [1/4*((A + 2*C)*b^(5/2)*cos(dx + c)^3*log(-(b*cos(dx + c))^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3) + 2*sqrt(b*cos(dx + c))*A*b^2*sqrt(cos(dx + c))*sin(dx + c)/(d*cos(dx + c)^3), -1/2*((A + 2*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c)^3 - sqrt(b*cos(dx + c))*A*b^2*sqrt(cos(dx + c))*sin(dx + c))/(d*cos(dx + c)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))**(5/2)*(A+C*cos(dx+c)**2)/cos(dx+c)**(11/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algo  
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2)  
, x)
```

$$3.114 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{13/2}(c+dx)} dx$$

Optimal. Leaf size=85

$$\frac{b^2(2A + 3C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d \cos^{3/2}(c + dx)} + \frac{Ab^2 \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d \cos^{7/2}(c + dx)}$$

[Out] (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^2*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0520294, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3012, 3767, 8}

$$\frac{b^2(2A + 3C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d \cos^{3/2}(c + dx)} + \frac{Ab^2 \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d \cos^{7/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2),x]

[Out] (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^2*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx))}{\cos^{13/2}(c + dx)} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{(b^2(2A + 3C) \sqrt{b \cos(c + dx)}) \int \sec^2(c + dx) dx}{3 \sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} - \frac{(b^2(2A + 3C) \sqrt{b \cos(c + dx)}) \text{Subst}}{3d \sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{b^2(2A + 3C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{3/2}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.217753, size = 51, normalized size = 0.6

$$\frac{\sin(c + dx)(b \cos(c + dx))^{5/2} (A \tan^2(c + dx) + 3(A + C))}{3d \cos^{7/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(13/2), x]

[Out] ((b*cos[c + d*x])^(5/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*cos[c + d*x]^(7/2))

Maple [A] time = 0.42, size = 54, normalized size = 0.6

$$\frac{(2A(\cos(dx + c))^2 + 3C(\cos(dx + c))^2 + A)\sin(dx + c)}{3d} (b \cos(dx + c))^{5/2} (\cos(dx + c))^{-11/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2), x)

[Out] 1/3/d*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)*sin(d*x+c)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2)

Maxima [B] time = 2.12351, size = 495, normalized size = 5.82

$$2 \left(\frac{3Cb^{5/2} \sin(2dx+2c)}{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2 \cos(2dx+2c) + 1} - \frac{2(3b^2 \cos(6dx+6c) \sin(2dx+2c))}{2(3 \cos(4dx+4c) + 3 \cos(2dx+2c) + 1) \cos(6dx+6c) + \cos(6dx+6c)^2 + 6(3 \cos(2dx+2c) + 1) \cos(4dx+4c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2), x, algorithm="maxima")

[Out] 2/3*(3*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) - 2*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*

$$b^2 \cos(4dx + 4c) \sin(2dx + 2c) - (3b^2 \cos(2dx + 2c) + b^2) \sin(6dx + 6c) - 3(3b^2 \cos(2dx + 2c) + b^2) \sin(4dx + 4c) \sqrt{b} / (2(3 \cos(4dx + 4c) + 3 \cos(2dx + 2c) + 1) \cos(6dx + 6c) + \cos(6dx + 6c)^2 + 6(3 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + 9 \cos(4dx + 4c)^2 + 9 \cos(2dx + 2c)^2 + 6(\sin(4dx + 4c) + \sin(2dx + 2c)) \sin(6dx + 6c) + \sin(6dx + 6c)^2 + 9 \sin(4dx + 4c)^2 + 18 \sin(4dx + 4c) \sin(2dx + 2c) + 9 \sin(2dx + 2c)^2 + 6 \cos(2dx + 2c) + 1) / d$$

Fricas [A] time = 1.44071, size = 139, normalized size = 1.64

$$\frac{((2A + 3C)b^2 \cos(dx + c)^2 + Ab^2) \sqrt{b \cos(dx + c)} \sin(dx + c)}{3d \cos(dx + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(13/2),x, algorithm="fricas")

[Out] 1/3*((2*A + 3*C)*b^2*cos(dx + c)^2 + A*b^2)*sqrt(b*cos(dx + c))*sin(dx + c)/(d*cos(dx + c)^(7/2))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))**(5/2)*(A+C*cos(dx+c)**2)/cos(dx+c)**(13/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+C*cos(dx+c)^2)/cos(dx+c)^(13/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + A)*(b*cos(dx + c))^(5/2)/cos(dx + c)^(13/2), x)

$$3.115 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+C \cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)} dx$$

Optimal. Leaf size=131

$$\frac{b^2(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))

Rubi [A] time = 0.0636691, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3012, 3768, 3770}

$$\frac{b^2(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2), x]

[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2))

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{5/2} (A + C \cos^2(c + dx))}{\cos^{15/2}(c + dx)} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{(b^2(3A + 4C) \sqrt{b \cos(c + dx)}) \int \sec^3(c + dx) dx}{4 \sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{b^2(3A + 4C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d \cos^{5/2}(c + dx)} \\
&= \frac{b^2(3A + 4C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{8d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.215726, size = 80, normalized size = 0.61

$$\frac{(b \cos(c + dx))^{5/2} (\sin(c + dx) ((3A + 4C) \cos^2(c + dx) + 2A) + (3A + 4C) \cos^4(c + dx) \tanh^{-1}(\sin(c + dx)))}{8d \cos^{13/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(15/2), x]

[Out] ((b*cos[c + d*x])^(5/2)*((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x]))/(8*d*Cos[c + d*x]^(13/2))

Maple [A] time = 0.231, size = 214, normalized size = 1.6

$$-\frac{1}{8d} \left(3A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) - 3A (\cos(dx + c))^4 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2), x)

[Out] -1/8/d*(3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*A*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*A*sin(d*x+c)*cos(d*x+c)^2-4*C*sin(d*x+c)*cos(d*x+c)^2-2*A*sin(d*x+c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2)

Maxima [B] time = 2.69141, size = 3594, normalized size = 27.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2), x, algorithm="maxima")

```
[Out] -1/16*((12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1) + 4*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*cos(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2
```

$$2*c) + b^2 + 2*(2*b^2*\cos(2*d*x + 2*c) + b^2)*\cos(4*d*x + 4*c))*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - 4*(b^2*\cos(4*d*x + 4*c) + 2*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(b^2*\cos(4*d*x + 4*c) + 2*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*C*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1))/d$$

Fricas [A] time = 1.73427, size = 714, normalized size = 5.45

$$\left[\frac{(3A + 4C)b^{\frac{5}{2}} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2((3A + 4C)b^2 \cos(dx + c)^2)}{16d \cos(dx + c)^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2), x, algorithm="fricas")

[Out] [1/16*((3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*b^2*cos(d*x + c)^2 + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(15/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{15}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2), x)

$$3.116 \quad \int \frac{\cos^5(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=113

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b \cos(c+dx)}} + \frac{(4A+3C) \sin(c+dx) \cos^3(c+dx)}{8d\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^7(c+dx)}{4d\sqrt{b \cos(c+dx)}}$$

[Out] $((4*A + 3*C)*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(8*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + ((4*A + 3*C)*\text{Cos}[c + d*x]^{3/2}*\text{Sin}[c + d*x])/(8*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (C*\text{Cos}[c + d*x]^{7/2}*\text{Sin}[c + d*x])/(4*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0593178, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3014, 2635, 8}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b \cos(c+dx)}} + \frac{(4A+3C) \sin(c+dx) \cos^3(c+dx)}{8d\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^7(c+dx)}{4d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^{5/2}*(A + C*\text{Cos}[c + d*x]^2))/\text{Sqrt}[b*\text{Cos}[c + d*x]], x]$

[Out] $((4*A + 3*C)*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(8*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + ((4*A + 3*C)*\text{Cos}[c + d*x]^{3/2}*\text{Sin}[c + d*x])/(8*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (C*\text{Cos}[c + d*x]^{7/2}*\text{Sin}[c + d*x])/(4*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a^{(m+1/2)}*b^{(n-1/2)}*\text{Sqrt}[b*v])/ \text{Sqrt}[a*v], \text{Int}[u*v^{(m+n)}, x], x] /;$ FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

$\text{Int}[(b_.*\sin[e_.] + (f_.)*(x_))^{(m_)}*((A_.) + (C_.)*\sin[e_.] + (f_.)*(x_))^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(m+2), \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

$\text{Int}[(b_.*\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx &= \frac{\sqrt{\cos(c+dx)} \int \cos^2(c+dx)(A+C\cos^2(c+dx)) dx}{\sqrt{b\cos(c+dx)}} \\
&= \frac{C \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right) \int \cos^2(c+dx) dx}{4\sqrt{b\cos(c+dx)}} \\
&= \frac{(4A+3C) \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{C \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{4d\sqrt{b\cos(c+dx)}} + \frac{\left((4A+3C)\sqrt{\cos(c+dx)}\right) \int \cos^2(c+dx) dx}{4\sqrt{b\cos(c+dx)}} \\
&= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8\sqrt{b\cos(c+dx)}} + \frac{(4A+3C) \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{8d\sqrt{b\cos(c+dx)}} + \frac{C \cos^{\frac{7}{2}}(c+dx) \sin(c+dx)}{4d\sqrt{b\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.133435, size = 67, normalized size = 0.59

$$\frac{\sqrt{\cos(c+dx)}(4(4A+3C)(c+dx) + 8(A+C)\sin(2(c+dx)) + C\sin(4(c+dx)))}{32d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)]))/(32*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.506, size = 88, normalized size = 0.8

$$\frac{2C(\cos(dx+c))^3 \sin(dx+c) + 4A \cos(dx+c) \sin(dx+c) + 3C \cos(dx+c) \sin(dx+c) + 4A(dx+c) + 3C(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/8/d*cos(d*x+c)^(1/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.1218, size = 101, normalized size = 0.89

$$\frac{\frac{8(2dx+2c+\sin(2dx+2c))A}{\sqrt{b}} + \frac{\left(12dx+12c+\sin(4dx+4c)+8\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))\right)\right)C}{\sqrt{b}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/sqr

$t(b)/d$

Fricas [A] time = 1.80576, size = 560, normalized size = 4.96

$$\left[\frac{2 \left(2 C \cos(dx + c)^2 + 4 A + 3 C \right) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) - (4 A + 3 C) \sqrt{-b} \log \left(2 b \cos(dx + c)^2 + \right)}{16 b d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/8*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))]/(b*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{5}{2}}}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)

$$3.117 \quad \int \frac{\cos^3(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=74

$$\frac{(A+C) \sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{b \cos(c+dx)}} - \frac{C \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3d \sqrt{b \cos(c+dx)}}$$

[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0321965, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3013}

$$\frac{(A+C) \sin(c+dx) \sqrt{\cos(c+dx)}}{d \sqrt{b \cos(c+dx)}} - \frac{C \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) - (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Dist[f^(-1), Subst[Int[(1 - x^2)^(m - 1/2)*(A + C - C*x^2), x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c+dx)(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx &= \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+C \cos^2(c+dx)) dx}{\sqrt{b \cos(c+dx)}} \\ &= -\frac{\sqrt{\cos(c+dx)} \text{Subst}\left(\int (A+C-Cx^2) dx, x, -\sin(c+dx)\right)}{d \sqrt{b \cos(c+dx)}} \\ &= \frac{(A+C) \sqrt{\cos(c+dx)} \sin(c+dx)}{d \sqrt{b \cos(c+dx)}} - \frac{C \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0864753, size = 52, normalized size = 0.7

$$\frac{\sin(c+dx) \sqrt{\cos(c+dx)} (6A + C \cos(2(c+dx)) + 5C)}{6d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.394, size = 47, normalized size = 0.6

$$\frac{(C(\cos(dx+c))^2 + 3A + 2C)\sin(dx+c)}{3d} \sqrt{\cos(dx+c)} \frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)
```

```
[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(1/2)*sin(d*x+c)/(b*cos(d*x+c))^(1/2)
```

Maxima [A] time = 2.70976, size = 77, normalized size = 1.04

$$\frac{C\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan\left(\frac{\sin(3dx+3c)}{\cos(3dx+3c)}\right)\right)\right)}{\sqrt{b}} + \frac{12A\sin(dx+c)}{\sqrt{b}}$$

12d

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")
```

```
[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b) + 12*A*sin(d*x + c)/sqrt(b))/d
```

Fricas [A] time = 1.38747, size = 128, normalized size = 1.73

$$\frac{(C\cos(dx+c)^2 + 3A + 2C)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3bd\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="fricas")
```

```
[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{3}{2}}}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)`

$$3.118 \quad \int \frac{\sqrt{\cos(c+dx)}(A+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=90

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0249023, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2635, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}(A+C\cos^2(c+dx))}{\sqrt{b}\cos(c+dx)} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C\cos^2(c+dx)) dx}{\sqrt{b}\cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b}\cos(c+dx)} + \frac{(C\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{\sqrt{b}\cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b}\cos(c+dx)} + \frac{(C\sqrt{\cos(c+dx)}) \int 1 dx}{2\sqrt{b}\cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b}\cos(c+dx)} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{b}\cos(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0633324, size = 52, normalized size = 0.58

$$\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b}\cos(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.421, size = 54, normalized size = 0.6

$$\frac{C\cos(dx+c)\sin(dx+c)+2A(dx+c)+C(dx+c)}{2d}\sqrt{\cos(dx+c)}\frac{1}{\sqrt{b}\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.48046, size = 70, normalized size = 0.78

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))C}{\sqrt{b}} + \frac{8A\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b))/d

Fricas [A] time = 1.63085, size = 470, normalized size = 5.22

$$\left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c) - (2A+C)\sqrt{-b}\log\left(2b\cos(dx+c)^2 + 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\right)}{4bd} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\sqrt{\cos(dx+c)}}{\sqrt{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)

$$3.119 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=68

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}}$$

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0353761, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3014, 3770}

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_)+(d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec(c+dx) dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{C\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{(A\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{A \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0448505, size = 44, normalized size = 0.65

$$\frac{\sqrt{\cos(c+dx)} \left(A \tanh^{-1}(\sin(c+dx)) + C \sin(c+dx) \right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]), x]

[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.47, size = 55, normalized size = 0.8

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right) - \sin(dx+c) C \right) \sqrt{\cos(dx+c)} \frac{1}{\sqrt{b\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.95395, size = 108, normalized size = 1.59

$$\frac{A \left(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1) \right)}{\sqrt{b}} + \frac{2 C \sin(dx+c)}{\sqrt{b}}$$

$$2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x, algorith="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 2*C*sin(d*x + c)/sqrt(b))/d

Fricas [A] time = 1.76659, size = 559, normalized size = 8.22

$$\left[\frac{A \sqrt{b} \cos(dx+c) \log \left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3} \right) + 2 \sqrt{b} \cos(dx+c) C \sqrt{\cos(dx+c)} \sin(dx+c)}{2 b d \cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x, algorith="fricas")

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x
+ c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*s
qrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c)
)*sin(d*x + c))/(b*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate(((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))),
x)
```

$$3.120 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=59

$$\frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}}$$

[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.035275, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3012, 8}

$$\frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A + C \cos^2(c+dx)) \sec^2(c+dx) dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{(C\sqrt{\cos(c+dx)}) \int 1 dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0466439, size = 45, normalized size = 0.76

$$\frac{A \sin(c + dx) + C dx \cos(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] (C*d*x*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.452, size = 45, normalized size = 0.8

$$\frac{C \cos(dx + c)(dx + c) + A \sin(dx + c)}{d} \frac{1}{\sqrt{b \cos(dx + c)}} \frac{1}{\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.38236, size = 115, normalized size = 1.95

$$\frac{2 \left(\frac{C \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{A \sqrt{b} \sin(2dx+2c)}{b \cos(2dx+2c)^2 + b \sin(2dx+2c)^2 + 2b \cos(2dx+2c)+b} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 2*(C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + A*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b))/d

Fricas [A] time = 1.66853, size = 525, normalized size = 8.9

$$\left[\frac{C \sqrt{-b} \cos(dx + c)^2 \log\left(2b \cos(dx + c)^2 + 2\sqrt{b} \cos(dx + c) \sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b\right) - 2\sqrt{b} \cos(dx + c) A \sqrt{\cos(dx + c)}}{2bd \cos(dx + c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2), x, algorithm="fricas")

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2), (C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(3/2)),x)
```

$$3.121 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=78

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0467411, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3012, 3770}

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec^3(c+dx) dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{((A+2C)\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{2\sqrt{b \cos(c+dx)}} \\ &= \frac{(A+2C) \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.079746, size = 59, normalized size = 0.76

$$\frac{(A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)) + A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 0.534, size = 135, normalized size = 1.7

$$-\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2), x)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))-A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)

Maxima [B] time = 2.99352, size = 983, normalized size = 12.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/4*(2*C*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) - (4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*

$d*x + 2*c))))*A/((2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\sqrt{b}))/d$

Fricas [A] time = 1.76089, size = 597, normalized size = 7.65

$$\left[\frac{(A + 2C)\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b} \cos(dx+c) A \sqrt{\cos(dx+c)}}{4bd \cos(dx+c)^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/4*((A + 2*C)*sqrt(b)*cos(d*x + c)^3*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^3 - sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(5/2)), x)

$$3.122 \quad \int \frac{A+C \cos^2(c+dx)}{7 \cos^2(c+dx) \sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=79

$$\frac{(2A+3C) \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[Out] (A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0608861, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3012, 3767, 8}

$$\frac{(2A+3C) \sin(c+dx)}{3d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_)] + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((2A + 3C)\sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{3\sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} - \frac{((2A + 3C)\sqrt{\cos(c + dx)}) \text{Subst}(\int 1 dx, x, -\tan(c + dx))}{3d\sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(2A + 3C) \sin(c + dx)}{3d\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.110338, size = 51, normalized size = 0.65

$$\frac{\sin(c + dx) (A \tan^2(c + dx) + 3(A + C))}{3d\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] (Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.396, size = 54, normalized size = 0.7

$$\frac{\sin(dx + c) (2A (\cos(dx + c))^2 + 3C (\cos(dx + c))^2 + A)}{3d} \frac{1}{\sqrt{b \cos(dx + c)}} (\cos(dx + c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2)

Maxima [B] time = 2.63428, size = 479, normalized size = 6.06

$$2 \left(\frac{3C\sqrt{b} \sin(2dx+2c)}{b \cos(2dx+2c)^2 + b \sin(2dx+2c)^2 + 2b \cos(2dx+2c) + b} + \frac{2((3 \cos(2dx+2c) + 1) \cos(6dx+6c) + \cos(6dx+6c)^2 + 6(3 \cos(2dx+2c) + 1) \cos(2dx+2c) + 3)}{(2(3 \cos(4dx+4c) + 3 \cos(2dx+2c) + 1) \cos(6dx+6c) + \cos(6dx+6c)^2 + 6(3 \cos(2dx+2c) + 1) \cos(2dx+2c) + 3)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b*cos(2*d*x + 2*c)^2 + b*sin(2*d*x + 2*c)^2 + 2*b*cos(2*d*x + 2*c) + b) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6

c) + 3(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d

Fricas [A] time = 1.48752, size = 131, normalized size = 1.66

$$\frac{((2A + 3C) \cos(dx + c)^2 + A) \sqrt{b \cos(dx + c)} \sin(dx + c)}{3bd \cos(dx + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^(7/2))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x)

$$3.123 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^2(c+dx)\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=122

$$\frac{(3A+4C)\sin(c+dx)}{8d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{(3A+4C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4d \cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0701011, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3012, 3768, 3770}

$$\frac{(3A+4C)\sin(c+dx)}{8d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{(3A+4C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4d \cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx}{\sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((3A + 4C) \sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{4 \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \sin(c + dx)}{8d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((3A + 4C) \sqrt{\cos(c + dx)}) \int \sec(c + dx) dx}{4 \sqrt{b \cos(c + dx)}}$$

$$= \frac{(3A + 4C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{8d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((3A + 4C) \sqrt{\cos(c + dx)}) \int \sec(c + dx) dx}{4 \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.153037, size = 80, normalized size = 0.66

$$\frac{\sin(c + dx) ((3A + 4C) \cos^2(c + dx) + 2A) + (3A + 4C) \cos^4(c + dx) \tanh^{-1}(\sin(c + dx))}{8d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 0.422, size = 214, normalized size = 1.8

$$-\frac{1}{8d} \left(3A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) - 3A (\cos(dx + c))^4 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2), x)

[Out] -1/8/d*(3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*A*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-3*A*sin(d*x+c)*cos(d*x+c)^2-4*C*sin(d*x+c)*cos(d*x+c)^2-2*A*sin(d*x+c))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)

Maxima [B] time = 3.00823, size = 3129, normalized size = 25.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2), x, algor ithm="maxima")

[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44

```

*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x
+ 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x
+ 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*co
s(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) +
4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)
^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*
cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(
4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*
x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(
3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x +
6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) +
3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos
(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x +
2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c)
+ 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*
(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x +
8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d
*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1
)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) +
6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos
(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c
) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))*A/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
+ 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(
2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c
)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x +
2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x +
2*c) + 1)*sqrt(b) + 4*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x +
2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2
*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(
cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/((2*(2*cos(2*d*x + 2*c) +
1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*

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$$\frac{(x + 4c)^2 + 4\sin(4dx + 4c)\sin(2dx + 2c) + 4\sin(2dx + 2c)^2 + 4\cos(2dx + 2c) + 1}{\sqrt{b}} \Big/ dx$$

Fricas [A] time = 1.75996, size = 693, normalized size = 5.68

$$\left[\frac{(3A + 4C)\sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c)\sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2((3A + 4C) \cos(dx + c) + 2A) \sqrt{b} \cos(dx + c)^5}{16bd \cos(dx + c)^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt(b)*cos(d*x + c)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(9/2)), x)

$$3.124 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=122

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b \cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b \cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b \cos(c+dx)}}$$

[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0592297, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3014, 2635, 8}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b \cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8bd\sqrt{b \cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4bd\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(3/2)),x]

[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{7}{2}}(c+dx) (A+C\cos^2(c+dx))}{(b\cos(c+dx))^{\frac{3}{2}}} dx &= \frac{\sqrt{\cos(c+dx)} \int \cos^2(c+dx) (A+C\cos^2(c+dx)) dx}{b\sqrt{b}\cos(c+dx)} \\
&= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b}\cos(c+dx)} + \frac{((4A+3C)\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{4b\sqrt{b}\cos(c+dx)} \\
&= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b}\cos(c+dx)} + \frac{((4A+3C)\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{4b\sqrt{b}\cos(c+dx)} \\
&= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b\sqrt{b}\cos(c+dx)} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8bd\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4bd\sqrt{b}\cos(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.12946, size = 67, normalized size = 0.55

$$\frac{\cos^{\frac{3}{2}}(c+dx)(4(4A+3C)(c+dx)+8(A+C)\sin(2(c+dx))+C\sin(4(c+dx)))}{32d(b\cos(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*cos[c + d*x])^(3/2), x]

[Out] (Cos[c + d*x]^(3/2)*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)]))/(32*d*(b*cos[c + d*x])^(3/2))

Maple [A] time = 0.362, size = 88, normalized size = 0.7

$$\frac{2C(\cos(dx+c))^3\sin(dx+c)+4A\cos(dx+c)\sin(dx+c)+3C\cos(dx+c)\sin(dx+c)+4A(dx+c)+3C(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/8/d*cos(d*x+c)^(3/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 3.20172, size = 101, normalized size = 0.83

$$\frac{\frac{8(2dx+2c+\sin(2dx+2c))A}{b^{\frac{3}{2}}} + \frac{\left(12dx+12c+\sin(4dx+4c)+8\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))\right)\right)C}{b^{\frac{3}{2}}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(3/2) + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b^(3/2))

$3/2)/d$

Fricas [A] time = 1.76487, size = 566, normalized size = 4.64

$$\left[\frac{2 \left(2 C \cos(dx + c)^2 + 4 A + 3 C \right) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c) \sin(dx + c)} - (4 A + 3 C) \sqrt{-b} \log(2 b \cos(dx + c)^2 + 2 \sqrt{b \cos(dx + c)})}{16 b^2 d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x)

$$3.125 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{\frac{3}{2}}} dx$$

Optimal. Leaf size=80

$$\frac{(A+C) \sin(c+dx) \sqrt{\cos(c+dx)}}{bd \sqrt{b \cos(c+dx)}} - \frac{C \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3bd \sqrt{b \cos(c+dx)}}$$

[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0335879, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3013}

$$\frac{(A+C) \sin(c+dx) \sqrt{\cos(c+dx)}}{bd \sqrt{b \cos(c+dx)}} - \frac{C \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3bd \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) - (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_.)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)], x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{\frac{3}{2}}} dx &= \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+C \cos^2(c+dx)) dx}{b \sqrt{b \cos(c+dx)}} \\ &= -\frac{\sqrt{\cos(c+dx)} \text{Subst}\left(\int (A+C-Cx^2) dx, x, -\sin(c+dx)\right)}{bd \sqrt{b \cos(c+dx)}} \\ &= \frac{(A+C) \sqrt{\cos(c+dx)} \sin(c+dx)}{bd \sqrt{b \cos(c+dx)}} - \frac{C \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3bd \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0943277, size = 52, normalized size = 0.65

$$\frac{\sin(c+dx) \cos^{\frac{3}{2}}(c+dx)(6A+C \cos(2(c+dx))+5C)}{6d(b \cos(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*cos[c + d*x]^2))/(b*cos[c + d*x])^(3/2), x]

[Out] (Cos[c + d*x]^(3/2)*(6*A + 5*C + C*cos[2*(c + d*x)])*Sin[c + d*x])/(6*d*(b*cos[c + d*x])^(3/2))

Maple [A] time = 0.245, size = 47, normalized size = 0.6

$$\frac{(C(\cos(dx+c))^2 + 3A + 2C)\sin(dx+c)}{3d} (\cos(dx+c))^{\frac{3}{2}} (b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(3/2)*sin(d*x+c)/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.87933, size = 77, normalized size = 0.96

$$\frac{C\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{b^{\frac{3}{2}}} + \frac{12A\sin(dx+c)}{b^{\frac{3}{2}}}$$

12d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2) + 12*A*sin(d*x + c)/b^(3/2))/d

Fricas [A] time = 1.41197, size = 131, normalized size = 1.64

$$\frac{(C\cos(dx+c)^2 + 3A + 2C)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3b^2d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="fricas")

[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2*d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x)

$$3.126 \quad \int \frac{\cos^3(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=99

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b \cos(c+dx)}}$$

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0283984, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2635, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(3/2)),x]

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx) (A + C \cos^2(c+dx))}{(b \cos(c+dx))^{\frac{3}{2}}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A + C \cos^2(c+dx)) dx}{b\sqrt{b} \cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b} \cos(c+dx)} + \frac{(C\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{b\sqrt{b} \cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b} \cos(c+dx)} + \frac{C \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b} \cos(c+dx)} + \frac{(C\sqrt{\cos(c+dx)}) \int 1 dx}{2b\sqrt{b} \cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b} \cos(c+dx)} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b} \cos(c+dx)} + \frac{C \cos^{\frac{3}{2}}(c+dx) \sin(c+dx)}{2bd\sqrt{b} \cos(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0768901, size = 52, normalized size = 0.53

$$\frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx) + C \sin(2(c+dx)))}{4d(b \cos(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(3/2)), x]

[Out] (Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + C*Sin[2*(c + d*x)]))/(4*d*(b*Cos[c + d*x]^(3/2)))

Maple [A] time = 0.273, size = 54, normalized size = 0.6

$$\frac{C \cos(dx+c) \sin(dx+c) + 2A(dx+c) + C(dx+c)}{2d} (\cos(dx+c))^{\frac{3}{2}} (b \cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/2/d*cos(d*x+c)^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.56736, size = 70, normalized size = 0.71

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))C}{b^{\frac{3}{2}}} + \frac{8A \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2))/d

Fricas [A] time = 1.66135, size = 475, normalized size = 4.8

$$\left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c) - (2A+C)\sqrt{-b}\log\left(2b\cos(dx+c)^2 + 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\right)}{4b^2d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^2*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^2*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A) \cos(dx+c)^{\frac{3}{2}}}{(b \cos(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x)

$$3.127 \quad \int \frac{\sqrt{\cos(c+dx)}(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=74

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{bd\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}}$$

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0392348, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3014, 3770}

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{bd\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_)] + (d_)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\cos(c+dx)}(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec(c+dx) dx}{b\sqrt{b \cos(c+dx)}} \\ &= \frac{C\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} + \frac{(A\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{b\sqrt{b \cos(c+dx)}} \\ &= \frac{A \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0497833, size = 44, normalized size = 0.59

$$\frac{\cos^3(c + dx) \left(A \tanh^{-1}(\sin(c + dx)) + C \sin(c + dx) \right)}{d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]

[Out] (Cos[c + d*x]^(3/2)*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.374, size = 55, normalized size = 0.7

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - \sin(dx + c) C \right) (\cos(dx + c))^{\frac{3}{2}} (b \cos(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*C)*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.79186, size = 108, normalized size = 1.46

$$\frac{A \left(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1) \right)}{b^{\frac{3}{2}}} + \frac{2 C \sin(dx+c)}{b^{\frac{3}{2}}}$$

$$2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 2*C*sin(d*x + c)/b^(3/2))/d

Fricas [A] time = 1.71586, size = 564, normalized size = 7.62

$$\left[\frac{A \sqrt{b} \cos(dx + c) \log \left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3} \right) + 2 \sqrt{b} \cos(dx + c) C \sqrt{\cos(dx + c)} \sin(dx + c)}{2 b^2 d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d
*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{\cos(dx + c)}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2),
x)
```

$$3.128 \quad \int \frac{A + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx$$

Optimal. Leaf size=65

$$\frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{Cx \sqrt{\cos(c + dx)}}{b \sqrt{b \cos(c + dx)}}$$

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0332848, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3012, 8}

$$\frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{Cx \sqrt{\cos(c + dx)}}{b \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^2(c + dx) dx}{b \sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{(C \sqrt{\cos(c + dx)}) \int 1 dx}{b \sqrt{b \cos(c + dx)}} \\ &= \frac{Cx \sqrt{\cos(c + dx)}}{b \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0554367, size = 45, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}(A \sin(c+dx) + Cdx \cos(c+dx))}{d(b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.414, size = 45, normalized size = 0.7

$$\frac{C \cos(dx+c)(dx+c) + A \sin(dx+c)}{d} \sqrt{\cos(dx+c)} (b \cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x)

[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.91733, size = 126, normalized size = 1.94

$$\frac{2 \left(\frac{A\sqrt{b} \sin(2dx+2c)}{b^2 \cos(2dx+2c)^2 + b^2 \sin(2dx+2c)^2 + 2b^2 \cos(2dx+2c) + b^2} + \frac{C \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] 2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2))/d

Fricas [A] time = 1.67772, size = 531, normalized size = 8.17

$$\left[\frac{C\sqrt{-b} \cos(dx+c)^2 \log\left(2b \cos(dx+c)^2 + 2\sqrt{b \cos(dx+c)}\sqrt{-b \cos(dx+c)} \sin(dx+c) - b\right) - 2\sqrt{b \cos(dx+c)}}{2b^2d \cos(dx+c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2), x, algorithm="fricas")

```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^2), (C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*sqrt(cos(d*x + c))), x)
```


$$3.129 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=84

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b} \cos(c+dx)} + \frac{A \sin(c+dx)}{2bd \cos^2(c+dx)\sqrt{b} \cos(c+dx)}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0447938, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3012, 3770}

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b} \cos(c+dx)} + \frac{A \sin(c+dx)}{2bd \cos^2(c+dx)\sqrt{b} \cos(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{3/2}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec^3(c+dx) dx}{b\sqrt{b} \cos(c+dx)} \\ &= \frac{A \sin(c+dx)}{2bd \cos^2(c+dx)\sqrt{b} \cos(c+dx)} + \frac{((A+2C)\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{2b\sqrt{b} \cos(c+dx)} \\ &= \frac{(A+2C) \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2bd\sqrt{b} \cos(c+dx)} + \frac{A \sin(c+dx)}{2bd \cos^2(c+dx)\sqrt{b} \cos(c+dx)} \end{aligned}$$

Mathematica [A] time = 0.071708, size = 59, normalized size = 0.7

$$\frac{(A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)) + A \sin(c + dx)}{2d\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)),x]
```

```
[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.345, size = 134, normalized size = 1.6

$$\frac{1}{2d} \left(-A (\cos(dx + c))^2 \ln \left(\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + A (\cos(dx + c))^2 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right) - 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/2/d*(-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+A*cos(d*x+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+A*sin(d*x+c)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2))
```

Maxima [B] time = 2.79174, size = 994, normalized size = 11.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorith="maxima")
```

```
[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*co
```

$$\frac{(2dx + 2c + b)\sqrt{b} - 2C(\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\sin(dx + c) + 1) - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2\sin(dx + c) + 1))/b^{3/2}}{d}$$

Fricas [A] time = 1.63159, size = 602, normalized size = 7.17

$$\frac{(A + 2C)\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b \cos(dx+c)} A \sqrt{\cos(dx+c)}}{4b^2 d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)/cos(dx+c)^(3/2)/(b*cos(dx+c))^(3/2),x, algorithm="fricas")

[Out] [1/4*((A + 2*C)*sqrt(b)*cos(dx + c)^3*log(-(b*cos(dx + c))^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3) + 2*sqrt(b*cos(dx + c))*A*sqrt(cos(dx + c))*sin(dx + c))/(b^2*d*cos(dx + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c)^3 - sqrt(b*cos(dx + c))*A*sqrt(cos(dx + c))*sin(dx + c))/(b^2*d*cos(dx + c)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)**2)/cos(dx+c)**(3/2)/(b*cos(dx+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)/cos(dx+c)^(3/2)/(b*cos(dx+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + A)/((b*cos(dx + c))^(3/2)*cos(dx + c)^(3/2)), x)

$$3.130 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=85

$$\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

[Out] (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0539444, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3012, 3767, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)),x]

[Out] (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx}{b\sqrt{b} \cos(c + dx)} \\
&= \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx)\sqrt{b} \cos(c + dx)} + \frac{((2A + 3C)\sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{3b\sqrt{b} \cos(c + dx)} \\
&= \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx)\sqrt{b} \cos(c + dx)} - \frac{((2A + 3C)\sqrt{\cos(c + dx)}) \text{Subst}(\int 1 dx, x)}{3bd\sqrt{b} \cos(c + dx)} \\
&= \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx)\sqrt{b} \cos(c + dx)} + \frac{(2A + 3C) \sin(c + dx)}{3bd\sqrt{\cos(c + dx)}\sqrt{b} \cos(c + dx)}
\end{aligned}$$

Mathematica [A] time = 0.128873, size = 51, normalized size = 0.6

$$\frac{\sin(c + dx)\sqrt{\cos(c + dx)}(A \tan^2(c + dx) + 3(A + C))}{3d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.276, size = 54, normalized size = 0.6

$$\frac{\sin(dx + c)(2A(\cos(dx + c))^2 + 3C(\cos(dx + c))^2 + A)}{3d} (b \cos(dx + c))^{-\frac{3}{2}} (\cos(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)

Maxima [B] time = 3.02972, size = 513, normalized size = 6.04

$$2 \left(\frac{3C\sqrt{b}\sin(2dx+2c)}{b^2\cos(2dx+2c)^2+b^2\sin(2dx+2c)^2+2b^2\cos(2dx+2c)+b^2} + \frac{2}{(b\cos(6dx+6c)^2+9b\cos(4dx+4c)^2+9b\cos(2dx+2c)^2+b\sin(6dx+6c)^2+9b\sin(4dx+4c)^2+b\sin(2dx+2c)^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6

```
*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x
+ 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x +
6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) +
b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*c
os(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c) + b)*sqrt(b))/d
```

Fricas [A] time = 1.39655, size = 134, normalized size = 1.58

$$\frac{\left((2A + 3C) \cos(dx + c)^2 + A\right) \sqrt{b \cos(dx + c)} \sin(dx + c)}{3b^2 d \cos(dx + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^2
*d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)
), x)
```

$$3.131 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=131

$$\frac{(3A+4C)\sin(c+dx)}{8bd \cos^3(c+dx)\sqrt{b \cos(c+dx)}} + \frac{(3A+4C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4bd \cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])]
```

Rubi [A] time = 0.0673471, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3012, 3768, 3770}

$$\frac{(3A+4C)\sin(c+dx)}{8bd \cos^3(c+dx)\sqrt{b \cos(c+dx)}} + \frac{(3A+4C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8bd\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4bd \cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)), x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*b*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])]
```

Rule 18

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx}{b \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{((3A + 4C) \sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{4b \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{(3A + 4C) \sin(c + dx)}{8bd \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{((3A + 4C) \tan^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)})}{8bd \sqrt{b} \cos(c + dx)} + \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)}$$

Mathematica [A] time = 0.156471, size = 80, normalized size = 0.61

$$\frac{\sin(c + dx) \left((3A + 4C) \cos^2(c + dx) + 2A \right) + (3A + 4C) \cos^4(c + dx) \tanh^{-1}(\sin(c + dx))}{8d \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)),x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.283, size = 214, normalized size = 1.6

$$\frac{1}{8d} \left(-3A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + 3A (\cos(dx + c))^4 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x)

[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*sin(d*x+c)*cos(d*x+c)^2+2*A*sin(d*x+c))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2)

Maxima [B] time = 3.0548, size = 3173, normalized size = 24.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44

$$\begin{aligned}
& *(\sin(8*d*x + 8*c) + 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x \\
& + 2*c))*\cos(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\sin(8*d*x \\
& + 8*c) + 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x + 2*c))*\cos \\
& (3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 12*(\sin(8*d*x + 8*c) + \\
& 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x + 2*c))*\cos(1/2*\arct \\
& an2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 3*(2*(4*\cos(6*d*x + 6*c) + 6*\cos \\
& (4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c) \\
& ^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16* \\
& \cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(\\
& 4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d* \\
& x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(\\
& 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + \\
& 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16* \\
& \sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x \\
& + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\
& + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) + \\
& 3*(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos \\
& (8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + \\
& 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) \\
& + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4* \\
& (2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + \\
& 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin \\
& (6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d* \\
& *x + 4*c)*\sin(2*d*x + 2*c) + 16*\sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1 \\
&)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arct \\
& an2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + \\
& 2*c), \cos(2*d*x + 2*c)))) + 1) - 12*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + \\
& 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(7/2*\arctan2(\sin(2*d*x + 2 \\
& *c), \cos(2*d*x + 2*c))) - 44*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos \\
& (4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), \cos \\
& (2*d*x + 2*c))) + 44*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x \\
& + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c))) + 12*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) \\
&) + 4*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\
& *c))))*A/((b*\cos(8*d*x + 8*c)^2 + 16*b*\cos(6*d*x + 6*c)^2 + 36*b*\cos(4*d*x \\
& + 4*c)^2 + 16*b*\cos(2*d*x + 2*c)^2 + b*\sin(8*d*x + 8*c)^2 + 16*b*\sin(6*d*x \\
& + 6*c)^2 + 36*b*\sin(4*d*x + 4*c)^2 + 48*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) \\
& + 16*b*\sin(2*d*x + 2*c)^2 + 2*(4*b*\cos(6*d*x + 6*c) + 6*b*\cos(4*d*x + 4*c) \\
& + 4*b*\cos(2*d*x + 2*c) + b)*\cos(8*d*x + 8*c) + 8*(6*b*\cos(4*d*x + 4*c) + 4 \\
& *b*\cos(2*d*x + 2*c) + b)*\cos(6*d*x + 6*c) + 12*(4*b*\cos(2*d*x + 2*c) + b)*\cos \\
& (4*d*x + 4*c) + 8*b*\cos(2*d*x + 2*c) + 4*(2*b*\sin(6*d*x + 6*c) + 3*b*\sin(\\
& 4*d*x + 4*c) + 2*b*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*b*\sin(4*d*x + \\
& 4*c) + 2*b*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + b)*\sqrt{b}) + 4*(4*(\sin(4* \\
& d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d* \\
& x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(\\
& 2*d*x + 2*c), \cos(2*d*x + 2*c)))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + \\
& 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*s \\
& in(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) \\
&) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2 \\
& *\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d \\
& *x + 2*c), \cos(2*d*x + 2*c)))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x \\
& + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4 \\
& *\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2 \\
& *c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1 \\
& /2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2 \\
& *d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + \\
& 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d \\
& *x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2 \\
& *d*x + 2*c))))*C/((b*\cos(4*d*x + 4*c)^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*
\end{aligned}$$

$$\frac{d^2x + 4c)^2 + 4b \sin(4dx + 4c) \sin(2dx + 2c) + 4b \sin(2dx + 2c)^2 + 2(2b \cos(2dx + 2c) + b) \cos(4dx + 4c) + 4b \cos(2dx + 2c) + b \sqrt{b}}{d}$$

Fricas [A] time = 2.01903, size = 698, normalized size = 5.33

$$\frac{\left((3A + 4C) \sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3} \right) + 2 \left((3A + 4C) \cos(dx + c)^2 - \dots \right) \right)}{16 b^2 d \cos(dx + c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt(b)*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3 + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate(((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(7/2))), x)

$$3.132 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=122

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b \cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b \cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b \cos(c+dx)}}$$

[Out] $((4*A + 3*C)*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(8*b^2*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + ((4*A + 3*C)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(8*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (C*\text{Cos}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x])/(4*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.0642664, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {17, 3014, 2635, 8}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b \cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b \cos(c+dx)}} + \frac{C\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{4b^2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c + d*x]^{(9/2)}*(A + C*\text{Cos}[c + d*x]^2))/(b*\text{Cos}[c + d*x]^{(5/2)}), x]$

[Out] $((4*A + 3*C)*x*\text{Sqrt}[\text{Cos}[c + d*x]])/(8*b^2*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + ((4*A + 3*C)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sin}[c + d*x])/(8*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (C*\text{Cos}[c + d*x]^{(7/2)}*\text{Sin}[c + d*x])/(4*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 17

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a^{(m+1/2)}*b^{(n-1/2)}*\text{Sqrt}[b*v])/ \text{Sqrt}[a*v], \text{Int}[u*v^{(m+n)}, x], x] /;$ FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((A_.) + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(m+2), \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2635

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(b*\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n-1)})/(d*n), x] + \text{Dist}[(b^2*(n-1))/n, \text{Int}[(b*\text{Sin}[c + d*x])^{(n-2)}, x], x] /;$ FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

$\text{Int}[a_, x_Symbol] \rightarrow \text{Simp}[a*x, x] /;$ FreeQ[a, x]

Rubi steps

$$\int \frac{\cos^{\frac{9}{2}}(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{\sqrt{\cos(c+dx)} \int \cos^2(c+dx)(A+C\cos^2(c+dx)) dx}{b^2\sqrt{b}\cos(c+dx)}$$

$$= \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b}\cos(c+dx)} + \frac{((4A+3C)\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{4b^2\sqrt{b}\cos(c+dx)}$$

$$= \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b}\cos(c+dx)} + \frac{((4A+3C)\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{4b^2\sqrt{b}\cos(c+dx)}$$

$$= \frac{(4A+3C)x\sqrt{\cos(c+dx)}}{8b^2\sqrt{b}\cos(c+dx)} + \frac{(4A+3C)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8b^2d\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4b^2d\sqrt{b}\cos(c+dx)}$$

Mathematica [A] time = 0.101435, size = 70, normalized size = 0.57

$$\frac{\sqrt{\cos(c+dx)}(4(4A+3C)(c+dx) + 8(A+C)\sin(2(c+dx)) + C\sin(4(c+dx)))}{32b^2d\sqrt{b}\cos(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(9/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(4*(4*A + 3*C)*(c + d*x) + 8*(A + C)*Sin[2*(c + d*x)] + C*Sin[4*(c + d*x)]))/(32*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.353, size = 88, normalized size = 0.7

$$\frac{2C(\cos(dx+c))^3\sin(dx+c) + 4A\cos(dx+c)\sin(dx+c) + 3C\cos(dx+c)\sin(dx+c) + 4A(dx+c) + 3C(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] 1/8/d*cos(d*x+c)^(5/2)*(2*C*cos(d*x+c)^3*sin(d*x+c)+4*A*cos(d*x+c)*sin(d*x+c)+3*C*cos(d*x+c)*sin(d*x+c)+4*A*(d*x+c)+3*C*(d*x+c))/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 3.01395, size = 101, normalized size = 0.83

$$\frac{\frac{8(2dx+2c+\sin(2dx+2c))A}{b^{\frac{5}{2}}} + \frac{(12dx+12c+\sin(4dx+4c)+8\sin(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))))C}{b^{\frac{5}{2}}}}{32d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 1/32*(8*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/b^(5/2) + (12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/b^(5/2))

5/2))/d

Fricas [A] time = 2.05693, size = 566, normalized size = 4.64

$$\left[\frac{2 \left(2 C \cos(dx + c)^2 + 4 A + 3 C \right) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) - (4 A + 3 C) \sqrt{-b} \log(2 b \cos(dx + c)^2 + 2 \sqrt{b \cos(dx + c)} \sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b)}{16 b^3 d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/16*(2*(2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (4*A + 3*C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/8*((2*C*cos(d*x + c)^2 + 4*A + 3*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(9/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{9}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(9/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)

$$3.133 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=80

$$\frac{(A+C) \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} - \frac{C \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0328293, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {17, 3013}

$$\frac{(A+C) \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} - \frac{C \sin^3(c+dx) \sqrt{\cos(c+dx)}}{3b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] ((A + C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) - (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3013

Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)], x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]

Rubi steps

$$\begin{aligned} \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx &= \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+C \cos^2(c+dx)) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= -\frac{\sqrt{\cos(c+dx)} \text{Subst}\left(\int (A+C-Cx^2) dx, x, -\sin(c+dx)\right)}{b^2 d \sqrt{b \cos(c+dx)}} \\ &= \frac{(A+C) \sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} - \frac{C \sqrt{\cos(c+dx)} \sin^3(c+dx)}{3b^2 d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0619518, size = 55, normalized size = 0.69

$$\frac{\sin(c+dx) \sqrt{\cos(c+dx)} (6A + C \cos(2(c+dx)) + 5C)}{6b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(7/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(6*A + 5*C + C*Cos[2*(c + d*x)])*Sin[c + d*x])/(6*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.256, size = 47, normalized size = 0.6

$$\frac{(C(\cos(dx+c))^2 + 3A + 2C)\sin(dx+c)}{3d} (\cos(dx+c))^{\frac{5}{2}} (b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] 1/3/d*(C*cos(d*x+c)^2+3*A+2*C)*cos(d*x+c)^(5/2)*sin(d*x+c)/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.69352, size = 77, normalized size = 0.96

$$\frac{C\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{b^{\frac{5}{2}}} + \frac{12A\sin(dx+c)}{b^{\frac{5}{2}}}$$

12d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 1/12*(C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2) + 12*A*sin(d*x + c)/b^(5/2))/d

Fricas [A] time = 1.64481, size = 131, normalized size = 1.64

$$\frac{(C\cos(dx+c)^2 + 3A + 2C)\sqrt{b\cos(dx+c)}\sin(dx+c)}{3b^3d\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="fricas")

[Out] 1/3*(C*cos(d*x + c)^2 + 3*A + 2*C)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3*d*sqrt(cos(d*x + c)))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x)

$$3.134 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=99

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b \cos(c+dx)}}$$

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0268343, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 2635, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C\cos^2(c+dx)) dx}{b^2\sqrt{b}\cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b}\cos(c+dx)} + \frac{(C\sqrt{\cos(c+dx)}) \int \cos^2(c+dx) dx}{b^2\sqrt{b}\cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b}\cos(c+dx)} + \frac{(C\sqrt{\cos(c+dx)}) \int 1 dx}{2b^2\sqrt{b}\cos(c+dx)} \\
&= \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b}\cos(c+dx)} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b}\cos(c+dx)} + \frac{C\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b^2d\sqrt{b}\cos(c+dx)}
\end{aligned}$$

Mathematica [A] time = 0.0583793, size = 55, normalized size = 0.56

$$\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+C\sin(2(c+dx)))}{4b^2d\sqrt{b}\cos(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + C*SIn[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.279, size = 54, normalized size = 0.6

$$\frac{C \cos(dx+c) \sin(dx+c) + 2A(dx+c) + C(dx+c)}{2d} (\cos(dx+c))^{\frac{5}{2}} (b \cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] 1/2/d*cos(d*x+c)^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.45914, size = 70, normalized size = 0.71

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))C}{b^{\frac{5}{2}}} + \frac{8A \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2))/d

Fricas [A] time = 1.98551, size = 475, normalized size = 4.8

$$\left[\frac{2\sqrt{b\cos(dx+c)}C\sqrt{\cos(dx+c)}\sin(dx+c) - (2A+C)\sqrt{-b}\log\left(2b\cos(dx+c)^2 + 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\right)}{4b^3d} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) - (2*A + C)*sqrt(-b)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b))/(b^3*d), 1/2*(sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c) + (2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2))))/(b^3*d)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A) \cos(dx+c)^{\frac{5}{2}}}{(b \cos(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x)

$$3.135 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=74

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0345567, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3014, 3770}

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)),x]

[Out] (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 3770

Int[csc[(c_)+(d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{C\sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{(A\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{A \tanh^{-1}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)} \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0447991, size = 47, normalized size = 0.64

$$\frac{\sqrt{\cos(c+dx)} \left(A \tanh^{-1}(\sin(c+dx)) + C \sin(c+dx) \right)}{b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]

[Out] (Sqrt[Cos[c + d*x]]*(A*ArcTanh[Sin[c + d*x]] + C*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.345, size = 55, normalized size = 0.7

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right) - \sin(dx+c) C \right) (\cos(dx+c))^{\frac{5}{2}} (b \cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-sin(d*x+c)*C)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.84041, size = 108, normalized size = 1.46

$$\frac{A \left(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1) \right)}{b^2} + \frac{2 C \sin(dx+c)}{b^2}$$

$$2 d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 2*C*sin(d*x + c)/b^(5/2))/d

Fricas [A] time = 1.77383, size = 564, normalized size = 7.62

$$\left[\frac{A \sqrt{b} \cos(dx+c) \log \left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3} \right) + 2 \sqrt{b \cos(dx+c)} C \sqrt{\cos(dx+c)} \sin(dx+c)}{2 b^3 d \cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

```
[Out] [1/2*(A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))
)*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^
3) + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d
*x + c)), -(A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b
*sqrt(cos(d*x + c))))*cos(d*x + c) - sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x +
c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(5/2),
x)
```

$$3.136 \quad \int \frac{\sqrt{\cos(c+dx)}(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=65

$$\frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{Cx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}}$$

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0326107, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {17, 3012, 8}

$$\frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{Cx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{\sqrt{\cos(c+dx)}(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec^2(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{(C \sqrt{\cos(c+dx)}) \int 1 dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{Cx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0554198, size = 45, normalized size = 0.69

$$\frac{\cos^3(c + dx)(A \sin(c + dx) + C dx \cos(c + dx))}{d(b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]

[Out] (Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.417, size = 45, normalized size = 0.7

$$\frac{C \cos(dx + c)(dx + c) + A \sin(dx + c)}{d} (\cos(dx + c))^{\frac{3}{2}} (b \cos(dx + c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)

[Out] 1/d*(C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.54233, size = 126, normalized size = 1.94

$$\frac{2 \left(\frac{A \sqrt{b} \sin(2 dx + 2 c)}{b^3 \cos(2 dx + 2 c)^2 + b^3 \sin(2 dx + 2 c)^2 + 2 b^3 \cos(2 dx + 2 c) + b^3} + \frac{C \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}} \right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 2*(A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + C*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2))/d

Fricas [A] time = 1.64077, size = 531, normalized size = 8.17

$$\left[\frac{C \sqrt{-b} \cos(dx + c)^2 \log(2 b \cos(dx + c)^2 + 2 \sqrt{b} \cos(dx + c) \sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b) - 2 \sqrt{b} \cos(dx + c) A \sqrt{-b}}{2 b^3 d \cos(dx + c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")


```
[Out] [-1/2*(C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^2), (C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sqrt{\cos(dx + c)}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(5/2), x)
```

$$3.137 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=84

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.041872, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {18, 3012, 3770}

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Ssin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx &= \frac{\sqrt{\cos(c+dx)} \int (A+C \cos^2(c+dx)) \sec^3(c+dx) dx}{b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{A \sin(c+dx)}{2b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{((A+2C)\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{2b^2 \sqrt{b \cos(c+dx)}} \\ &= \frac{(A+2C) \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{2b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.0737511, size = 59, normalized size = 0.7

$$\frac{\sqrt{\cos(c+dx)} \left((A+2C) \cos^2(c+dx) \tanh^{-1}(\sin(c+dx)) + A \sin(c+dx) \right)}{2d(b \cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]

[Out] (Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + A*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.392, size = 135, normalized size = 1.6

$$-\frac{1}{2d} \left(A (\cos(dx+c))^2 \ln \left(-\frac{-1 + \cos(dx+c) + \sin(dx+c)}{\sin(dx+c)} \right) - A (\cos(dx+c))^2 \ln \left(\frac{1 - \cos(dx+c) + \sin(dx+c)}{\sin(dx+c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))-A*sin(d*x+c)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2))

Maxima [B] time = 2.92449, size = 1018, normalized size = 12.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] -1/4*((4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(

$$2dx + 2c) + b^2) \cos(4dx + 4c) \sqrt{b}) - 2C(\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2\sin(dx + c) + 1) - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2\sin(dx + c) + 1))/b^{5/2})/d$$

Fricas [A] time = 1.66281, size = 602, normalized size = 7.17

$$\frac{(A + 2C)\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2\sqrt{b} \cos(dx + c) A \sqrt{\cos(dx + c)}}{4b^3 d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)/(b*cos(dx+c))^(5/2)/cos(dx+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*((A + 2*C)*sqrt(b)*cos(dx + c)^3*log(-(b*cos(dx + c)^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3) + 2*sqrt(b*cos(dx + c))*A*sqrt(cos(dx + c))*sin(dx + c))/(b^3*d*cos(dx + c)^3), -1/2*((A + 2*C)*sqrt(-b)*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c)^3 - sqrt(b*cos(dx + c))*A*sqrt(cos(dx + c))*sin(dx + c))/(b^3*d*cos(dx + c)^3)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)**2)/(b*cos(dx+c))**(5/2)/cos(dx+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(dx+c)^2)/(b*cos(dx+c))^(5/2)/cos(dx+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + A)/((b*cos(dx + c))^(5/2)*sqrt(cos(dx + c))), x)

$$3.138 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=85

$$\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

[Out] (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0469938, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3012, 3767, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3b^2d\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)), x]

[Out] (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3767

Int[csc[(c_)] + (d_)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^4(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((2A + 3C)\sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{3b^2 \sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} - \frac{((2A + 3C)\sqrt{\cos(c + dx)}) \text{Subst}(\int 1 dx, x, -)}{3b^2 d \sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(2A + 3C) \sin(c + dx)}{3b^2 d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.111845, size = 51, normalized size = 0.6

$$\frac{\sin(c + dx) \cos^{\frac{3}{2}}(c + dx) (A \tan^2(c + dx) + 3(A + C))}{3d(b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)), x]

[Out] (Cos[c + d*x]^(3/2)*Sin[c + d*x]*(3*(A + C) + A*Tan[c + d*x]^2))/(3*d*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.256, size = 54, normalized size = 0.6

$$\frac{\sin(dx + c) (2A (\cos(dx + c))^2 + 3C (\cos(dx + c))^2 + A)}{3d} (b \cos(dx + c))^{-\frac{5}{2}} \frac{1}{\sqrt{\cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2), x)

[Out] 1/3/d*sin(d*x+c)*(2*A*cos(d*x+c)^2+3*C*cos(d*x+c)^2+A)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2)

Maxima [B] time = 2.93057, size = 556, normalized size = 6.54

$$2 \left(\frac{3C\sqrt{b}\sin(2dx+2c)}{b^3 \cos(2dx+2c)^2 + b^3 \sin(2dx+2c)^2 + 2b^3 \cos(2dx+2c) + b^3} + \frac{1}{(b^2 \cos(6dx+6c)^2 + 9b^2 \cos(4dx+4c)^2 + 9b^2 \cos(2dx+2c)^2 + b^2 \sin(6dx+6c)^2 + 9b^2 \sin(4dx+4c)^2 + 9b^2 \sin(2dx+2c)^2 + b^2)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 2/3*(3*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 2*((3*cos(2*d*x + 2*c) + 1)*sin(6

```
*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d
*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin
(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3
*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*
(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) +
b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b))/d
```

Fricas [A] time = 1.34058, size = 134, normalized size = 1.58

$$\frac{\left((2A + 3C) \cos(dx + c)^2 + A \right) \sqrt{b \cos(dx + c)} \sin(dx + c)}{3b^3 d \cos(dx + c)^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="fricas")
```

```
[Out] 1/3*((2*A + 3*C)*cos(d*x + c)^2 + A)*sqrt(b*cos(d*x + c))*sin(d*x + c)/(b^3
*d*cos(d*x + c)^(7/2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algo
rithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)
), x)
```

$$3.139 \quad \int \frac{A+C \cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=131

$$\frac{(3A+4C)\sin(c+dx)}{8b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{(3A+4C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4b^2d \cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0624157, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {18, 3012, 3768, 3770}

$$\frac{(3A+4C)\sin(c+dx)}{8b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{(3A+4C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4b^2d \cos^{\frac{7}{2}}(c+dx)\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)), x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*b^2*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + C \cos^2(c + dx)) \sec^5(c + dx) dx}{b^2 \sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((3A + 4C) \sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{4b^2 \sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \sin(c + dx)}{8b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{((3A + 4C) \tan^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)})}{8b^2 d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.121423, size = 80, normalized size = 0.61

$$\frac{\sin(c + dx) \left((3A + 4C) \cos^2(c + dx) + 2A \right) + (3A + 4C) \cos^4(c + dx) \tanh^{-1}(\sin(c + dx))}{8d \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)),x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + (2*A + (3*A + 4*C)*Cos[c + d*x]^2)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.246, size = 214, normalized size = 1.6

$$\frac{1}{8d} \left(-3A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) + 3A (\cos(dx + c))^4 \ln \left(\frac{1 - \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x)

[Out] 1/8/d*(-3*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^4*ln((1-cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+3*A*sin(d*x+c)*cos(d*x+c)^2+4*C*sin(d*x+c)*cos(d*x+c)^2+2*A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)

Maxima [B] time = 2.64667, size = 3264, normalized size = 24.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x,algor
ithm="maxima")

[Out] -1/16*((12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44

$$\begin{aligned}
& *(\sin(8*d*x + 8*c) + 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x \\
& + 2*c))*\cos(5/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 44*(\sin(8*d*x \\
& + 8*c) + 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x + 2*c))*\co \\
& s(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 12*(\sin(8*d*x + 8*c) + \\
& 4*\sin(6*d*x + 6*c) + 6*\sin(4*d*x + 4*c) + 4*\sin(2*d*x + 2*c))*\cos(1/2*\arct \\
& an2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 3*(2*(4*\cos(6*d*x + 6*c) + 6*\cos \\
& (4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(8*d*x + 8*c) + \cos(8*d*x + 8*c) \\
& ^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + 16* \\
& \cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 36*\cos(\\
& 4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4*(2*\sin(6*d*x + 6*c) + 3*\sin(4*d* \\
& x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + 8*c) + \sin(8*d*x + 8*c)^2 + 16*(\\
& 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + 16*\sin(6*d*x + \\
& 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 16* \\
& \sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x \\
& + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x \\
& + 2*c))))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + \\
& 3*(2*(4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\cos \\
& (8*d*x + 8*c) + \cos(8*d*x + 8*c)^2 + 8*(6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + \\
& 2*c) + 1)*\cos(6*d*x + 6*c) + 16*\cos(6*d*x + 6*c)^2 + 12*(4*\cos(2*d*x + 2*c) \\
& + 1)*\cos(4*d*x + 4*c) + 36*\cos(4*d*x + 4*c)^2 + 16*\cos(2*d*x + 2*c)^2 + 4* \\
& (2*\sin(6*d*x + 6*c) + 3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\sin(8*d*x + \\
& 8*c) + \sin(8*d*x + 8*c)^2 + 16*(3*\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\si \\
& n(6*d*x + 6*c) + 16*\sin(6*d*x + 6*c)^2 + 36*\sin(4*d*x + 4*c)^2 + 48*\sin(4*d \\
& *x + 4*c)*\sin(2*d*x + 2*c) + 16*\sin(2*d*x + 2*c)^2 + 8*\cos(2*d*x + 2*c) + 1 \\
&)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arct \\
& an2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + \\
& 2*c), \cos(2*d*x + 2*c))) + 1) - 12*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + \\
& 6*\cos(4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(7/2*\arctan2(\sin(2*d*x + 2 \\
& *c), \cos(2*d*x + 2*c))) - 44*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos \\
& (4*d*x + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(5/2*\arctan2(\sin(2*d*x + 2*c), c \\
& os(2*d*x + 2*c))) + 44*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x \\
& + 4*c) + 4*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d \\
& *x + 2*c))) + 12*(\cos(8*d*x + 8*c) + 4*\cos(6*d*x + 6*c) + 6*\cos(4*d*x + 4*c \\
&) + 4*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2 \\
& *c))))*A/((b^2*\cos(8*d*x + 8*c)^2 + 16*b^2*\cos(6*d*x + 6*c)^2 + 36*b^2*\cos(\\
& 4*d*x + 4*c)^2 + 16*b^2*\cos(2*d*x + 2*c)^2 + b^2*\sin(8*d*x + 8*c)^2 + 16*b^ \\
& 2*\sin(6*d*x + 6*c)^2 + 36*b^2*\sin(4*d*x + 4*c)^2 + 48*b^2*\sin(4*d*x + 4*c)* \\
& \sin(2*d*x + 2*c) + 16*b^2*\sin(2*d*x + 2*c)^2 + 8*b^2*\cos(2*d*x + 2*c) + b^2 \\
& + 2*(4*b^2*\cos(6*d*x + 6*c) + 6*b^2*\cos(4*d*x + 4*c) + 4*b^2*\cos(2*d*x + 2 \\
& *c) + b^2)*\cos(8*d*x + 8*c) + 8*(6*b^2*\cos(4*d*x + 4*c) + 4*b^2*\cos(2*d*x + \\
& 2*c) + b^2)*\cos(6*d*x + 6*c) + 12*(4*b^2*\cos(2*d*x + 2*c) + b^2)*\cos(4*d*x \\
& + 4*c) + 4*(2*b^2*\sin(6*d*x + 6*c) + 3*b^2*\sin(4*d*x + 4*c) + 2*b^2*\sin(2* \\
& d*x + 2*c))*\sin(8*d*x + 8*c) + 16*(3*b^2*\sin(4*d*x + 4*c) + 2*b^2*\sin(2*d*x \\
& + 2*c))*\sin(6*d*x + 6*c))*\sqrt{b}) + 4*(4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x \\
& + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x \\
& + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
& 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 \\
& + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x \\
& + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan \\
& 2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c) \\
& , \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2* \\
& c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c) \\
& ^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d \\
& *x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arct \\
& an2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2* \\
& c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + \\
& 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2 \\
& (\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + \\
& 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*C/((b^2*\co
\end{aligned}$$

$$\frac{\sin(4dx + 4c)^2 + 4b^2 \cos(2dx + 2c)^2 + b^2 \sin(4dx + 4c)^2 + 4b^2 \sin(4dx + 4c) \sin(2dx + 2c) + 4b^2 \sin(2dx + 2c)^2 + 4b^2 \cos(2dx + 2c) + b^2 + 2(2b^2 \cos(2dx + 2c) + b^2) \cos(4dx + 4c) \sqrt{\sin(2dx + 2c)}}{d}$$

Fricas [A] time = 1.7362, size = 698, normalized size = 5.33

$$\frac{(3A + 4C)\sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2((3A + 4C) \cos(dx + c) \sqrt{\cos(dx+c)})}{16b^3 d \cos(dx + c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [1/16*((3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt(b)*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3 + 2*((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^5), -1/8*((3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - ((3*A + 4*C)*cos(d*x + c)^2 + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(5/2)), x)

3.140 $\int \cos^2(c+dx) \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{10/3}}{13b^3d} - \frac{3(13A+10C) \sin(c+dx)(b \cos(c+dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right)}{130b^3d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^3*d) - (3*(13*A + 10*C)*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(130*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0709075, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{10/3}}{13b^3d} - \frac{3(13A+10C) \sin(c+dx)(b \cos(c+dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right)}{130b^3d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^3*d) - (3*(13*A + 10*C)*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(130*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Ssin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Ssin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Ssin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(c+dx) \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{7/3} (A + C \cos^2(c+dx)) dx}{b^2} \\ &= \frac{3C(b \cos(c+dx))^{10/3} \sin(c+dx)}{13b^3d} + \frac{(13A+10C) \int (b \cos(c+dx))^{7/3} dx}{13b^2} \\ &= \frac{3C(b \cos(c+dx))^{10/3} \sin(c+dx)}{13b^3d} - \frac{3(13A+10C)(b \cos(c+dx))^{7/3}}{13b^2} \end{aligned}$$

Mathematica [A] time = 0.111706, size = 96, normalized size = 1.01

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) \sqrt[3]{b \cos(c+dx)} \left(8A \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right) + 5C \cos^4(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{8}{3}; \frac{11}{3}; \cos^2(c+dx)\right) \right)}{80d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(8*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*d)

Maple [F] time = 0.408, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 \sqrt[3]{b \cos(dx+c)} (A + C (\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{1/3} \cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^4 + A \cos(dx+c)^2\right) (b \cos(dx+c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^2, x)
```

3.141 $\int \cos(c+dx) \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{7/3}}{10b^2d} - \frac{3(10A+7C) \sin(c+dx)(b \cos(c+dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{70b^2d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^2*d) - (3*(10*A + 7*C)*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(70*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0716823, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{7/3}}{10b^2d} - \frac{3(10A+7C) \sin(c+dx)(b \cos(c+dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{70b^2d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^2*d) - (3*(10*A + 7*C)*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(70*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos(c+dx) \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{4/3} (A + C \cos^2(c+dx)) dx}{b} \\ &= \frac{3C(b \cos(c+dx))^{7/3} \sin(c+dx)}{10b^2d} + \frac{(10A+7C) \int (b \cos(c+dx))^{4/3} dx}{10b} \\ &= \frac{3C(b \cos(c+dx))^{7/3} \sin(c+dx)}{10b^2d} - \frac{3(10A+7C)(b \cos(c+dx))^{7/3}}{70b^2} \end{aligned}$$

Mathematica [A] time = 0.166515, size = 91, normalized size = 0.96

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) (b \cos(c+dx))^{4/3} \left(13A {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right) + 7C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c+dx)\right) \right)}{91bd}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(13*A*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*b*d)

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int \cos(dx+c) \sqrt[3]{b \cos(dx+c)} (A + C (\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{1/3} \cos(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^3 + A \cos(dx+c)\right) (b \cos(dx+c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c), x)
```

3.142 $\int \sqrt[3]{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{4/3}}{7bd} - \frac{3(7A + 4C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{28bd \sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/((28*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0563284, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{4/3}}{7bd} - \frac{3(7A + 4C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{28bd \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/((28*b*d*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{1}{7}(7A + 4C) \int \sqrt[3]{b \cos(c + dx)} dx \\ &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} - \frac{3(7A + 4C)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{28bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.111655, size = 88, normalized size = 0.93

$$\frac{3\sqrt{\sin^2(c + dx)} \cot(c + dx) \sqrt[3]{b \cos(c + dx)} \left(5A {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) + 2C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(1/3)*(A + C*cos[c + d*x]^2),x]

[Out] $(-3*(b*\cos[c + d*x])^{1/3}*\cot[c + d*x]*(5*A*\text{Hypergeometric2F1}[1/2, 2/3, 5/3, \cos[c + d*x]^2] + 2*C*\cos[c + d*x]^2*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \cos[c + d*x]^2])*\sqrt{\sin[c + d*x]^2})/(20*d)$

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \cos(dx + c)} (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3), x)

3.143 $\int \sqrt[3]{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=87

$$\frac{3C \sin(c + dx) \sqrt[3]{b \cos(c + dx)}}{4d} - \frac{3(4A + C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0758734, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c + dx) \sqrt[3]{b \cos(c + dx)}}{4d} - \frac{3(4A + C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec(c+dx) dx &= b \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx \\ &= \frac{3C \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{4d} + \frac{1}{4}(b(4A + C)) \int \frac{1}{(b \cos(c+dx))} dx \\ &= \frac{3C \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{4d} - \frac{3(4A + C) \sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{4d \sqrt{\sin(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.124798, size = 88, normalized size = 1.01

$$\frac{3b \sqrt{\sin^2(c+dx)} \cot(c+dx) \left(7A {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right) + C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)\right)}{7d(b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (-3*b*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \cos(dx+c)} (A + C (\cos(dx+c))^2) \sec(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x)

[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{1/3} \sec(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^2 + A\right) (b \cos(dx+c))^{1/3} \sec(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)
```

3.144 $\int \sqrt[3]{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$

Optimal. Leaf size=91

$$\frac{3(A - 2C) \sin(c + dx) (b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8bd\sqrt{\sin^2(c + dx)}} + \frac{3Ab \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}}$$

[Out] (3*A*b*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0933749, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3(A - 2C) \sin(c + dx) (b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8bd\sqrt{\sin^2(c + dx)}} + \frac{3Ab \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (3*A*b*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_)*((A_) + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^2(c+dx) dx &= b^2 \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{5/3}} dx \\ &= \frac{3Ab \sin(c+dx)}{2d(b \cos(c+dx))^{2/3}} + \frac{1}{2}(-A + 2C) \int \sqrt[3]{b \cos(c+dx)} dx \\ &= \frac{3Ab \sin(c+dx)}{2d(b \cos(c+dx))^{2/3}} + \frac{3(A-2C)(b \cos(c+dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{8bd \sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.163227, size = 88, normalized size = 0.97

$$\frac{3b \sqrt{\sin^2(c+dx)} \csc(c+dx) \left(C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right) - 2A {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right) \right)}{4d(b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(1/3)*(A + C*cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (-3*b*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] + C*cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*cos[c + d*x])^(2/3))

Maple [F] time = 0.346, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \cos(dx+c)} (A + C (\cos(dx+c))^2) (\sec(dx+c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{1/3} \sec(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^2 + A\right) (b \cos(dx+c))^{1/3} \sec(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

3.145 $\int \sqrt[3]{b \cos(c + dx)} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$

Optimal. Leaf size=92

$$\frac{3Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} - \frac{3(2A + 5C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0952027, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} - \frac{3(2A + 5C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{5d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) \sec^3(c+dx) dx &= b^3 \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{8/3}} dx \\ &= \frac{3Ab^2 \sin(c+dx)}{5d(b \cos(c+dx))^{5/3}} + \frac{1}{5}(b(2A+5C)) \int \frac{1}{(b \cos(c+dx))^{2/3}} dx \\ &= \frac{3Ab^2 \sin(c+dx)}{5d(b \cos(c+dx))^{5/3}} - \frac{3(2A+5C)\sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.116762, size = 96, normalized size = 1.04

$$\frac{3\sqrt{\sin^2(c+dx)} \csc(c+dx) \sec^2(c+dx) \sqrt[3]{b \cos(c+dx)} \left(5C \cos^2(c+dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right) - A {}_2F_1\left(-\frac{5}{6}, \frac{1}{2}; \frac{1}{6}; \cos^2(c+dx)\right)\right)}{5d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(1/3)*(A + C*cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (-3*(b*cos[c + d*x])^(1/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]) + 5*C*cos[c + d*x]^2*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2]))/(5*d)

Maple [F] time = 0.454, size = 0, normalized size = 0.

$$\int \sqrt[3]{b \cos(dx+c)} (A + C (\cos(dx+c))^2) (\sec(dx+c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] int((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{1/3} \sec(dx+c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^2 + A\right) (b \cos(dx+c))^{1/3} \sec(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)
```

3.146 $\int \cos^2(c+dx)(b \cos(c+dx))^{2/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{11/3}}{14b^3d} - \frac{3(14A+11C) \sin(c+dx)(b \cos(c+dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c+dx)\right)}{154b^3d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(11/3)*Sin[c + d*x])/(14*b^3*d) - (3*(14*A + 11*C)*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(154*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0698885, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{11/3}}{14b^3d} - \frac{3(14A+11C) \sin(c+dx)(b \cos(c+dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c+dx)\right)}{154b^3d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(11/3)*Sin[c + d*x])/(14*b^3*d) - (3*(14*A + 11*C)*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(154*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Ssin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Ssin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Ssin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(c+dx)(b \cos(c+dx))^{2/3} (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{8/3} (A+C \cos^2(c+dx)) dx}{b^2} \\ &= \frac{3C(b \cos(c+dx))^{11/3} \sin(c+dx)}{14b^3d} + \frac{(14A+11C) \int (b \cos(c+dx))^{8/3} dx}{14b^2} \\ &= \frac{3C(b \cos(c+dx))^{11/3} \sin(c+dx)}{14b^3d} - \frac{3(14A+11C)(b \cos(c+dx))^{5/3}}{14b^2} \end{aligned}$$

Mathematica [A] time = 0.109981, size = 96, normalized size = 1.01

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx)(b \cos(c+dx))^{2/3} \left(17A \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c+dx)\right) + 11C \cos^4(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{17}{6}; \frac{23}{6}; \cos^2(c+dx)\right)\right)}{187d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(17*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2] + 11*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 17/6, 23/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(187*d)

Maple [F] time = 0.354, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b \cos(dx+c))^{2/3} (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{2/3} \cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^4 + A \cos(dx+c)^2\right) (b \cos(dx+c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```


3.147 $\int \cos(c+dx)(b \cos(c+dx))^{2/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{8/3}}{11b^2d} - \frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0700274, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{8/3}}{11b^2d} - \frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos(c+dx)(b \cos(c+dx))^{2/3} (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{5/3} (A+C \cos^2(c+dx)) dx}{b} \\ &= \frac{3C(b \cos(c+dx))^{8/3} \sin(c+dx)}{11b^2d} + \frac{(11A+8C) \int (b \cos(c+dx))^{5/3} dx}{11b} \\ &= \frac{3C(b \cos(c+dx))^{8/3} \sin(c+dx)}{11b^2d} - \frac{3(11A+8C)(b \cos(c+dx))^{5/3}}{11b} \end{aligned}$$

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Mathematica [A] time = 0.166098, size = 91, normalized size = 0.96

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx)(b \cos(c+dx))^{5/3} \left(7A {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right) + 4C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{3}; \frac{10}{3}; \cos^2(c+dx)\right)\right)}{56bd}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*(b*Cos[c + d*x])^(5/3)*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/3, 10/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*b*d)

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b \cos(dx+c))^{2/3} (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A)(b \cos(dx+c))^{2/3} \cos(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^3 + A \cos(dx+c)\right)(b \cos(dx+c))^{2/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)
```

3.148 $\int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{5/3}}{8bd} - \frac{3(8A + 5C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40bd\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0576242, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{5/3}}{8bd} - \frac{3(8A + 5C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40bd\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{1}{8}(8A + 5C) \int (b \cos(c + dx))^{2/3} dx \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} - \frac{3(8A + 5C)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40bd\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.115125, size = 88, normalized size = 0.93

$$\frac{3\sqrt{\sin^2(c + dx)} \cot(c + dx)(b \cos(c + dx))^{2/3} \left(11A {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) + 5C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)\right)}{55d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(2/3)*(A + C*cos[c + d*x]^2),x]

[Out] $(-3*(b*\cos[c + d*x])^{2/3}*\cot[c + d*x]*(11*A*\text{Hypergeometric2F1}[1/2, 5/6, 11/6, \cos[c + d*x]^2] + 5*C*\cos[c + d*x]^2*\text{Hypergeometric2F1}[1/2, 11/6, 17/6, \cos[c + d*x]^2])*sqrt[\sin[c + d*x]^2])/(55*d)$

Maple [F] time = 0.269, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{\frac{2}{3}} (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3), x)

$$3.149 \quad \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal. Leaf size=89

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{2/3}}{5d} - \frac{3(5A + 2C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{10d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0905986, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{2/3}}{5d} - \frac{3(5A + 2C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{10d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \frac{A + C \cos^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5d} + \frac{1}{5}(b(5A + 2C)) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5d} - \frac{3(5A + 2C)(b \cos(c + dx))^{2/3}}{5d} \end{aligned}$$

Mathematica [A] time = 0.120756, size = 88, normalized size = 0.99

$$\frac{3b\sqrt{\sin^2(c + dx) \cot(c + dx)} \left(4A {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) + C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) \right)}{8d\sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (-3*b*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))

Maple [F] time = 0.339, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x)

[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{2/3} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{2/3} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)
```

$$3.150 \quad \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal. Leaf size=91

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd \sqrt{\sin^2(c + dx)}} + \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}}$$

[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.104575, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd \sqrt{\sin^2(c + dx)}} + \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^(2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx \\ &= \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + (-2A + C) \int (b \cos(c + dx))^{2/3} dx \\ &= \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + \frac{3(2A - C)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.1588, size = 88, normalized size = 0.97

$$\frac{3b \sqrt{\sin^2(c + dx)} \csc(c + dx) \left(C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) - 5A {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \right)}{5d \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (-3*b*Csc[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(1/3))

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{2/3} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{2/3} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

$$3.151 \quad \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal. Leaf size=90

$$\frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/((8*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.100401, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/((8*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{1}{4}(b(A + 4C)) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{1}{3}, \frac{1}{2}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.125305, size = 96, normalized size = 1.07

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^2(c + dx) (b \cos(c + dx))^{2/3} \left(2C \cos^2(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) - A {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)\right)}{4d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]))*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)

Maple [F] time = 0.409, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] int((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{2/3} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{2/3} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)
```

3.152 $\int \cos^2(c+dx)(b \cos(c+dx))^{4/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{13/3}}{16b^3d} - \frac{3(16A+13C) \sin(c+dx)(b \cos(c+dx))^{13/3} {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c+dx)\right)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(13/3)*Sin[c + d*x])/(16*b^3*d) - (3*(16*A + 13*C)*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(208*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0701794, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{13/3}}{16b^3d} - \frac{3(16A+13C) \sin(c+dx)(b \cos(c+dx))^{13/3} {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c+dx)\right)}{208b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(13/3)*Sin[c + d*x])/(16*b^3*d) - (3*(16*A + 13*C)*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(208*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(c+dx)(b \cos(c+dx))^{4/3} (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{10/3} (A+C \cos^2(c+dx)) dx}{b^2} \\ &= \frac{3C(b \cos(c+dx))^{13/3} \sin(c+dx)}{16b^3d} + \frac{(16A+13C) \int (b \cos(c+dx))^{10/3} dx}{16b^2} \\ &= \frac{3C(b \cos(c+dx))^{13/3} \sin(c+dx)}{16b^3d} - \frac{3(16A+13C)(b \cos(c+dx))^{10/3}}{16b^2d} \end{aligned}$$

Mathematica [A] time = 0.261575, size = 96, normalized size = 1.01

$$\frac{3\sqrt{\sin^2(c+dx)} \cos^2(c+dx) \cot(c+dx) (b \cos(c+dx))^{4/3} \left(19A {}_2F_1\left(\frac{1}{2}, \frac{13}{6}, \frac{19}{6}; \cos^2(c+dx)\right) + 13C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{19}{6}, \frac{25}{6}; \cos^2(c+dx)\right)\right)}{247d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^2*(b*Cos[c + d*x])^(4/3)*Cot[c + d*x]*(19*A*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2] + 13*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 19/6, 25/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(247*d)

Maple [F] time = 0.349, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b \cos(dx+c))^{4/3} (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{4/3} \cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx+c)^5 + Ab \cos(dx+c)^3\right) (b \cos(dx+c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^5 + A*b*cos(d*x + c)^3)*(b*cos(d*x + c))^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

3.153 $\int \cos(c+dx)(b \cos(c+dx))^{4/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{10/3}}{13b^2d} - \frac{3(13A + 10C) \sin(c + dx)(b \cos(c + dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)}{130b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0696035, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{10/3}}{13b^2d} - \frac{3(13A + 10C) \sin(c + dx)(b \cos(c + dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)}{130b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos(c+dx)(b \cos(c+dx))^{4/3} (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{7/3} (A+C \cos^2(c+dx)) dx}{b} \\ &= \frac{3C(b \cos(c+dx))^{10/3} \sin(c+dx)}{13b^2d} + \frac{(13A+10C) \int (b \cos(c+dx))^{7/3} dx}{13b} \\ &= \frac{3C(b \cos(c+dx))^{10/3} \sin(c+dx)}{13b^2d} - \frac{3(13A+10C)(b \cos(c+dx))^{7/3}}{13b} \end{aligned}$$

Mathematica [A] time = 0.163785, size = 91, normalized size = 0.96

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx)(b \cos(c+dx))^{7/3} \left(8A {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right) + 5C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{8}{3}; \frac{11}{3}; \cos^2(c+dx)\right)\right)}{80bd}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*(b*Cos[c + d*x])^(7/3)*Cot[c + d*x]*(8*A*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 8/3, 11/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(80*b*d)

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int \cos(dx+c)(b \cos(dx+c))^{4/3} (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A)(b \cos(dx+c))^{4/3} \cos(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx+c)^4 + Ab \cos(dx+c)^2\right)(b \cos(dx+c))^{1/3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^4 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

3.154 $\int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=95

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{7/3}}{10bd} - \frac{3(10A + 7C) \sin(c + dx)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{70bd \sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0591191, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{7/3}}{10bd} - \frac{3(10A + 7C) \sin(c + dx)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{70bd \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(70*b*d*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) dx &= \frac{3C(b \cos(c + dx))^{7/3} \sin(c + dx)}{10bd} + \frac{1}{10}(10A + 7C) \int (b \cos(c + dx))^{4/3} dx \\ &= \frac{3C(b \cos(c + dx))^{7/3} \sin(c + dx)}{10bd} - \frac{3(10A + 7C)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{70bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.147148, size = 88, normalized size = 0.93

$$\frac{3\sqrt{\sin^2(c + dx)} \cot(c + dx)(b \cos(c + dx))^{4/3} \left(13A {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right) + 7C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c + dx)\right)\right)}{91d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(4/3)*(A + C*cos[c + d*x]^2),x]

[Out] $(-3*(b*\cos[c + d*x])^{4/3}*\cot[c + d*x]*(13*A*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + d*x]^2] + 7*C*\cos[c + d*x]^2*\text{Hypergeometric2F1}[1/2, 13/6, 19/6, \cos[c + d*x]^2])*Sqrt[\sin[c + d*x]^2])/(91*d)$

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{\frac{4}{3}} (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3), x)

$$3.155 \quad \int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal. Leaf size=89

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{4/3}}{7d} - \frac{3(7A + 4C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{28d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0751851, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{4/3}}{7d} - \frac{3(7A + 4C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{28d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :=> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :=> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \sqrt[3]{b \cos(c + dx)} (A + C \cos^2(c + dx)) dx \\ &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7d} + \frac{1}{7}(b(7A + 4C)) \int \sqrt[3]{b \cos(c + dx)} dx \\ &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7d} - \frac{3(7A + 4C)(b \cos(c + dx))^{1/3}}{28d} \end{aligned}$$

Mathematica [A] time = 0.0682225, size = 89, normalized size = 1.

$$\frac{3b\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(5A{}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right) + 2C\cos^2(c+dx){}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right)\right)}{20d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d)

Maple [F] time = 0.374, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{4/3} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{1/3} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="
giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)
```

3.156 $\int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$

Optimal. Leaf size=89

$$\frac{3bC \sin(c + dx) \sqrt[3]{b \cos(c + dx)}}{4d} - \frac{3b(4A + C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0949197, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3bC \sin(c + dx) \sqrt[3]{b \cos(c + dx)}}{4d} - \frac{3b(4A + C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3bC \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{1}{4} (b^2(4A + C)) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3bC \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{4d} - \frac{3b(4A + C) \sqrt[3]{b \cos(c + dx)}}{4d} \end{aligned}$$

Mathematica [A] time = 0.103264, size = 90, normalized size = 1.01

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \cot(c + dx) \left(7A {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) + C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right) \right)}{7d(b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (-3*b^2*Cot[c + d*x]*(7*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(2/3))

Maple [F] time = 0.361, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{4/3} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{1/3} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

$$3.157 \quad \int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal. Leaf size=90

$$\frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3(A - 2C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/((8*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0978578, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3(A - 2C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/((8*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} - \frac{1}{2}(b(A - 2C)) \int \sqrt[3]{b \cos(c + dx)} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3(A - 2C)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.163305, size = 90, normalized size = 1.

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \csc(c + dx) \left(C \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) - 2A {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right) \right)}{4d(b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(4/3)*(A + C*cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2] + C*cos[c + d*x]^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(4*d*(b*cos[c + d*x])^(2/3))

Maple [F] time = 0.393, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{4/3} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{1/3} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec
(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)
```

$$3.158 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{8/3}}{11b^3d} - \frac{3(11A+8C) \sin(c+dx)(b \cos(c+dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right)}{88b^3d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.068204, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{8/3}}{11b^3d} - \frac{3(11A+8C) \sin(c+dx)(b \cos(c+dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right)}{88b^3d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \frac{\int (b\cos(c+dx))^{5/3} (A+C\cos^2(c+dx)) dx}{b^2}$$

$$= \frac{3C(b\cos(c+dx))^{8/3} \sin(c+dx)}{11b^3d} + \frac{(11A+8C) \int (b\cos(c+dx))^{5/3} dx}{11b^2}$$

$$= \frac{3C(b\cos(c+dx))^{8/3} \sin(c+dx)}{11b^3d} - \frac{3(11A+8C)(b\cos(c+dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right)}{88b^3d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.111496, size = 96, normalized size = 1.01

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) \left(7A \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right) + 4C \cos^4(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{3}; \frac{10}{3}; \cos^2(c+dx)\right)\right)}{56d\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(1/3), x]

[Out] (-3*Cot[c + d*x]*(7*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 7/3, 10/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(56*d*(b*Cos[c + d*x])^(1/3))

Maple [F] time = 0.299, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (A+C(\cos(dx+c))^2) \frac{1}{\sqrt[3]{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A) \cos(dx+c)^2}{(b \cos(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx+c)^3 + A \cos(dx+c))(b \cos(dx+c))^{\frac{2}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

$$3.159 \quad \int \frac{\cos(c+dx)(A+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{5/3}}{8b^2d} - \frac{3(8A+5C) \sin(c+dx)(b \cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^2d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0674989, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{5/3}}{8b^2d} - \frac{3(8A+5C) \sin(c+dx)(b \cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^2d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)])^2, x_Symbol] :=> -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] :=> Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx &= \frac{\int (b\cos(c+dx))^{2/3} (A+C\cos^2(c+dx)) dx}{b} \\ &= \frac{3C(b\cos(c+dx))^{5/3} \sin(c+dx)}{8b^2d} + \frac{(8A+5C) \int (b\cos(c+dx))^{2/3} dx}{8b} \\ &= \frac{3C(b\cos(c+dx))^{5/3} \sin(c+dx)}{8b^2d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.106723, size = 91, normalized size = 0.96

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) (b\cos(c+dx))^{2/3} \left(11A {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right) + 5C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c+dx)\right)\right)}{55bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (-3*(b*Cos[c + d*x])^(2/3)*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*b*d)

Maple [F] time = 0.382, size = 0, normalized size = 0.

$$\int \cos(dx+c) (A+C(\cos(dx+c))^2) \frac{1}{\sqrt[3]{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)(b\cos(dx+c))^{\frac{2}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(1/3), x)
```

$$3.160 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{2/3}}{5bd} - \frac{3(5A+2C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10bd \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/ (10*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0547281, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{2/3}}{5bd} - \frac{3(5A+2C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10bd \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/ (10*b*d*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{1}{5}(5A + 2C) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{3(5A + 2C)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) \sin(c + dx)}{10bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.088347, size = 87, normalized size = 0.92

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) \left(4A {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right) + C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right) \right)}{8d\sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3), x]

[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(8*d*(b*Cos[c + d*x])^(1/3))

Maple [F] time = 0.237, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

[Out] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}}}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(1/3), x)

$$3.161 \quad \int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=90

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^2 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}}$$

[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0770518, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3012, 2643}

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^2 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx = b \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx$$

$$= \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} - \frac{(2A - C) \int (b \cos(c + dx))^{2/3} dx}{b}$$

$$= \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + \frac{3(2A - C)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \sin(c + dx)}{5b^2 d \sqrt{\sin^2(c + dx)}}$$

Mathematica [C] time = 3.57046, size = 283, normalized size = 3.14

$$3 \csc(c) e^{-idx} \sqrt[3]{\cos(c + dx)} (\cos(dx) + i \sin(dx)) \left(2(2A - C)(\cos(dx) - i \sin(dx)) \sqrt[3]{i \sin(2(c + dx)) + \cos(2(c + dx))} + 1 \right) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \sin(c + dx)$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3), x]

[Out] (-3*Cos[c + d*x]^(1/3)*Csc[c]*(Cos[d*x] + I*Sin[d*x])*(-8*A*Cos[d*x] + 2*C*Cos[d*x] + 2*C*Cos[2*c + d*x] + 2*(2*A - C)*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*(1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(1/3) + (2*A - C)*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*(1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(1/3)))/(4*2^(2/3)*d*E^(I*d*x)*(b*Cos[c + d*x])^(1/3)*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(1/3))

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) \sec(dx + c) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)

$$3.162 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=91

$$\frac{3Ab \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} - \frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8bd \sqrt{\sin^2(c+dx)}}$$

[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0920171, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} - \frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8bd \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^(n_)), x_Symbol] := Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx = b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/3}} dx$$

$$= \frac{3Ab \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{1}{4}(A + 4C) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx$$

$$= \frac{3Ab \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{8bd \sqrt{\sin^2(c + dx)}}$$

Mathematica [A] time = 0.764362, size = 101, normalized size = 1.11

$$\frac{6A \tan(c + dx) \sqrt[3]{\sin^2(dx - \tan^{-1}(\cot(c)))} - (A + 4C) \sin(2dx - 2 \tan^{-1}(\cot(c))) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \cos^2(dx - \tan^{-1}(\cot(c)))\right)}{8d \sqrt[3]{b \cos(c + dx)} \sqrt[3]{\sin^2(dx - \tan^{-1}(\cot(c)))}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x]^(1/3), x]

[Out] (-(A + 4*C)*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[2*d*x - 2*ArcTan[Cot[c]]]) + 6*A*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)*Tan[c + d*x]/(8*d*(b*Cos[c + d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3))

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (\sec(dx + c))^2 \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)

$$3.163 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^3(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=92

$$\frac{3Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/3}} + \frac{3(4A+7C) \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0972061, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/3}} + \frac{3(4A+7C) \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{10/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{1}{7}(b(4A + 7C)) \int \frac{1}{(b \cos(c + dx))^{4/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3(4A + 7C) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sin(c + dx)}{7d \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [C] time = 5.98642, size = 404, normalized size = 4.39

$$3b^2 \csc(c) \sec(c) e^{-idx} (A + C \cos^2(c + dx)) \left(2 \cos(c) \left(e^{idx} \sqrt[3]{e^{-idx} (i \sin(c) (-1 + e^{2idx}) + \cos(c) (1 + e^{2idx}))} \right) ((2A + 7C) \cos(c) \right.$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*b^2*(A + C*Cos[c + d*x]^2)*Csc[c]*Sec[c]*(-((4*A + 7*C)*E^((2*I)*d*x)*Cos[c]*Cos[c + d*x]^(7/3)*Csc[c/2]*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sec[c/2]*Sin[c]*(2 + 2*E^((2*I)*d*x)*Cos[2*c] + (2*I)*E^((2*I)*d*x)*Sin[2*c])^(1/3)) + 2*Cos[c]*(E^(I*d*x)*(2*(5*A + 7*C)*Cos[d*x] + (2*A + 7*C)*Cos[2*c + d*x] + (4*A + 7*C)*Cos[2*c + 3*d*x]))*((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(1/3) - 2*2^(2/3)*(4*A + 7*C)*Cos[c + d*x]^(7/3)*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[c + d*x]*(Cos[c + d*x] + I*Sin[c + d*x]))^(1/3)))/(28*d*E^(I*d*x)*(b*Cos[c + d*x])^(7/3)*(2*A + C + C*Cos[2*(c + d*x)])*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(1/3))

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (\sec(dx + c))^3 \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)

$$3.164 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{7/3}}{10b^3d} - \frac{3(10A+7C) \sin(c+dx)(b \cos(c+dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{70b^3d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^3*d) - (3*(10*A + 7*C)*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(70*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0684882, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{7/3}}{10b^3d} - \frac{3(10A+7C) \sin(c+dx)(b \cos(c+dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{70b^3d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x])/(10*b^3*d) - (3*(10*A + 7*C)*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(70*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_)^(n_)), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Ssin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Ssin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Ssin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}} dx &= \frac{\int (b\cos(c+dx))^{4/3}(A+C\cos^2(c+dx)) dx}{b^2} \\ &= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} + \frac{(10A+7C)\int (b\cos(c+dx))^{4/3} dx}{10b^2} \\ &= \frac{3C(b\cos(c+dx))^{7/3}\sin(c+dx)}{10b^3d} - \frac{3(10A+7C)(b\cos(c+dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)}{70b^3d\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.119558, size = 96, normalized size = 1.01

$$\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\left(13A\cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right) + 7C\cos^4(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c+dx)\right)\right)}{91d(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(2/3), x]

[Out] (-3*Cot[c + d*x]*(13*A*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^4*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(91*d*(b*Cos[c + d*x]^(2/3)))

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (A+C(\cos(dx+c))^2) (b\cos(dx+c))^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)^2}{(b\cos(dx+c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)^3 + A\cos(dx+c))(b\cos(dx+c))^{1/3}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)
```

$$3.165 \quad \int \frac{\cos(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{4/3}}{7b^2d} - \frac{3(7A+4C) \sin(c+dx)(b \cos(c+dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{28b^2d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b^2*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0640115, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{4/3}}{7b^2d} - \frac{3(7A+4C) \sin(c+dx)(b \cos(c+dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{28b^2d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b^2*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e+f*x]*(b*Ssin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Ssin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] :> Simp[(Cos[c+d*x]*(b*Ssin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}} dx &= \frac{\int \sqrt[3]{b\cos(c+dx)}(A+C\cos^2(c+dx)) dx}{b} \\ &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d} + \frac{(7A+4C)\int \sqrt[3]{b\cos(c+dx)} dx}{7b} \\ &= \frac{3C(b\cos(c+dx))^{4/3}\sin(c+dx)}{7b^2d} - \frac{3(7A+4C)(b\cos(c+dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{28b^2d\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.112143, size = 91, normalized size = 0.96

$$\frac{3\sqrt{\sin^2(c+dx)}\cot(c+dx)\sqrt[3]{b\cos(c+dx)}\left(5A {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right) + 2C\cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c+dx)\right)\right)}{20bd}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3), x]

[Out] (-3*(b*Cos[c + d*x])^(1/3)*Cot[c + d*x]*(5*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*b*d)

Maple [F] time = 0.343, size = 0, normalized size = 0.

$$\int \cos(dx+c)(A+C(\cos(dx+c))^2)(b\cos(dx+c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C\cos(dx+c)^2 + A)(b\cos(dx+c))^{\frac{1}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(2/3), x)
```

$$3.166 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=93

$$\frac{3C \sin(c+dx) \sqrt[3]{b \cos(c+dx)}}{4bd} - \frac{3(4A+C) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{4bd \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0614186, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3014, 2643}

$$\frac{3C \sin(c+dx) \sqrt[3]{b \cos(c+dx)}}{4bd} - \frac{3(4A+C) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{4bd \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(4*b*d*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx &= \frac{3C \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{4bd} + \frac{1}{4}(4A+C) \int \frac{1}{(b \cos(c+dx))^{2/3}} dx \\ &= \frac{3C \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{4bd} - \frac{3(4A+C) \sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right) \sin(c+dx)}{4bd \sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.109764, size = 87, normalized size = 0.94

$$\frac{3 \sqrt{\sin^2(c+dx)} \cot(c+dx) \left(7A {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right) + C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c+dx)\right)\right)}{7d(b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(2/3),x]

[Out] $(-3*\cot[c + d*x]*(7*A*\text{Hypergeometric2F1}[1/6, 1/2, 7/6, \cos[c + d*x]^2] + C*\cos[c + d*x]^2*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \cos[c + d*x]^2])*\sqrt{\sin[c + d*x]^2})/(7*d*(b*\cos[c + d*x])^{2/3})$

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int (A + C(\cos(dx + c))^2)(b \cos(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)

[Out] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{1}{3}}}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(2/3), x)

$$3.167 \quad \int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=90

$$\frac{3(A-2C) \sin(c+dx)(b \cos(c+dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{8b^2 d \sqrt{\sin^2(c+dx)}} + \frac{3A \sin(c+dx)}{2d(b \cos(c+dx))^{2/3}}$$

[Out] (3*A*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0813656, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3012, 2643}

$$\frac{3(A-2C) \sin(c+dx)(b \cos(c+dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c+dx)\right)}{8b^2 d \sqrt{\sin^2(c+dx)}} + \frac{3A \sin(c+dx)}{2d(b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*A*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] :> Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{2/3}} dx &= b \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/3}} dx \\ &= \frac{3A \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} - \frac{(A - 2C) \int \sqrt[3]{b \cos(c + dx)} dx}{2b} \\ &= \frac{3A \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3(A - 2C)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [C] time = 3.64923, size = 277, normalized size = 3.08

$$3 \csc(c) e^{-idx} \cos^{\frac{2}{3}}(c + dx) (\cos(dx) + i \sin(dx)) \left(5(A - 2C)(\cos(dx) - i \sin(dx))(i \sin(2(c + dx)) + \cos(2(c + dx)) + 1)^{2/3} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(2/3), x]

[Out] (-3*Cos[c + d*x]^(2/3)*Csc[c]*(Cos[d*x] + I*Sin[d*x])*(10*((-A + C)*Cos[d*x] + C*Cos[2*c + d*x]) + 5*(A - 2*C)*Hypergeometric2F1[-1/6, 2/3, 5/6, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] - I*Sin[d*x])*(1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(2/3) + (A - 2*C)*Hypergeometric2F1[2/3, 5/6, 1 1/6, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[d*x] + I*Sin[d*x])*(1 + Cos[2*(c + d*x)] + I*Sin[2*(c + d*x)])^(2/3)))/(10*2^(1/3)*d*E^(I*d*x)*(b*Cos[c + d*x])^(2/3)*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(2/3))

Maple [F] time = 0.329, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) \sec(dx + c) (b \cos(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(2/3), x)

$$3.168 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=93

$$\frac{3Ab \sin(c+dx)}{5d(b \cos(c+dx))^{5/3}} - \frac{3(2A+5C) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{5bd \sqrt{\sin^2(c+dx)}}$$

[Out] (3*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/ (5*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.101386, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab \sin(c+dx)}{5d(b \cos(c+dx))^{5/3}} - \frac{3(2A+5C) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c+dx)\right)}{5bd \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) - (3*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/ (5*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx = b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{8/3}} dx$$

$$= \frac{3Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} + \frac{1}{5}(2A + 5C) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx$$

$$= \frac{3Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} - \frac{3(2A + 5C)\sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}}$$

Mathematica [A] time = 0.675028, size = 103, normalized size = 1.11

$$\frac{6A \tan(c + dx) \sqrt[6]{\sin^2(dx - \tan^{-1}(\cot(c)))} - (2A + 5C) \sin(2dx - 2 \tan^{-1}(\cot(c))) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \cos^2(dx - \tan^{-1}(\cot(c)))\right)}{10d(b \cos(c + dx))^{2/3} \sqrt[6]{\sin^2(dx - \tan^{-1}(\cot(c)))}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x]^(2/3), x]

[Out] (-((2*A + 5*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[2*d*x - 2*ArcTan[Cot[c]]]) + 6*A*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/6)*Tan[c + d*x])/(10*d*(b*Cos[c + d*x])^(2/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/6))

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (\sec(dx + c))^2 (b \cos(dx + c))^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^2}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(2/3), x)

$$3.169 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=92

$$\frac{3Ab^2 \sin(c+dx)}{8d(b \cos(c+dx))^{8/3}} + \frac{3(5A+8C) \sin(c+dx) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{16d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(8*d*(b*Cos[c + d*x])^(8/3)) + (3*(5*A + 8*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.100614, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab^2 \sin(c+dx)}{8d(b \cos(c+dx))^{8/3}} + \frac{3(5A+8C) \sin(c+dx) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c+dx)\right)}{16d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*A*b^2*Sin[c + d*x])/(8*d*(b*Cos[c + d*x])^(8/3)) + (3*(5*A + 8*C)*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(16*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[e_] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[e_] + (f_)*(x_))^(2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2) + C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e + f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[c_] + (d_)*(x_))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{2/3}} dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{11/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{8d(b \cos(c + dx))^{8/3}} + \frac{1}{8}(b(5A + 8C)) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{8d(b \cos(c + dx))^{8/3}} + \frac{3(5A + 8C) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right) \sin(c + dx)}{16d(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [C] time = 6.13016, size = 402, normalized size = 4.37

$$3b^2 \csc(c) \sec(c) e^{-idx} (A + C \cos^2(c + dx)) \left(5 \cos(c) \left(e^{idx} \left(e^{-idx} (i \sin(c) (-1 + e^{2idx}) + \cos(c) (1 + e^{2idx}))\right)\right)^{2/3} ((A + 8C) \cos(c + dx))^{2/3}\right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x]^(2/3), x]

[Out] (3*b^2*(A + C*Cos[c + d*x]^2)*Csc[c]*Sec[c]*(-(5*A + 8*C)*E^((2*I)*d*x)*Cos[c]*Cos[c + d*x]^(8/3)*Csc[c/2]*Hypergeometric2F1[2/3, 5/6, 11/6, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sec[c/2]*Sin[c]*(2 + 2*E^((2*I)*d*x)*Cos[2*c] + (2*I)*E^((2*I)*d*x)*Sin[2*c])^(2/3)) + 5*Cos[c]*(E^(I*d*x)*(2*(7*A + 8*C)*Cos[d*x] + (A + 8*C)*Cos[2*c + d*x] + (5*A + 8*C)*Cos[2*c + 3*d*x]))*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(2/3) - 4*2^(1/3)*(5*A + 8*C)*Cos[c + d*x]^(8/3)*Hypergeometric2F1[-1/6, 2/3, 5/6, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*(Cos[c + d*x]*(Cos[c + d*x] + I*Sin[c + d*x]))^(2/3)))/(160*d*E^(I*d*x)*(b*Cos[c + d*x])^(8/3)*(2*A + C + C*Cos[2*(c + d*x)])*(((1 + E^((2*I)*d*x))*Cos[c] + I*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x))^(2/3))

Maple [F] time = 0.398, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (\sec(dx + c))^3 (b \cos(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(2/3), x)

$$3.170 \quad \int \frac{\cos^2(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{5/3}}{8b^3d} - \frac{3(8A+5C) \sin(c+dx)(b \cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^3d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0668854, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{5/3}}{8b^3d} - \frac{3(8A+5C) \sin(c+dx)(b \cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^3d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx &= \frac{\int (b\cos(c+dx))^{2/3} (A+C\cos^2(c+dx)) dx}{b^2} \\ &= \frac{3C(b\cos(c+dx))^{5/3} \sin(c+dx)}{8b^3d} + \frac{(8A+5C) \int (b\cos(c+dx))^{2/3} dx}{8b^2} \\ &= \frac{3C(b\cos(c+dx))^{5/3} \sin(c+dx)}{8b^3d} - \frac{3(8A+5C)(b\cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^3d\sqrt{\sin^2(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.192695, size = 96, normalized size = 1.01

$$\frac{3\sqrt{\sin^2(c+dx)} \cos^2(c+dx) \cot(c+dx) \left(11A {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right) + 5C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c+dx)\right)\right)}{55d(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(4/3), x]

[Out] (-3*Cos[c + d*x]^2*Cot[c + d*x]*(11*A*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(55*d*(b*Cos[c + d*x]^(4/3)))

Maple [F] time = 0.348, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (A+C(\cos(dx+c))^2) (b\cos(dx+c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A) \cos(dx+c)^2}{(b \cos(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx+c)^2 + A) (b \cos(dx+c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```


$$3.171 \quad \int \frac{\cos(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=95

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{2/3}}{5b^2d} - \frac{3(5A+2C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10b^2d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0658111, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{3C \sin(c+dx)(b \cos(c+dx))^{2/3}}{5b^2d} - \frac{3(5A+2C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10b^2d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e+f*x]*(b*Ssin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Ssin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] :> Simp[(Cos[c+d*x]*(b*Ssin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{\cos(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \frac{\int \frac{A+C\cos^2(c+dx)}{\sqrt[3]{b\cos(c+dx)}} dx}{b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3} \sin(c+dx)}{5b^2d} + \frac{(5A+2C) \int \frac{1}{\sqrt[3]{b\cos(c+dx)}} dx}{5b}$$

$$= \frac{3C(b\cos(c+dx))^{2/3} \sin(c+dx)}{5b^2d} - \frac{3(5A+2C)(b\cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A] time = 0.0918609, size = 90, normalized size = 0.95

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) \left(4A {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right) + C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right)\right)}{8bd\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (-3*Cot[c + d*x]*(4*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(8*b*d*(b*Cos[c + d*x])^(1/3))

Maple [F] time = 0.326, size = 0, normalized size = 0.

$$\int \cos(dx+c)(A+C(\cos(dx+c))^2)(b\cos(dx+c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)}{(b\cos(dx+c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{2}{3}}}{b^2 \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)/(b*cos(d*x + c))^(4/3), x)

$$3.172 \quad \int \frac{A+C \cos^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=93

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^3 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}}$$

[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.062424, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.08$, Rules used = {3012, 2643}

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^3 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx &= \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} - \frac{(2A - C) \int (b \cos(c + dx))^{2/3} dx}{b^2} \\ &= \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} + \frac{3(2A - C)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \sin(c + dx)}{5b^3 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.146138, size = 87, normalized size = 0.94

$$\frac{3\sqrt{\sin^2(c+dx)} \cot(c+dx) \left(C \cos^2(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right) - 5A {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right) \right)}{5d(b \cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3), x]

[Out] (-3*Cot[c + d*x]*(-5*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] + C*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(5*d*(b*Cos[c + d*x])^(4/3))

Maple [F] time = 0.234, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}}}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c))^(4/3), x)

$$3.173 \quad \int \frac{(A+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=90

$$\frac{3A \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} - \frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8b^2 d \sqrt{\sin^2(c+dx)}}$$

[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0814465, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3012, 2643}

$$\frac{3A \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} - \frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8b^2 d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> Simp[(A*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+1)), x] + Dist[(A*(m+2)+C*(m+1))/(b^2*(m+1)), Int[(b*Sin[e+f*x])^(m+2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] :> Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(A + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = b \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/3}} dx$$

$$= \frac{3A \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{(A + 4C) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx}{4b}$$

$$= \frac{3A \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{8b^2 d \sqrt{\sin^2(c + dx)}}$$

Mathematica [A] time = 0.653806, size = 104, normalized size = 1.16

$$\frac{6A \tan(c + dx) \sqrt[3]{\sin^2(dx - \tan^{-1}(\cot(c)))} - (A + 4C) \sin(2dx - 2 \tan^{-1}(\cot(c))) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \cos^2(dx - \tan^{-1}(\cot(c)))\right)}{8bd \sqrt[3]{b \cos(c + dx)} \sqrt[3]{\sin^2(dx - \tan^{-1}(\cot(c)))}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3), x]

[Out] (-((A + 4*C)*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*Sin[2*d*x - 2*ArcTan[Cot[c]]]) + 6*A*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)*Tan[c + d*x])/(8*b*d*(b*Cos[c + d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3))

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) \sec(dx + c) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)

$$3.174 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=93

$$\frac{3(4A + 7C) \sin(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{7bd \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}} + \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}}$$

[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.100125, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3(4A + 7C) \sin(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{7bd \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}} + \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_)*sin[(e_) + (f_)*(x_)]^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx &= b^2 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{10/3}} dx \\ &= \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{1}{7}(4A + 7C) \int \frac{1}{(b \cos(c + dx))^{4/3}} dx \\ &= \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3(4A + 7C) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sin(c + dx)}{7bd \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.208289, size = 90, normalized size = 0.97

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \cot(c + dx) \left(A {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right) + 7C \cos^2(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \right)}{7d(b \cos(c + dx))^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x]^(4/3), x]

[Out] (3*b^2*Cot[c + d*x]*(A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2] + 7*C*Cos[c + d*x]^2*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(7*d*(b*Cos[c + d*x])^(10/3))

Maple [F] time = 0.333, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (\sec(dx + c))^2 (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

$$3.175 \quad \int \frac{(A+C \cos^2(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=92

$$\frac{3Ab^2 \sin(c+dx)}{10d(b \cos(c+dx))^{10/3}} + \frac{3(7A+10C) \sin(c+dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{40d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{4/3}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(10*d*(b*Cos[c + d*x])^(10/3)) + (3*(7*A + 10*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(40*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.104965, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {16, 3012, 2643}

$$\frac{3Ab^2 \sin(c+dx)}{10d(b \cos(c+dx))^{10/3}} + \frac{3(7A+10C) \sin(c+dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{40d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*b^2*Sin[c + d*x])/(10*d*(b*Cos[c + d*x])^(10/3)) + (3*(7*A + 10*C)*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(40*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3012

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{(A + C \cos^2(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{4/3}} dx &= b^3 \int \frac{A + C \cos^2(c + dx)}{(b \cos(c + dx))^{13/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{10d(b \cos(c + dx))^{10/3}} + \frac{1}{10}(b(7A + 10C)) \int \frac{1}{(b \cos(c + dx))^{7/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{10d(b \cos(c + dx))^{10/3}} + \frac{3(7A + 10C) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) \sin(c + dx)}{40d(b \cos(c + dx))^{4/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.209038, size = 91, normalized size = 0.99

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \csc(c + dx) \left(2A {}_2F_1\left(-\frac{5}{3}, \frac{1}{2}; -\frac{2}{3}; \cos^2(c + dx)\right) + 5C \cos^2(c + dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) \right)}{20d(b \cos(c + dx))^{10/3}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(4/3),x]

[Out] (3*b^2*Csc[c + d*x]*(2*A*Hypergeometric2F1[-5/3, 1/2, -2/3, Cos[c + d*x]^2] + 5*C*Cos[c + d*x]^2*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(20*d*(b*Cos[c + d*x])^(10/3))

Maple [F] time = 0.393, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (\sec(dx + c))^3 (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)

[Out] int((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x)

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^3}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)
```

3.176 $\int \cos^m(c+dx)(b \cos(c+dx))^{4/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=148

$$\frac{3bC \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \cos^{m+2}(c + dx)}{d(3m + 10)} - \frac{3b(A(3m + 10) + C(3m + 7)) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \cos^{m+2}(c + dx)}{d(3m + 7)(3m + 10) \sqrt{\sin^2(c + dx)}}$$

[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3*m)) - (3*b*(C*(7 + 3*m) + A*(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.113143, antiderivative size = 138, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{3bC \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \cos^{m+2}(c + dx)}{d(3m + 10)} - \frac{3b \left(\frac{A}{3m+7} + \frac{C}{3m+10} \right) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \cos^{m+2}(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{1}{6} \right)}{d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3*m)) - (3*b*(A/(7 + 3*m) + C/(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^m(c+dx)(b \cos(c+dx))^{4/3} (A+C \cos^2(c+dx)) dx &= \frac{(b\sqrt[3]{b \cos(c+dx)}) \int \cos^{\frac{4}{3}+m}(c+dx) (A+C \cos^2(c+dx)) dx}{\sqrt[3]{\cos(c+dx)}} \\ &= \frac{3bC \cos^{2+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{d(10+3m)} + \frac{b \left(C \int \cos^{m+1}(c+dx) dx \right)}{d(10+3m)} \\ &= \frac{3bC \cos^{2+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{d(10+3m)} - \frac{3b(C \int \cos^{m+1}(c+dx) dx)}{d(10+3m)} \end{aligned}$$

Mathematica [A] time = 0.279265, size = 142, normalized size = 0.96

$$\frac{3\sqrt{\sin^2(c+dx)} \csc(c+dx) (b \cos(c+dx))^{4/3} \cos^{m+1}(c+dx) \left(A(3m+13) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+13); \cos^2(c+dx)\right) + C(7+3m) \cos(c+dx) \right)}{d(3m+7)(3m+13)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(A*(13 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2] + C*(7 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(7 + 3*m)*(13 + 3*m))

Maple [F] time = 0.309, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b \cos(dx+c))^{\frac{4}{3}} (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{\frac{4}{3}} \cos(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)

$$3.177 \quad \int \cos^m(c+dx)(b \cos(c+dx))^{2/3} (A + C \cos^2(c + dx)) dx$$

Optimal. Leaf size=146

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{2/3} \cos^{m+1}(c + dx)}{d(3m + 8)} - \frac{3(A(3m + 8) + C(3m + 5)) \sin(c + dx)(b \cos(c + dx))^{2/3} \cos^{m+1}(c + dx)}{d(3m + 5)(3m + 8)\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)) - (3*(C*(5 + 3*m) + A*(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.116504, antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{3C \sin(c + dx)(b \cos(c + dx))^{2/3} \cos^{m+1}(c + dx)}{d(3m + 8)} - \frac{3\left(\frac{A}{3m+5} + \frac{C}{3m+8}\right) \sin(c + dx)(b \cos(c + dx))^{2/3} \cos^{m+1}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5 + 3m}{6}, \frac{11 + 3m}{6}, \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)) - (3*(A/(5 + 3*m) + C/(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(a*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \cos^m(c+dx)(b \cos(c+dx))^{2/3} (A+C \cos^2(c+dx)) dx = \frac{(b \cos(c+dx))^{2/3} \int \cos^{\frac{2}{3}+m}(c+dx) (A+C \cos^2(c+dx)) dx}{\cos^{\frac{2}{3}}(c+dx)}$$

$$= \frac{3C \cos^{1+m}(c+dx)(b \cos(c+dx))^{2/3} \sin(c+dx)}{d(8+3m)} + \frac{\left(C \left(\frac{5}{3} + \dots \right) \right)}{d(8+3m)}$$

$$= \frac{3C \cos^{1+m}(c+dx)(b \cos(c+dx))^{2/3} \sin(c+dx)}{d(8+3m)} - \frac{3(C(5+3\dots))}{d(8+3m)}$$

Mathematica [A] time = 0.210663, size = 142, normalized size = 0.97

$$\frac{3\sqrt{\sin^2(c+dx)} \csc(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+1}(c+dx) \left(A(3m+11) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+5); \frac{1}{6}(3m+11); \cos^2(c+dx)\right) \right)}{d(3m+5)(3m+11)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(A*(11 + 3*m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(5 + 3*m)*(11 + 3*m))

Maple [F] time = 0.306, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (b \cos(dx+c))^{2/3} (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{2/3} \cos(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

3.178 $\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx$

Optimal. Leaf size=146

$$\frac{3C \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx)}{d(3m+7)} - \frac{3(A(3m+7) + C(3m+4)) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{4+3m}{6}, \frac{10+3m}{6}, \cos^2(c+dx)\right)}{d(3m+4)(3m+7) \sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m))
- (3*(C*(4 + 3*m) + A*(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*(4 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.108096, antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{3C \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx)}{d(3m+7)} - \frac{3\left(\frac{A}{3m+4} + \frac{C}{3m+7}\right) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{4+3m}{6}, \frac{10+3m}{6}, \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m))
- (3*(A/(4 + 3*m) + C/(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m +
n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(
x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*
(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A + C \cos^2(c+dx)) dx = \frac{\sqrt[3]{b \cos(c+dx)} \int \cos^{\frac{1}{3}+m}(c+dx) (A + C \cos^2(c+dx)) dx}{\sqrt[3]{\cos(c+dx)}}$$

$$= \frac{3C \cos^{1+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{d(7+3m)} + \frac{\left(C \left(\frac{4}{3} + m \right) \right)}{d(7+3m)}$$

$$= \frac{3C \cos^{1+m}(c+dx) \sqrt[3]{b \cos(c+dx)} \sin(c+dx)}{d(7+3m)} - \frac{3(C(4+3m))}{d(7+3m)}$$

Mathematica [A] time = 0.299688, size = 142, normalized size = 0.97

$$\frac{3\sqrt{\sin^2(c+dx)} \csc(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx) \left(A(3m+10) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+4); \frac{m}{2} + \frac{5}{3}; \cos^2(c+dx)\right) + C \right)}{d(3m+4)(3m+10)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(4 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + A*(10 + 3*m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + 3*m)*(10 + 3*m))

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m \sqrt[3]{b \cos(dx+c)} (A + C (\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^{\frac{1}{3}} \cos(dx+c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

$$3.179 \quad \int \frac{\cos^m(c+dx)(A+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=146

$$\frac{3C \sin(c+dx) \cos^{m+1}(c+dx)}{d(3m+5)\sqrt[3]{b \cos(c+dx)}} - \frac{3(A(3m+5) + C(3m+2)) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \frac{\sin^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}}\right)}{d(3m+2)(3m+5)\sqrt{\sin^2(c+dx)\sqrt[3]{b \cos(c+dx)}}$$

[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.105282, antiderivative size = 136, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{3C \sin(c+dx) \cos^{m+1}(c+dx)}{d(3m+5)\sqrt[3]{b \cos(c+dx)}} - \frac{3\left(\frac{A}{3m+2} + \frac{C}{3m+5}\right) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)\sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)) - (3*(A/(2 + 3*m) + C/(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^(2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)] + (d_)*(x_))^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx = \frac{\sqrt[3]{\cos(c+dx)} \int \cos^{-\frac{1}{3}+m}(c+dx)(A+C\cos^2(c+dx)) dx}{\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C \cos^{1+m}(c+dx) \sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} + \frac{\left(\left(C\left(\frac{2}{3}+m\right)+A\left(\frac{5}{3}+m\right)\right)\sqrt[3]{\cos(c+dx)}\right) \int \cos^{-\frac{1}{3}+m}(c+dx) dx}{\left(\frac{5}{3}+m\right)\sqrt[3]{b\cos(c+dx)}}$$

$$= \frac{3C \cos^{1+m}(c+dx) \sin(c+dx)}{d(5+3m)\sqrt[3]{b\cos(c+dx)}} - \frac{3(C(2+3m)+A(5+3m)) \cos^{1+m}(c+dx)}{d(2+3m)(5+3m)\sqrt[3]{b\cos(c+dx)}}$$

Mathematica [A] time = 0.270992, size = 142, normalized size = 0.97

$$\frac{3\sqrt{\sin^2(c+dx)} \csc(c+dx) \cos^{m+1}(c+dx) \left(A(3m+8) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \cos^2(c+dx)\right) + C(3m+2) \cos^2(c+dx) \right)}{d(3m+2)(3m+8)\sqrt[3]{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(1/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(8 + 3*m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(2 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (A+C(\cos(dx+c))^2) \frac{1}{\sqrt[3]{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A) \cos(dx+c)^m}{(b \cos(dx+c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + C \cos^2(c + dx)) \cos^m(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)

[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)

$$3.180 \quad \int \frac{\cos^m(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=144

$$\frac{3C \sin(c+dx) \cos^{m+1}(c+dx)}{d(3m+4)(b \cos(c+dx))^{2/3}} - \frac{3(A(3m+4) + 3Cm + C) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+1); \frac{1}{6}(3m+7); \cos^2(c+dx)\right)}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)(b \cos(c+dx))^{2/3}}}$$

[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.104, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{3C \sin(c+dx) \cos^{m+1}(c+dx)}{d(3m+4)(b \cos(c+dx))^{2/3}} - \frac{3(A(3m+4) + 3Cm + C) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+1); \frac{1}{6}(3m+7); \cos^2(c+dx)\right)}{d(3m+1)(3m+4)\sqrt{\sin^2(c+dx)(b \cos(c+dx))^{2/3}}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{2/3}} dx = \frac{\cos^{2/3}(c+dx) \int \cos^{-2/3+m}(c+dx)(A+C\cos^2(c+dx)) dx}{(b\cos(c+dx))^{2/3}}$$

$$= \frac{3C \cos^{1+m}(c+dx) \sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} + \frac{\left(\left(C\left(\frac{1}{3}+m\right)+A\left(\frac{4}{3}+m\right)\right)\cos^{2/3}(c+dx)\right)}{\left(\frac{4}{3}+m\right)(b\cos(c+dx))^{2/3}}$$

$$= \frac{3C \cos^{1+m}(c+dx) \sin(c+dx)}{d(4+3m)(b\cos(c+dx))^{2/3}} - \frac{3(C+3Cm+A(4+3m))\cos^{1+m}(c+dx)}{d(1+3m)(4+3m)}$$

Mathematica [A] time = 0.241487, size = 142, normalized size = 0.99

$$\frac{3\sqrt{\sin^2(c+dx)} \csc(c+dx) \cos^{m+1}(c+dx) \left(A(3m+7) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+1); \frac{1}{6}(3m+7); \cos^2(c+dx)\right) + C(3m+1) \right)}{d(3m+1)(3m+7)(b\cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(2/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(7 + 3*m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + C*(1 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(1 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x]^(2/3))

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (A+C(\cos(dx+c))^2) (b\cos(dx+c))^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + A) \cos(dx+c)^m}{(b \cos(dx+c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + C \cos^2(c + dx)) \cos^m(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)

[Out] Integral((A + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

$$3.181 \quad \int \frac{\cos^m(c+dx)(A+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=149

$$\frac{3C \sin(c+dx) \cos^m(c+dx)}{bd(3m+2)\sqrt[3]{b \cos(c+dx)}} - \frac{3(C(1-3m) - A(3m+2)) \sin(c+dx) \cos^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m-1); \frac{1}{6}(3m+5); \cos^2(c+dx)\right)}{bd(1-3m)(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}}$$

[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) - (3*(C*(1 - 3*m) - A*(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.122399, antiderivative size = 139, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{3\left(\frac{A}{1-3m} - \frac{C}{3m+2}\right) \sin(c+dx) \cos^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m-1); \frac{1}{6}(3m+5); \cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}} + \frac{3C \sin(c+dx) \cos^m(c+dx)}{bd(3m+2)\sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) + (3*(A/(1 - 3*m) - C/(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_)*sin[(e_)] + (f_)*(x_))^(m_)*((A_) + (C_)*sin[(e_)] + (f_)*(x_))^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)] + (d_)*(x_))^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{\cos^m(c+dx)(A+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx = \frac{\sqrt[3]{\cos(c+dx)} \int \cos^{-\frac{4}{3}+m}(c+dx)(A+C\cos^2(c+dx)) dx}{b\sqrt[3]{b}\cos(c+dx)}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b}\cos(c+dx)} + \frac{\left(\left(C\left(-\frac{1}{3}+m\right)+A\left(\frac{2}{3}+m\right)\right)\sqrt[3]{\cos(c+dx)}\right) \int}{b\left(\frac{2}{3}+m\right)\sqrt[3]{b}\cos(c+dx)}$$

$$= \frac{3C\cos^m(c+dx)\sin(c+dx)}{bd(2+3m)\sqrt[3]{b}\cos(c+dx)} - \frac{3(C(1-3m)-A(2+3m))\cos^m(c+dx) {}_2F_1\left(\right)}{bd(1-3m)(2+3m)\sqrt[3]{b}}$$

Mathematica [A] time = 0.25823, size = 142, normalized size = 0.95

$$\frac{3\sqrt{\sin^2(c+dx)}\csc(c+dx)\cos^{m+1}(c+dx)\left(A(3m+5) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m-1); \frac{1}{6}(3m+5); \cos^2(c+dx)\right) + C(3m-1)\cos^2(c+dx)\right)}{d(3m-1)(3m+5)(b\cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(A + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(4/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Csc[c + d*x]*(A*(5 + 3*m)*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + C*(-1 + 3*m)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^m (A+C(\cos(dx+c))^2) (b\cos(dx+c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + A)\cos(dx+c)^m}{(b\cos(dx+c))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(A+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

3.182 $\int (a \cos(c+dx))^m (b \cos(c+dx))^n (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=144

$$\frac{C \sin(c + dx)(a \cos(c + dx))^{m+1}(b \cos(c + dx))^n}{ad(m + n + 2)} - \frac{(A(m + n + 2) + C(m + n + 1)) \sin(c + dx)(a \cos(c + dx))^{m+1}(b \cos(c + dx))^n}{ad(m + n + 1)(m + n + 2)}$$

[Out] (C*(a*cos[c + d*x])^(1 + m)*(b*cos[c + d*x])^n*sin[c + d*x])/(a*d*(2 + m + n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*cos[c + d*x])^(1 + m)*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.111634, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{C \sin(c + dx)(a \cos(c + dx))^{m+1}(b \cos(c + dx))^n}{ad(m + n + 2)} - \frac{(A(m + n + 2) + C(m + n + 1)) \sin(c + dx)(a \cos(c + dx))^{m+1}(b \cos(c + dx))^n}{ad(m + n + 1)(m + n + 2)}$$

Antiderivative was successfully verified.

[In] Int[(a*cos[c + d*x])^m*(b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2), x]

[Out] (C*(a*cos[c + d*x])^(1 + m)*(b*cos[c + d*x])^n*sin[c + d*x])/(a*d*(2 + m + n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*cos[c + d*x])^(1 + m)*(b*cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*cos[e + f*x]*(b*sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int (a \cos(c + dx))^m (b \cos(c + dx))^n (A + C \cos^2(c + dx)) dx = ((a \cos(c + dx))^{-n} (b \cos(c + dx))^n) \int (a \cos(c + dx))^{m+n} dx$$

$$= \frac{C(a \cos(c + dx))^{1+m} (b \cos(c + dx))^n \sin(c + dx)}{ad(2 + m + n)} + \left(\frac{A}{ad(2 + m + n)} \right) \int (a \cos(c + dx))^{m+n} dx$$

$$= \frac{C(a \cos(c + dx))^{1+m} (b \cos(c + dx))^n \sin(c + dx)}{ad(2 + m + n)} - \frac{A}{ad(2 + m + n)} \int (a \cos(c + dx))^{m+n} dx$$

Mathematica [A] time = 0.239387, size = 132, normalized size = 0.92

$$\frac{\sqrt{\sin^2(c + dx)} \cot(c + dx) (a \cos(c + dx))^m (b \cos(c + dx))^n \left(A(m + n + 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m + n + 1); \frac{1}{2}(m + n + 3); \cos^2(c + dx)\right) + C(1 + m + n) \cos(c + dx) {}_2F_1\left[\frac{1}{2}, \frac{3 + m + n}{2}, \frac{5 + m + n}{2}, \cos(c + dx)\right] \right) \sqrt{\sin(c + dx)^2}}{d(m + n + 1)(m + n + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]

[Out] -(((a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + m + n)*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2] + C*(1 + m + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + m + n)*(3 + m + n)))

Maple [F] time = 2.474, size = 0, normalized size = 0.

$$\int (\cos(dx + c) a)^m (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(d*x+c)*a)^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)

[Out] int((cos(d*x+c)*a)^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (a \cos(dx + c))^m (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right) (a \cos(dx + c))^m (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (a \cos(dx + c))^m (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

3.183 $\int \cos^2(c+dx)(b \cos(c+dx))^n (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=117

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{n+3}}{b^3 d(n + 4)} - \frac{(A(n + 4) + C(n + 3)) \sin(c + dx)(b \cos(c + dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c + dx)\right)}{b^3 d(n + 3)(n + 4) \sqrt{\sin^2(c + dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.105195, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{n+3}}{b^3 d(n + 4)} - \frac{(A(n + 4) + C(n + 3)) \sin(c + dx)(b \cos(c + dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c + dx)\right)}{b^3 d(n + 3)(n + 4) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^2(c+dx)(b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{2+n} (A+C \cos^2(c+dx)) dx}{b^2} \\ &= \frac{C(b \cos(c+dx))^{3+n} \sin(c+dx)}{b^3 d(4+n)} + \frac{\left(A + \frac{C(3+n)}{4+n}\right) \int (b \cos(c+dx))^{2+n} dx}{b^2} \\ &= \frac{C(b \cos(c+dx))^{3+n} \sin(c+dx)}{b^3 d(4+n)} - \frac{\left(A + \frac{C(3+n)}{4+n}\right) (b \cos(c+dx))^{2+n}}{b^3 d} \end{aligned}$$

Mathematica [A] time = 0.199681, size = 122, normalized size = 1.04

$$\frac{\sqrt{\sin^2(c+dx)} \cos^2(c+dx) \cot(c+dx) (b \cos(c+dx))^n \left(A(n+5) {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right) + C(n+3) \cos^2(c+dx) \right)}{d(n+3)(n+5)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(5 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + n)*(5 + n))

Maple [F] time = 1.911, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^2 (b \cos(dx+c))^n (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^n \cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx+c)^2 + A) (b \cos(dx+c))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)
```

3.184 $\int \cos(c+dx)(b \cos(c+dx))^n (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=117

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{n+2}}{b^2 d(n + 3)} - \frac{(A(n + 3) + C(n + 2)) \sin(c + dx)(b \cos(c + dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c + dx)\right)}{b^2 d(n + 2)(n + 3) \sqrt{\sin^2(c + dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.10299, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3014, 2643}

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{n+2}}{b^2 d(n + 3)} - \frac{(A(n + 3) + C(n + 2)) \sin(c + dx)(b \cos(c + dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c + dx)\right)}{b^2 d(n + 2)(n + 3) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^(2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Ssin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos(c+dx)(b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx &= \frac{\int (b \cos(c+dx))^{1+n} (A+C \cos^2(c+dx)) dx}{b} \\ &= \frac{C(b \cos(c+dx))^{2+n} \sin(c+dx)}{b^2 d(3+n)} + \frac{\left(A + \frac{C(2+n)}{3+n}\right) \int (b \cos(c+dx))^n dx}{b} \\ &= \frac{C(b \cos(c+dx))^{2+n} \sin(c+dx)}{b^2 d(3+n)} - \frac{\left(A + \frac{C(2+n)}{3+n}\right) (b \cos(c+dx))^n}{b} \end{aligned}$$

Mathematica [A] time = 0.175528, size = 120, normalized size = 1.03

$$\frac{\sqrt{\sin^2(c+dx)} \cos(c+dx) \cot(c+dx) (b \cos(c+dx))^n \left(A(n+4) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right) + C(n+2) \cos^2(c+dx) \right)}{d(n+2)(n+4)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2),x]

[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(A*(4 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + n)*(4 + n))

Maple [F] time = 1.463, size = 0, normalized size = 0.

$$\int \cos(dx+c) (b \cos(dx+c))^n (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)

[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^n \cos(dx+c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx+c)^3 + A \cos(dx+c)) (b \cos(dx+c))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)
```

3.185 $\int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=117

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{n+1}}{bd(n+2)} - \frac{(A(n+2) + C(n+1)) \sin(c + dx)(b \cos(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c + dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0717034, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3014, 2643}

$$\frac{C \sin(c + dx)(b \cos(c + dx))^{n+1}}{bd(n+2)} - \frac{(A(n+2) + C(n+1)) \sin(c + dx)(b \cos(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])

Rule 3014

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) dx &= \frac{C(b \cos(c + dx))^{1+n} \sin(c + dx)}{bd(2 + n)} + \left(A + \frac{C(1 + n)}{2 + n}\right) \int (b \cos(c + dx))^n dx \\ &= \frac{C(b \cos(c + dx))^{1+n} \sin(c + dx)}{bd(2 + n)} - \frac{\left(A + \frac{C(1+n)}{2+n}\right) (b \cos(c + dx))^{1+n} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right)}{bd(1 + n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.162769, size = 114, normalized size = 0.97

$$\frac{\sqrt{\sin^2(c + dx)} \cot(c + dx)(b \cos(c + dx))^n \left(A(n + 3) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) + C(n + 1) \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right)\right)}{d(n + 1)(n + 3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2),x]

[Out] -(((b*cos[c + d*x])^n*Cot[c + d*x]*(A*(3 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n)*(3 + n))

Maple [F] time = 1.42, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) (b \cos(dx + c))^n, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n, x)

3.186 $\int (b \cos(c+dx))^n (A + C \cos^2(c + dx)) \sec(c+dx) dx$

Optimal. Leaf size=100

$$\frac{C \sin(c + dx)(b \cos(c + dx))^n}{d(n + 1)} - \frac{(An + A + Cn) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right)}{dn(n + 1)\sqrt{\sin^2(c + dx)}}$$

[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.0943137, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3014, 2643}

$$\frac{C \sin(c + dx)(b \cos(c + dx))^n}{d(n + 1)} - \frac{(An + A + Cn) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right)}{dn(n + 1)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int (b \cos(c + dx))^{-1+n} (A + C \cos^2(c + dx)) dx \\ &= \frac{C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + n)} + \frac{(b(A + An + Cn)) \int (b \cos(c + dx))^{-1+n} dx}{1 + n} \\ &= \frac{C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + n)} - \frac{(A + An + Cn)(b \cos(c + dx))^{-1+n}}{dn} \end{aligned}$$

Mathematica [A] time = 0.226902, size = 111, normalized size = 1.11

$$\frac{b \sqrt{\sin^2(c + dx)} \cot(c + dx) (b \cos(c + dx))^{n-1} \left(A(n+2) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right) + Cn \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right) \right)}{dn(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x],x]

[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(A*(2 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(2 + n)))

Maple [F] time = 1.338, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)
```


$$3.187 \quad \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec^2(c + dx) dx$$

Optimal. Leaf size=112

$$\frac{bC \sin(c + dx)(b \cos(c + dx))^{n-1}}{dn} - \frac{b(C(1-n) - An) \sin(c + dx)(b \cos(c + dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right)}{d(1-n)n\sqrt{\sin^2(c + dx)}}$$

[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.118526, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{bC \sin(c + dx)(b \cos(c + dx))^{n-1}}{dn} - \frac{b(C(1-n) - An) \sin(c + dx)(b \cos(c + dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right)}{d(1-n)n\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int (b \cos(c + dx))^{-2+n} (A + C \cos^2(c + dx)) dx \\ &= \frac{bC(b \cos(c + dx))^{-1+n} \sin(c + dx)}{dn} - \frac{(b^2(C(1 - n) - An)) \int (b \cos(c + dx))^{-2+n} dx}{n} \\ &= \frac{bC(b \cos(c + dx))^{-1+n} \sin(c + dx)}{dn} - \frac{b(C(1 - n) - An)(b \cos(c + dx))^{-2+n}}{d(n - 1)(n + 1)} \end{aligned}$$

Mathematica [A] time = 0.195237, size = 117, normalized size = 1.04

$$\frac{b\sqrt{\sin^2(c + dx)} \csc(c + dx) (b \cos(c + dx))^{n-1} \left(A(n + 1) {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right) + C(n - 1) \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right) \right)}{d(n - 1)(n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(A*(1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n)*(1 + n))

Maple [F] time = 1.25, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)
```

3.188 $\int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec^3(c + dx) dx$

Optimal. Leaf size=125

$$\frac{b^2(A(1-n) + C(2-n)) \sin(c+dx)(b \cos(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}} - \frac{b^2C \sin(c+dx)(b \cos(c+dx))^{n-2}}{d(1-n)}$$

[Out] $-\left(\frac{b^2 C (b \cos[c + dx])^{-2+n} \sin[c + dx]}{d(1-n)}\right) + \left(\frac{b^2 (A(1-n) + C(2-n)) (b \cos[c + dx])^{-2+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, (-2+n)/2, n/2, \cos^2[c + dx]\right] \sin[c + dx]}{d(1-n)(2-n)\sqrt{\sin^2[c + dx]}}\right)$

Rubi [A] time = 0.13244, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{b^2(A(1-n) + C(2-n)) \sin(c+dx)(b \cos(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}} - \frac{b^2C \sin(c+dx)(b \cos(c+dx))^{n-2}}{d(1-n)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \cos[c + dx])^n (A + C \cos^2[c + dx]) \sec^3[c + dx], x]$

[Out] $-\left(\frac{b^2 C (b \cos[c + dx])^{-2+n} \sin[c + dx]}{d(1-n)}\right) + \left(\frac{b^2 (A(1-n) + C(2-n)) (b \cos[c + dx])^{-2+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, (-2+n)/2, n/2, \cos^2[c + dx]\right] \sin[c + dx]}{d(1-n)(2-n)\sqrt{\sin^2[c + dx]}}\right)$

Rule 16

$\text{Int}[(u_.) \cdot (v_.)^{(m_.)} \cdot ((b_.) \cdot (v_))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u \cdot (b \cdot v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

$\text{Int}[(b_.) \cdot \sin[(e_.) + (f_.) \cdot (x_.)]^{(m_.)} \cdot ((A_.) + (C_.) \cdot \sin[(e_.) + (f_.) \cdot (x_.)]^{(n_.)}), x_Symbol] \rightarrow -\text{Simp}[(C \cos[e + f \cdot x] \cdot (b \sin[e + f \cdot x])^{(m+1)}) / (b \cdot f \cdot (m+2)), x] + \text{Dist}[(A \cdot (m+2) + C \cdot (m+1)) / (m+2), \text{Int}[(b \sin[e + f \cdot x])^m, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

$\text{Int}[(b_.) \cdot \sin[(c_.) + (d_.) \cdot (x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d \cdot x] \cdot (b \sin[c + d \cdot x])^{(n+1)} \cdot \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, (n+1)/2, (n+3)/2, \sin^2[c + d \cdot x]\right]) / (b \cdot d \cdot (n+1) \cdot \sqrt{\cos^2[c + d \cdot x]}), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int (b \cos(c + dx))^{-3+n} (A + C \cos^2(c + dx)) dx \\ &= -\frac{b^2 C (b \cos(c + dx))^{-2+n} \sin(c + dx)}{d(1-n)} + \left(b^3 \left(A + \frac{C(2-n)}{1-n} \right) \right) \\ &= -\frac{b^2 C (b \cos(c + dx))^{-2+n} \sin(c + dx)}{d(1-n)} + \frac{b^2 \left(A + \frac{C(2-n)}{1-n} \right) (b \cos(c + dx))^{-3+n}}{d(1-n)} \end{aligned}$$

Mathematica [A] time = 0.154865, size = 114, normalized size = 0.91

$$\frac{\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^2(c + dx) (b \cos(c + dx))^n \left(A n {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c + dx)\right) + C(n-2) \cos^2(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c + dx)\right) \right)}{d(n-2)n}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(A*n*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*sqrt[Sin[c + d*x]^2])/(d*(-2 + n)*n))

Maple [F] time = 1.506, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)
```

$$3.189 \quad \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec^4(c + dx) dx$$

Optimal. Leaf size=127

$$\frac{b^3(A(2-n) + C(3-n)) \sin(c + dx)(b \cos(c + dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{n-3}{2}; \frac{n-1}{2}; \cos^2(c + dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c + dx)}} - \frac{b^3 C \sin(c + dx)(b \cos(c + dx))}{d(2-n)}$$

[Out] $-\left(\frac{b^3 C (b \cos[c + dx])^{-3+n} \sin[c + dx]}{d(2-n)}\right) + \left(\frac{b^3 (A(2-n) + C(3-n)) (b \cos[c + dx])^{-3+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, (-3+n)/2, (-1+n)/2, \cos[c + dx]^2\right] \sin[c + dx]}{d(2-n)(3-n)\sqrt{\sin^2[c + dx]}}\right)$

Rubi [A] time = 0.125602, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {16, 3014, 2643}

$$\frac{b^3(A(2-n) + C(3-n)) \sin(c + dx)(b \cos(c + dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{n-3}{2}; \frac{n-1}{2}; \cos^2(c + dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c + dx)}} - \frac{b^3 C \sin(c + dx)(b \cos(c + dx))}{d(2-n)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \cos[c + dx])^n (A + C \cos^2[c + dx]) \sec^4[c + dx], x]$

[Out] $-\left(\frac{b^3 C (b \cos[c + dx])^{-3+n} \sin[c + dx]}{d(2-n)}\right) + \left(\frac{b^3 (A(2-n) + C(3-n)) (b \cos[c + dx])^{-3+n} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, (-3+n)/2, (-1+n)/2, \cos[c + dx]^2\right] \sin[c + dx]}{d(2-n)(3-n)\sqrt{\sin^2[c + dx]}}\right)$

Rule 16

$\text{Int}[(u_.) (v_.)^{(m_.)} ((b_.) (v_.)^{(n_.)}), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3014

$\text{Int}[(b_.) \sin[(e_.) + (f_.) (x_.)]^{(m_.)} ((A_.) + (C_.) \sin[(e_.) + (f_.) (x_.)]^{(2)}), x_Symbol] \rightarrow -\text{Simp}[(C \cos[e + f*x] (b \sin[e + f*x])^{(m+1)}) / (b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1)) / (m+2), \text{Int}[(b \sin[e + f*x])^m, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

$\text{Int}[(b_.) \sin[(c_.) + (d_.) (x_.)]^{(n_.)}, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d*x] (b \sin[c + d*x])^{(n+1)} \operatorname{Hypergeometric2F1}\left[\frac{1}{2}, (n+1)/2, (n+3)/2, \sin^2[c + d*x]\right]) / (b*d*(n+1)\sqrt{\cos^2[c + d*x]}), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int (b \cos(c + dx))^{-4+n} (A + C \cos^2(c + dx)) dx \\ &= -\frac{b^3 C (b \cos(c + dx))^{-3+n} \sin(c + dx)}{d(2-n)} + \left(b^4 \left(A + \frac{C(3-n)}{2-n} \right) \right) \int (b \cos(c + dx))^{-4+n} dx \\ &= -\frac{b^3 C (b \cos(c + dx))^{-3+n} \sin(c + dx)}{d(2-n)} + \frac{b^3 \left(A + \frac{C(3-n)}{2-n} \right) (b \cos(c + dx))^{-4+n}}{d(2-n)} \end{aligned}$$

Mathematica [A] time = 0.158083, size = 122, normalized size = 0.96

$$\frac{\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^3(c + dx) (b \cos(c + dx))^n \left(A(n-1) {}_2F_1\left(\frac{1}{2}, \frac{n-3}{2}; \frac{n-1}{2}; \cos^2(c + dx)\right) + C(n-3) \cos^2(c + dx) \right)}{d(n-3)(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] -(((b*cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + n)*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2] + C*(-3 + n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^3*sqrt[Sin[c + d*x]^2])/(d*(-3 + n)*(-1 + n)))

Maple [F] time = 1.289, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\sec(dx + c))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)
```

3.190 $\int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=142

$$\frac{2C \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n}{d(2n + 9)} - \frac{2(A(2n + 9) + C(2n + 7)) \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{4}, \frac{11 + 2n}{4}, \cos^2(c + dx)\right)}{d(2n + 7)(2n + 9)\sqrt{\sin^2(c + dx)}}$$

[Out] (2*C*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(9 + 2*n)) - (2*(C*(7 + 2*n) + A*(9 + 2*n))*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*(9 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.112071, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2C \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n}{d(2n + 9)} - \frac{2\left(\frac{A}{2n+7} + \frac{C}{2n+9}\right) \sin(c + dx) \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{11 + 2n}{4}, \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (2*C*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(9 + 2*n)) - (2*(A/(7 + 2*n) + C/(9 + 2*n))*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx &= (\cos^{-n}(c+dx)(b \cos(c+dx))^n) \int \cos^{\frac{5}{2}+n}(c+dx) (A+C \cos^2(c+dx)) dx \\ &= \frac{2C \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n \sin(c+dx)}{d(9+2n)} + \frac{\left(C \left(\frac{7}{2} + n \right) \right)}{d(9+2n)} \\ &= \frac{2C \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n \sin(c+dx)}{d(9+2n)} - \frac{2(C(7+2n))}{d(9+2n)} \end{aligned}$$

Mathematica [A] time = 0.213727, size = 140, normalized size = 0.99

$$\frac{2\sqrt{\sin^2(c+dx)} \cos^{\frac{7}{2}}(c+dx) \csc(c+dx)(b \cos(c+dx))^n \left(A(2n+1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+7); \frac{1}{4}(2n+1); \cos^2(c+dx)\right) + C \cos^2(c+dx) \right)}{d(2n+7)(2n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(11 + 2*n)*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(7 + 2*n)*(11 + 2*n))

Maple [F] time = 0.605, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{5}{2}} (b \cos(dx+c))^n (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^n \cos(dx+c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^4 + A \cos(dx+c)^2\right) (b \cos(dx+c))^n \sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + A*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)
```

$$3.191 \quad \int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \left(A + C \cos^2(c + dx) \right) dx$$

Optimal. Leaf size=142

$$\frac{2C \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n}{d(2n + 7)} - \frac{2(A(2n + 7) + C(2n + 5)) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1}{d(2n + 5)(2n + 7)\sqrt{\sin^2(c + dx)}}$$

[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2*(C*(5 + 2*n) + A*(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.110245, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2C \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n}{d(2n + 7)} - \frac{2\left(\frac{A}{2n+5} + \frac{C}{2n+7}\right) \sin(c + dx) \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 1), \frac{3}{4}(2n + 1), \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2*(A/(5 + 2*n) + C/(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n (A+C \cos^2(c+dx)) dx = (\cos^{-n}(c+dx)(b \cos(c+dx))^n) \int \cos^{\frac{3}{2}+n}(c+dx) (A+C \cos^2(c+dx)) dx$$

$$= \frac{2C \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \sin(c+dx)}{d(7+2n)} + \frac{\left(C \left(\frac{5}{2} + n \right) + \dots \right)}{d(7+2n)}$$

$$= \frac{2C \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n \sin(c+dx)}{d(7+2n)} - \frac{2(C(5+2n) + \dots)}{d(7+2n)}$$

Mathematica [A] time = 0.190696, size = 140, normalized size = 0.99

$$\frac{2\sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx) \csc(c+dx) (b \cos(c+dx))^n \left(A(2n+9) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+5); \frac{1}{4}(2n+9); \cos^2(c+dx)\right) + C(2n+9) \right)}{d(2n+5)(2n+9)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(9 + 2*n)*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(5 + 2*n)*(9 + 2*n))

Maple [F] time = 0.582, size = 0, normalized size = 0.

$$\int (\cos(dx+c))^{\frac{3}{2}} (b \cos(dx+c))^n (A+C(\cos(dx+c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^n \cos(dx+c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^3 + A \cos(dx+c)\right) (b \cos(dx+c))^n \sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.192 $\int \sqrt{\cos(c + dx)}(b \cos(c + dx))^n (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=142

$$\frac{2C \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n}{d(2n + 5)} - \frac{2(A(2n + 5) + C(2n + 3)) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{3 + 2n}{4}, \frac{7 + 2n}{4}, \cos^2(c + dx)\right)}{d(2n + 3)(2n + 5)\sqrt{\sin^2(c + dx)}}$$

[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2*(C*(3 + 2*n) + A*(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.105783, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2C \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n}{d(2n + 5)} - \frac{2\left(\frac{A}{2n+3} + \frac{C}{2n+5}\right) \sin(c + dx) \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}, \frac{1}{4}, \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2*(A/(3 + 2*n) + C/(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e+f*x]*(b*Sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*Sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n (A + C \cos^2(c+dx)) dx &= (\cos^{-n}(c+dx)(b \cos(c+dx))^n) \int \cos^{\frac{1}{2}+n}(c+dx) (A + C \cos^2(c+dx)) dx \\ &= \frac{2C \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \sin(c+dx)}{d(5+2n)} + \frac{\left(C \left(\frac{3}{2} + n \right) \right)}{d(5+2n)} \\ &= \frac{2C \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \sin(c+dx)}{d(5+2n)} - \frac{2(C(3+2n))}{d(5+2n)} \end{aligned}$$

Mathematica [A] time = 0.170159, size = 140, normalized size = 0.99

$$\frac{2\sqrt{\sin^2(c+dx)} \cos^{\frac{3}{2}}(c+dx) \csc(c+dx)(b \cos(c+dx))^n \left(A(2n+7) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \cos^2(c+dx)\right) + C \right)}{d(2n+3)(2n+7)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2), x]

[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(7 + 2*n)*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(3 + 2*n)*(7 + 2*n))

Maple [F] time = 0.606, size = 0, normalized size = 0.

$$\int (b \cos(dx+c))^n (A + C (\cos(dx+c))^2) \sqrt{\cos(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx+c)^2 + A) (b \cos(dx+c))^n \sqrt{\cos(dx+c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^2 + A\right) (b \cos(dx+c))^n \sqrt{\cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A) (b \cos(dx + c))^n \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

$$3.193 \quad \int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=140

$$\frac{2C \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n}{d(2n+3)} - \frac{2(A(2n+3) + 2Cn + C) \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1+2n}{4}, \frac{5+2n}{4}, \cos^2(c+dx)\right) \sin(c+dx)}{d(2n+1)(2n+3) \sqrt{\sin^2(c+dx)}}$$

[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.101933, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2C \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n}{d(2n+3)} - \frac{2(A(2n+3) + 2Cn + C) \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1+2n}{4}, \frac{5+2n}{4}, \cos^2(c+dx)\right) \sin(c+dx)}{d(2n+1)(2n+3) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]

[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] := -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2) + C*(m+1))/(m+2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (A + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) (A + C \cos^2(c + dx)) dx \\ &= \frac{2C\sqrt{\cos(c + dx)}(b \cos(c + dx))^n \sin(c + dx)}{d(3 + 2n)} + \frac{\left(\left(C\left(\frac{1}{2} + n\right) + A\left(\frac{3}{2} + n\right)\right)\right)}{d(3 + 2n)} \\ &= \frac{2C\sqrt{\cos(c + dx)}(b \cos(c + dx))^n \sin(c + dx)}{d(3 + 2n)} - \frac{2(C + 2Cn + A(3 + 2n))\sqrt{\cos(c + dx)}}{d(3 + 2n)} \end{aligned}$$

Mathematica [A] time = 0.170331, size = 140, normalized size = 1.

$$\frac{2\sqrt{\sin^2(c + dx)}\sqrt{\cos(c + dx)}\csc(c + dx)(b \cos(c + dx))^n \left(A(2n + 5) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 1); \frac{1}{4}(2n + 5); \cos^2(c + dx)\right) + C(2n + 5) \right)}{d(2n + 1)(2n + 5)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(A*(5 + 2*n)*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(1 + 2*n)*(5 + 2*n))

Maple [F] time = 0.74, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) \frac{1}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

$$3.194 \quad \int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=136

$$\frac{2(2An + A - C(1 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right)}{d(1 - 4n^2) \sqrt{\sin^2(c + dx)} \sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^n}{d(2n + 1) \sqrt{\cos(c + dx)}}$$

[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.099564, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2(2An + A - C(1 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right)}{d(1 - 4n^2) \sqrt{\sin^2(c + dx)} \sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)(b \cos(c + dx))^n}{d(2n + 1) \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3014

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((A_.) + (C_)*sin[(e_.) + (f_)*(x_)]^(n_)), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_)), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(b \cos(c + dx))^n (A + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) (A + C \cos^2(c + dx)) dx$$

$$= \frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + 2n)\sqrt{\cos(c + dx)}} + \frac{\left(\left(C\left(-\frac{1}{2} + n\right) + A\left(\frac{1}{2} + n\right)\right) \cos^{-n}(c + dx)\right)}{d(1 - 4n)}$$

$$= \frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + 2n)\sqrt{\cos(c + dx)}} + \frac{2(A - C(1 - 2n) + 2An)(b \cos(c + dx))^n}{d(1 - 4n)}$$

Mathematica [A] time = 0.164108, size = 140, normalized size = 1.03

$$\frac{2\sqrt{\sin^2(c + dx)} \csc(c + dx) (b \cos(c + dx))^n \left(A(2n + 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right) + C(2n - 1) \cos^2(c + dx) \right)}{d(2n - 1)(2n + 3)\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (-2*(b*cos[c + d*x])^n*Csc[c + d*x]*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(-1 + 2*n)*(3 + 2*n)*Sqrt[Cos[c + d*x]])

Maple [F] time = 0.702, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

$$3.195 \quad \int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=140

$$\frac{2(-2An + A + C(3 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right)}{d(1 - 2n)(3 - 2n)\sqrt{\sin^2(c + dx)} \cos^{\frac{3}{2}}(c + dx)} - \frac{2C \sin(c + dx)(b \cos(c + dx))^n}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)}$$

[Out] $(-2*C*(b*\text{Cos}[c + d*x])^n*\text{Sin}[c + d*x])/(d*(1 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}) + (2*(A + C*(3 - 2*n) - 2*A*n)*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rubi [A] time = 0.112909, antiderivative size = 132, normalized size of antiderivative = 0.94, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2\left(\frac{A}{3-2n} + \frac{C}{1-2n}\right) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)} \cos^{\frac{3}{2}}(c + dx)} - \frac{2C \sin(c + dx)(b \cos(c + dx))^n}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^n*(A + C*\text{Cos}[c + d*x]^2)/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(-2*C*(b*\text{Cos}[c + d*x])^n*\text{Sin}[c + d*x])/(d*(1 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}) + (2*(C/(1 - 2*n) + A/(3 - 2*n))*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3014

$\text{Int}[(b_.*\sin[e_.] + (f_.)*(x_))]^{(m_)}*((A_.) + (C_.)*\sin[e_.] + (f_.)*(x_))^{(n_)}, x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(m+2), \text{Int}[(b*\text{Sin}[e + f*x])^{(m)}, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

$\text{Int}[(b_.*\sin[c_.] + (d_.)*(x_))]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(b \cos(c + dx))^n (A + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) (A + C \cos^2(c + dx)) dx$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)} + \frac{\left(\left(C\left(-\frac{3}{2} + n\right) + A\left(-\frac{1}{2} + n\right)\right) \cos^{-n}(c + dx)\right)}{d(1 - 2n)(3 - 2n)}$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)} + \frac{2(A(1 - 2n) + C(3 - 2n))(b \cos(c + dx))^n}{d(1 - 2n)(3 - 2n)}$$

Mathematica [A] time = 0.166223, size = 140, normalized size = 1.

$$\frac{2\sqrt{\sin^2(c + dx)} \operatorname{csc}(c + dx) (b \cos(c + dx))^n \left(A(2n + 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right) + C(2n - 3) \cos^2(c + dx) \right)}{d(2n - 3)(2n + 1) \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (-2*(b*cos[c + d*x])^n*Csc[c + d*x]*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-3 + 2*n)*(1 + 2*n)*Cos[c + d*x]^(3/2))

Maple [F] time = 0.612, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

$$3.196 \quad \int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=142

$$\frac{2(A(3-2n) + C(5-2n)) \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d(3-2n)(5-2n)\sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx)} - \frac{2C \sin(c+dx)(b \cos(c+dx))^n}{d(3-2n) \cos^{\frac{5}{2}}(c+dx)}$$

[Out] $(-2*C*(b*\text{Cos}[c + d*x])^n*\text{Sin}[c + d*x])/(d*(3 - 2*n)*\text{Cos}[c + d*x]^{(5/2)}) + (2*(A*(3 - 2*n) + C*(5 - 2*n))*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(d*(3 - 2*n)*(5 - 2*n)*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rubi [A] time = 0.115321, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$\frac{2\left(\frac{A}{5-2n} + \frac{C}{3-2n}\right) \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx)} - \frac{2C \sin(c+dx)(b \cos(c+dx))^n}{d(3-2n) \cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2))/cos[c + d*x]^(7/2), x]

[Out] $(-2*C*(b*\text{Cos}[c + d*x])^n*\text{Sin}[c + d*x])/(d*(3 - 2*n)*\text{Cos}[c + d*x]^{(5/2)}) + (2*(C/(3 - 2*n) + A/(5 - 2*n))*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(d*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((A_)+(C_)*sin[(e_)+(f_)*(x_)]^(n_)), x_Symbol] := -Simp[(C*cos[e+f*x]*(b*sin[e+f*x])^(m+1))/(b*f*(m+2)), x] + Dist[(A*(m+2)+C*(m+1))/(m+2), Int[(b*sin[e+f*x])^m, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_)), x_Symbol] := Simp[(cos[c+d*x]*(b*sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, sin[c+d*x]^2])/(b*d*(n+1)*Sqrt[cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(b \cos(c + dx))^n (A + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) (A + C \cos^2(c + dx)) dx$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(3 - 2n) \cos^{\frac{5}{2}}(c + dx)} + \frac{\left(\left(C \left(-\frac{5}{2} + n \right) + A \left(-\frac{3}{2} + n \right) \right) \cos^{-n}(c + dx) \right)}{d(3 - 2n)}$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(3 - 2n) \cos^{\frac{5}{2}}(c + dx)} + \frac{2(A(3 - 2n) + C(5 - 2n))(b \cos(c + dx))^n}{d(3 - 2n)}$$

Mathematica [A] time = 0.173177, size = 140, normalized size = 0.99

$$\frac{2\sqrt{\sin^2(c + dx)} \operatorname{csc}(c + dx) (b \cos(c + dx))^n \left(A(2n - 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 5); \frac{1}{4}(2n - 1); \cos^2(c + dx)\right) + C(2n - 5) \cos^2(c + dx) \right)}{d(2n - 5)(2n - 1) \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (-2*(b*cos[c + d*x])^n*Csc[c + d*x]*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-5 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(5/2))

Maple [F] time = 0.631, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

$$3.197 \quad \int \frac{(b \cos(c+dx))^n (A+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=142

$$\frac{2(A(5-2n) + C(7-2n)) \sin(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-7); \frac{1}{4}(2n-3); \cos^2(c+dx)\right)}{d(5-2n)(7-2n) \sqrt{\sin^2(c+dx)} \cos^{\frac{7}{2}}(c+dx)} - \frac{2C \sin(c+dx) (b \cos(c+dx))^n}{d(5-2n) \cos^{\frac{7}{2}}(c+dx)}$$

[Out] $(-2*C*(b*\text{Cos}[c+d*x])^n*\text{Sin}[c+d*x])/(d*(5-2*n)*\text{Cos}[c+d*x]^{(7/2)}) + (2*(A*(5-2*n) + C*(7-2*n))*(b*\text{Cos}[c+d*x])^n*\text{Hypergeometric2F1}[1/2, (-7+2*n)/4, (-3+2*n)/4, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(5-2*n)*(7-2*n)*\text{Cos}[c+d*x]^{(7/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rubi [A] time = 0.112824, antiderivative size = 132, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {20, 3014, 2643}

$$2 \left(\frac{A}{7-2n} + \frac{C}{5-2n} \right) \frac{\sin(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-7); \frac{1}{4}(2n-3); \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)} \cos^{\frac{7}{2}}(c+dx)} - \frac{2C \sin(c+dx) (b \cos(c+dx))^n}{d(5-2n) \cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c+d*x])^n*(A + C*\text{Cos}[c+d*x]^2)/\text{Cos}[c+d*x]^{(9/2)}, x]$

[Out] $(-2*C*(b*\text{Cos}[c+d*x])^n*\text{Sin}[c+d*x])/(d*(5-2*n)*\text{Cos}[c+d*x]^{(7/2)}) + (2*(C/(5-2*n) + A/(7-2*n))*(b*\text{Cos}[c+d*x])^n*\text{Hypergeometric2F1}[1/2, (-7+2*n)/4, (-3+2*n)/4, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*\text{Cos}[c+d*x]^{(7/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_*))^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3014

$\text{Int}[(b_* \sin(e_*) + (f_*)*(x_*))^{(m_*)}((A_*) + (C_*) \sin(e_*) + (f_*)*(x_*))^2), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e+f*x]*(b*\text{Sin}[e+f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[(A*(m+2) + C*(m+1))/(m+2), \text{Int}[(b*\text{Sin}[e+f*x])^{(m)}, x], x] /;$ FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2643

$\text{Int}[(b_* \sin(c_*) + (d_*)*(x_*))^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\text{Sin}[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(b \cos(c + dx))^n (A + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{9}{2}+n}(c + dx) (A + C \cos^2(c + dx)) dx$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(5 - 2n) \cos^{\frac{7}{2}}(c + dx)} + \frac{\left(\left(C\left(-\frac{7}{2} + n\right) + A\left(-\frac{5}{2} + n\right)\right) \cos^{-n}(c + dx)\right)}{d(5 - 2n) \cos^{\frac{7}{2}}(c + dx)}$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(5 - 2n) \cos^{\frac{7}{2}}(c + dx)} + \frac{2(A(5 - 2n) + C(7 - 2n))(b \cos(c + dx))^n}{d(5 - 2n) \cos^{\frac{7}{2}}(c + dx)}$$

Mathematica [A] time = 0.171402, size = 140, normalized size = 0.99

$$\frac{2\sqrt{\sin^2(c + dx)} \csc(c + dx) (b \cos(c + dx))^n \left(A(2n - 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 7); \frac{1}{4}(2n - 3); \cos^2(c + dx)\right) + C(2n - 7) \cos^2(c + dx) \right)}{d(2n - 7)(2n - 3) \cos^{\frac{7}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^n*(A + C*cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (-2*(b*cos[c + d*x])^n*Csc[c + d*x]*(A*(-3 + 2*n)*Hypergeometric2F1[1/2, (-7 + 2*n)/4, (-3 + 2*n)/4, Cos[c + d*x]^2] + C*(-7 + 2*n)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-7 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(7/2))

Maple [F] time = 0.644, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x)

[Out] int((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)

3.198 $\int (a + a \cos(e + fx))^m (A + C \cos^2(e + fx)) dx$

Optimal. Leaf size=170

$$\frac{2^{m+\frac{1}{2}} \left(A(m^2 + 3m + 2) + C(m^2 + m + 1) \right) \sin(e + fx) (\cos(e + fx) + 1)^{-m-\frac{1}{2}} (a \cos(e + fx) + a)^m {}_2F_1 \left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2} (1 - \cos(e + fx)) \right)}{f(m+1)(m+2)}$$

[Out] -((C*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(2 + 3*m + m^2))) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rubi [A] time = 0.208652, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.16$, Rules used = {3024, 2751, 2652, 2651}

$$\frac{2^{m+\frac{1}{2}} \left(A(m^2 + 3m + 2) + C(m^2 + m + 1) \right) \sin(e + fx) (\cos(e + fx) + 1)^{-m-\frac{1}{2}} (a \cos(e + fx) + a)^m {}_2F_1 \left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2} (1 - \cos(e + fx)) \right)}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2), x]

[Out] -((C*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(2 + 3*m + m^2))) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Sinp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2751

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m]/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n +
1/2)*a^(n - 1/2)*b*cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
- (b*sin[c + d*x])/a))/2])/(d*Sqrt[a + b*sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \cos(e + fx))^m (A + C \cos^2(e + fx)) dx &= \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} + \frac{\int (a + a \cos(e + fx))^m (aC \cos^2(e + fx) + A) dx}{af(2 + m)} \\ &= -\frac{C(a + a \cos(e + fx))^m \sin(e + fx)}{f(2 + 3m + m^2)} + \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} \\ &= -\frac{C(a + a \cos(e + fx))^m \sin(e + fx)}{f(2 + 3m + m^2)} + \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} \\ &= -\frac{C(a + a \cos(e + fx))^m \sin(e + fx)}{f(2 + 3m + m^2)} + \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} \end{aligned}$$

Mathematica [C] time = 1.4133, size = 242, normalized size = 1.42

$$i4^{-m-1} (1 + e^{i(e+fx)}) e^{-i(m+2)(e+fx)} \left(e^{-\frac{1}{2}i(e+fx)} (1 + e^{i(e+fx)}) \right)^{2m} \cos^{-2m} \left(\frac{1}{2}(e + fx) \right) (a(\cos(e + fx) + 1))^m ((m + 2)e^{i(m+2)(e+fx)})$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + a*cos[e + f*x])^m*(A + C*cos[e + f*x]^2), x]

[Out] (I*4^(-1 - m)*(1 + E^(I*(e + f*x))))*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*(a*(1 + Cos[e + f*x]))^m*(C*E^(I*m*(e + f*x))*(-2 + m)*Hypergeometric2F1[1, -1 + m, -1 - m, -E^(I*(e + f*x))] + E^(I*(2 + m)*(e + f*x))*(2 + m)*(2*(2*A + C)*(-2 + m)*Hypergeometric2F1[1, 1 + m, 1 - m, -E^(I*(e + f*x))] + C*E^((2*I)*(e + f*x))*Hypergeometric2F1[1, 3 + m, 3 - m, -E^(I*(e + f*x))]))/(E^(I*(2 + m)*(e + f*x))*f*(-2 + m)*m*(2 + m)*Cos[(e + f*x)/2]^(2*m))

Maple [F] time = 1.687, size = 0, normalized size = 0.

$$\int (a + a \cos(fx + e))^m (A + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2), x)

[Out] int((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + A)(a \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")

[Out] integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(fx + e)^2 + A\right)\left(a \cos(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")

[Out] integral((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \left(\cos(e + fx) + 1\right)\right)^m \left(A + C \cos^2(e + fx)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)

[Out] Integral((a*(cos(e + f*x) + 1))**m*(A + C*cos(e + f*x)**2), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(fx + e)^2 + A\right)\left(a \cos(fx + e) + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")

[Out] integrate((C*cos(f*x + e)^2 + A)*(a*cos(f*x + e) + a)^m, x)

3.199 $\int (a + a \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=135

$$\frac{(40A + 19C) \sin(c + dx)(a \cos(c + dx) + a)^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{10 \cdot 2^{5/6} d (\cos(c + dx) + 1)^{7/6}} + \frac{3C \sin(c + dx)(a \cos(c + dx) + a)^{5/3}}{8ad}$$

[Out] $(-9*C*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Sin}[c + d*x])/(40*d) + (3*C*(a + a*\text{Cos}[c + d*x])^{(5/3)}*\text{Sin}[c + d*x])/(8*a*d) + ((40*A + 19*C)*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Hypergeometric2F1}[-1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(10*2^{(5/6)}*d*(1 + \text{Cos}[c + d*x])^{(7/6)})$

Rubi [A] time = 0.167684, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3024, 2751, 2652, 2651}

$$\frac{(40A + 19C) \sin(c + dx)(a \cos(c + dx) + a)^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{10 \cdot 2^{5/6} d (\cos(c + dx) + 1)^{7/6}} + \frac{3C \sin(c + dx)(a \cos(c + dx) + a)^{5/3}}{8ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(2/3)}*(A + C*\text{Cos}[c + d*x]^2), x]$

[Out] $(-9*C*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Sin}[c + d*x])/(40*d) + (3*C*(a + a*\text{Cos}[c + d*x])^{(5/3)}*\text{Sin}[c + d*x])/(8*a*d) + ((40*A + 19*C)*(a + a*\text{Cos}[c + d*x])^{(2/3)}*\text{Hypergeometric2F1}[-1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(10*2^{(5/6)}*d*(1 + \text{Cos}[c + d*x])^{(7/6)})$

Rule 3024

$\text{Int}[(a + b*\sin[(e + f*x)])^{(m)}*((A + C*\sin[(e + f*x)] + (f*x)^2)], x_Symbol] := -\text{Simp}[(C*\cos[e + f*x]*(a + b*\sin[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\sin[e + f*x])^{(m)}*\text{Simp}[A*b*(m+2) + b*C*(m+1) - a*C*\sin[e + f*x], x], x] /; \text{FreeQ}\{a, b, e, f, A, C, m\}, x] \&\amp; !\text{LtQ}[m, -1]$

Rule 2751

$\text{Int}[(a + b*\sin[(e + f*x)])^{(m)}*((c + d*\sin[(e + f*x)] + (f*x)^2)], x_Symbol] := -\text{Simp}[(d*\cos[e + f*x]*(a + b*\sin[e + f*x])^{(m+1)})/(f*(m+1)), x] + \text{Dist}[(a*d*m + b*c*(m+1))/(b*(m+1)), \text{Int}[(a + b*\sin[e + f*x])^{(m)}, x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\amp; \text{NeQ}[b*c - a*d, 0] \&\amp; \text{EqQ}[a^2 - b^2, 0] \&\amp; !\text{LtQ}[m, -2^{(-1)}]$

Rule 2652

$\text{Int}[(a + b*\sin[(c + d*x)])^{(n)}], x_Symbol] := \text{Dist}[(a^{\text{IntPart}[n]}*(a + b*\sin[c + d*x])^{\text{FracPart}[n]})/(1 + (b*\sin[c + d*x])/a)^{\text{FracPart}[n]}, \text{Int}[(1 + (b*\sin[c + d*x])/a)^n, x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\amp; \text{EqQ}[a^2 - b^2, 0] \&\amp; !\text{IntegerQ}[2*n] \&\amp; !\text{GtQ}[a, 0]$

Rule 2651

$\text{Int}[(a + b*\sin[(c + d*x)])^{(n)}], x_Symbol] := -\text{Simp}[(2^{(n+1/2)}*a^{(n-1/2)}*b*\cos[c + d*x]*\text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1*(1$

- (b*Sin[c + d*x])/a))/2)]/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx &= \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{8ad} + \frac{3 \int (a + a \cos(c + dx))^{2/3} \left(\frac{1}{3}a(8\right)}{8} \\ &= -\frac{9C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{40d} + \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{8ad} \\ &= -\frac{9C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{40d} + \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{8ad} \\ &= -\frac{9C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{40d} + \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{8ad} \end{aligned}$$

Mathematica [A] time = 0.793286, size = 175, normalized size = 1.3

$$\frac{\sec^2\left(\frac{1}{2}(c + dx)\right) (a(\cos(c + dx) + 1))^{2/3} \left(6 \cdot 2^{5/6} \sin(c + dx)(40A + 14C \cos(c + dx) + 5C \cos(2(c + dx)) + 28C) \sqrt[6]{1 - \cos(dx - 2 \operatorname{ArcTan}[\cot(c/2)])}\right)}{320 \cdot 2^{5/6} d \sqrt[6]{1 - \cos(dx - 2 \operatorname{ArcTan}[\cot(c/2)])}}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] ((a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*(6*2^(5/6)*(40*A + 28*C + 14*C*Cos[c + d*x] + 5*C*Cos[2*(c + d*x)]))*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6)*Sin[c + d*x] - 4*(40*A + 19*C)*Hypergeometric2F1[1/2, 5/6, 3/2, Cos[(d*x)/2 - ArcTan[Cot[c/2]]]^2]*Sin[d*x - 2*ArcTan[Cot[c/2]]]))/(320*2^(5/6)*d*(1 - Cos[d*x - 2*ArcTan[Cot[c/2]]])^(1/6))

Maple [F] time = 0.371, size = 0, normalized size = 0.

$$\int (a + \cos(dx + c) a)^{2/3} (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cos(d*x+c)*a)^(2/3)*(A+C*cos(d*x+c)^2), x)

[Out] int((a+cos(d*x+c)*a)^(2/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(a \cos(dx + c) + a)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)\left(a \cos(dx + c) + a\right)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right)\left(a \cos(dx + c) + a\right)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(2/3), x)

3.200 $\int \sqrt[3]{a + a \cos(c + dx)} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=135

$$\frac{(28A + 13C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{14\sqrt[6]{2d}(\cos(c + dx) + 1)^{5/6}} + \frac{3C \sin(c + dx)(a \cos(c + dx) + a)^{4/3}}{7ad} - \frac{9C}{7ad}$$

[Out] $(-9*C*(a + a*\text{Cos}[c + d*x])^{(1/3)}*\text{Sin}[c + d*x])/(28*d) + (3*C*(a + a*\text{Cos}[c + d*x])^{(4/3)}*\text{Sin}[c + d*x])/(7*a*d) + ((28*A + 13*C)*(a + a*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(14*2^{(1/6)}*d*(1 + \text{Cos}[c + d*x])^{(5/6)})$

Rubi [A] time = 0.160788, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3024, 2751, 2652, 2651}

$$\frac{(28A + 13C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{14\sqrt[6]{2d}(\cos(c + dx) + 1)^{5/6}} + \frac{3C \sin(c + dx)(a \cos(c + dx) + a)^{4/3}}{7ad} - \frac{9C}{7ad}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a + a*\text{Cos}[c + d*x])^{(1/3)}*(A + C*\text{Cos}[c + d*x]^2), x]$

[Out] $(-9*C*(a + a*\text{Cos}[c + d*x])^{(1/3)}*\text{Sin}[c + d*x])/(28*d) + (3*C*(a + a*\text{Cos}[c + d*x])^{(4/3)}*\text{Sin}[c + d*x])/(7*a*d) + ((28*A + 13*C)*(a + a*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/6, 1/2, 3/2, (1 - \text{Cos}[c + d*x])/2]*\text{Sin}[c + d*x])/(14*2^{(1/6)}*d*(1 + \text{Cos}[c + d*x])^{(5/6)})$

Rule 3024

$\text{Int}[(a + b*\sin[(e + f*x)])^m * ((A + C*\sin[(e + f*x)] + (f*x)^2)], x_Symbol] :> -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m * \text{Simp}[A*b*(m+2) + b*C*(m+1) - a*C*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}\{a, b, e, f, A, C, m\}, x] \&\amp; !\text{LtQ}[m, -1]$

Rule 2751

$\text{Int}[(a + b*\sin[(e + f*x)])^m * ((c + d*\sin[(e + f*x)] + (f*x)^2)], x_Symbol] :> -\text{Simp}[(d*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^m)/(f*(m+1)), x] + \text{Dist}[(a*d*m + b*c*(m+1))/(b*(m+1)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\amp; \text{NeQ}[b*c - a*d, 0] \&\amp; \text{EqQ}[a^2 - b^2, 0] \&\amp; !\text{LtQ}[m, -2^{(-1)}]$

Rule 2652

$\text{Int}[(a + b*\sin[(c + d*x)])^n], x_Symbol] :> \text{Dist}[(a^{\text{IntPart}[n]} * (a + b*\text{Sin}[c + d*x])^{\text{FracPart}[n]}) / (1 + (b*\text{Sin}[c + d*x])/a)^{\text{FracPart}[n]}, \text{Int}[(1 + (b*\text{Sin}[c + d*x])/a)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\amp; \text{EqQ}[a^2 - b^2, 0] \&\amp; !\text{IntegerQ}[2*n] \&\amp; !\text{GtQ}[a, 0]$

Rule 2651

$\text{Int}[(a + b*\sin[(c + d*x)])^n], x_Symbol] :> -\text{Simp}[(2^{(n+1/2)} * a^{(n-1/2)} * b * \text{Cos}[c + d*x] * \text{Hypergeometric2F1}[1/2, 1/2 - n, 3/2, (1*(1$

- (b*Sin[c + d*x])/a))/2))/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \sqrt[3]{a + a \cos(c + dx)} (A + C \cos^2(c + dx)) dx &= \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{7ad} + \frac{3 \int \sqrt[3]{a + a \cos(c + dx)} \left(\frac{1}{3}a(7\right)}{7ad} \\ &= -\frac{9C\sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{28d} + \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{7ad} \\ &= -\frac{9C\sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{28d} + \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{7ad} \\ &= -\frac{9C\sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{28d} + \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{7ad} \end{aligned}$$

Mathematica [C] time = 0.83763, size = 240, normalized size = 1.78

$$3\sqrt[3]{a(\cos(c + dx) + 1)} \left(\frac{(28A+13C) \csc\left(\frac{c}{4}\right) \sec\left(\frac{c}{4}\right) \sqrt[3]{i \sin(c) e^{idx} + \cos(c) e^{idx} + 1} \left({}_2F_1\left(-\frac{1}{3}, \frac{1}{3}, \frac{2}{3}; -e^{idx}(\cos(c) + i \sin(c))\right) + e^{idx} {}_2F_1\left(\frac{1}{3}, \frac{2}{3}, \frac{5}{3}; -e^{idx}(\cos(c) + i \sin(c))\right) \right)}{i \sin\left(\frac{c}{2}\right) (-1 + e^{idx}) + \cos\left(\frac{c}{2}\right) (1 + e^{idx})} \right)$$

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Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*(a*(1 + Cos[c + d*x]))^(1/3)*(-4*(28*A + 13*C)*Cot[c/2] + 4*C*Cos[d*x]*Sin[c] + ((28*A + 13*C)*Csc[c/4]*(2*Hypergeometric2F1[-1/3, 1/3, 2/3, -(E^(I*d*x)*(Cos[c] + I*Sin[c]))] + E^(I*d*x)*Hypergeometric2F1[1/3, 2/3, 5/3, -(E^(I*d*x)*(Cos[c] + I*Sin[c]))])*Sec[c/4]*(1 + E^(I*d*x)*Cos[c] + I*E^(I*d*x)*Sin[c])^(1/3))/((1 + E^(I*d*x))*Cos[c/2] + I*(-1 + E^(I*d*x))*Sin[c/2]) + 8*C*Cos[2*d*x]*Sin[2*c] + 4*C*Cos[c]*Sin[d*x] + 8*C*Cos[2*c]*Sin[2*d*x]))/(112*d)

Maple [F] time = 0.302, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + \cos(dx + c)a} (A + C(\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cos(d*x+c)*a)^(1/3)*(A+C*cos(d*x+c)^2), x)

[Out] int((a+cos(d*x+c)*a)^(1/3)*(A+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(a \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)\left(a \cos(dx + c) + a\right)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + A\right)\left(a \cos(dx + c) + a\right)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)*(a*cos(d*x + c) + a)^(1/3), x)

$$3.201 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx$$

Optimal. Leaf size=135

$$\frac{(10A+7C) \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1-\cos(c+dx))\right)}{5 \cdot 2^{5/6} d \sqrt[6]{\cos(c+dx)+1} \sqrt[3]{a \cos(c+dx)+a}} + \frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{2/3}}{5ad} - \frac{9C \sin(c+dx)}{10d \sqrt[3]{a \cos(c+dx)+a}}$$

[Out] $(-9*C*\text{Sin}[c+d*x])/(10*d*(a+a*\text{Cos}[c+d*x])^{(1/3)}) + (3*C*(a+a*\text{Cos}[c+d*x])^{(2/3)}*\text{Sin}[c+d*x])/(5*a*d) + ((10*A+7*C)*\text{Hypergeometric2F1}[1/2, 5/6, 3/2, (1-\text{Cos}[c+d*x])/2]*\text{Sin}[c+d*x])/(5*2^{(5/6)}*d*(1+\text{Cos}[c+d*x])^{(1/6)}*(a+a*\text{Cos}[c+d*x])^{(1/3)})$

Rubi [A] time = 0.16098, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3024, 2751, 2652, 2651}

$$\frac{(10A+7C) \sin(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1-\cos(c+dx))\right)}{5 \cdot 2^{5/6} d \sqrt[6]{\cos(c+dx)+1} \sqrt[3]{a \cos(c+dx)+a}} + \frac{3C \sin(c+dx)(a \cos(c+dx)+a)^{2/3}}{5ad} - \frac{9C \sin(c+dx)}{10d \sqrt[3]{a \cos(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A+C*\text{Cos}[c+d*x]^2)/(a+a*\text{Cos}[c+d*x])^{(1/3)}, x]$

[Out] $(-9*C*\text{Sin}[c+d*x])/(10*d*(a+a*\text{Cos}[c+d*x])^{(1/3)}) + (3*C*(a+a*\text{Cos}[c+d*x])^{(2/3)}*\text{Sin}[c+d*x])/(5*a*d) + ((10*A+7*C)*\text{Hypergeometric2F1}[1/2, 5/6, 3/2, (1-\text{Cos}[c+d*x])/2]*\text{Sin}[c+d*x])/(5*2^{(5/6)}*d*(1+\text{Cos}[c+d*x])^{(1/6)}*(a+a*\text{Cos}[c+d*x])^{(1/3)})$

Rule 3024

$\text{Int}[(a_+ + (b_+)*\text{sin}[(e_+) + (f_+)*(x_+)])^{(m_+)}*((A_+) + (C_+)*\text{sin}[(e_+) + (f_+)*(x_+)]^2), x_Symbol] := -\text{Simp}[(C_+\text{Cos}[e_+ + f_+*x]*(a_+ + b_+\text{Sin}[e_+ + f_+*x])^{(m_+ + 1)})/(b_+*f_+(m_+ + 2)), x] + \text{Dist}[1/(b_+(m_+ + 2)), \text{Int}[(a_+ + b_+\text{Sin}[e_+ + f_+*x])^{m_+} \text{Simp}[A_+*b_+(m_+ + 2) + b_+*C_+(m_+ + 1) - a_+*C_+\text{Sin}[e_+ + f_+*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, C, m\}, x] \&\amp; \text{!LtQ}[m, -1]$

Rule 2751

$\text{Int}[(a_+ + (b_+)*\text{sin}[(e_+) + (f_+)*(x_+)])^{(m_+)}*((c_+) + (d_+)*\text{sin}[(e_+) + (f_+)*(x_+)]), x_Symbol] := -\text{Simp}[(d_+\text{Cos}[e_+ + f_+*x]*(a_+ + b_+\text{Sin}[e_+ + f_+*x])^m)/(f_+(m_+ + 1)), x] + \text{Dist}[(a_+*d_+*m_+ + b_+*c_+(m_+ + 1))/(b_+(m_+ + 1)), \text{Int}[(a_+ + b_+\text{Sin}[e_+ + f_+*x])^m, x], x] /; \text{FreeQ}\{a, b, c, d, e, f, m\}, x] \&\amp; \text{NeQ}[b_+*c_+ - a_+*d_+, 0] \&\amp; \text{EqQ}[a_+^2 - b_+^2, 0] \&\amp; \text{!LtQ}[m, -2^{(-1)}]$

Rule 2652

$\text{Int}[(a_+ + (b_+)*\text{sin}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x_Symbol] := \text{Dist}[(a_+^{\text{IntPart}[n_+]}*(a_+ + b_+\text{Sin}[c_+ + d_+*x])^{\text{FracPart}[n_+]})/(1 + (b_+\text{Sin}[c_+ + d_+*x])/a_+)^{\text{FracPart}[n_+]}, \text{Int}[(1 + (b_+\text{Sin}[c_+ + d_+*x])/a_+)^n, x], x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\amp; \text{EqQ}[a_+^2 - b_+^2, 0] \&\amp; \text{!IntegerQ}[2*n_+] \&\amp; \text{!GtQ}[a_+, 0]$

Rule 2651

$\text{Int}[(a_+ + (b_+)*\text{sin}[(c_+) + (d_+)*(x_+)])^{(n_+)}, x_Symbol] := -\text{Simp}[(2^{(n_+ + 1/2)}*a_+^{(n_+ - 1/2)}*b_+\text{Cos}[c_+ + d_+*x]*\text{Hypergeometric2F1}[1/2, 1/2 - n_+, 3/2, (1*(1$

$-(b \sin(c + dx)/a)/2)/(d \sqrt{a + b \sin(c + dx)}), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx &= \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{3 \int \frac{\frac{1}{3}a(5A+2C)-aC \cos(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx}{5a} \\ &= -\frac{9C \sin(c + dx)}{10d \sqrt[3]{a + a \cos(c + dx)}} + \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{1}{10}(10A + 7C) \int \frac{1}{\sqrt[3]{a + a \cos(c + dx)}} dx \\ &= -\frac{9C \sin(c + dx)}{10d \sqrt[3]{a + a \cos(c + dx)}} + \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{((10A + 7C) \sqrt[3]{1 + \cos(c + dx)})}{10 \sqrt[3]{a + a \cos(c + dx)}} \\ &= -\frac{9C \sin(c + dx)}{10d \sqrt[3]{a + a \cos(c + dx)}} + \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{(10A + 7C) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1 + \cos(c + dx)}{2}\right)}{5 \cdot 2^{5/6} d \sqrt[6]{1 + \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.440855, size = 144, normalized size = 1.07

$$\frac{2(10A + 7C) \sin\left(dx - 2 \tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \cos^2\left(\frac{dx}{2} - \tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)\right) + 3 \cdot 2^{5/6} C (\sin(c + dx) - \sin(2(c + dx)))}{20d \sqrt[3]{a(\cos(c + dx) + 1)} \sqrt[6]{\sin^2\left(\frac{dx}{2} - \tan^{-1}\left(\cot\left(\frac{c}{2}\right)\right)\right)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3), x]

[Out] $-(3 \cdot 2^{5/6} \cdot C \cdot (1 - \cos[dx - 2 \text{ArcTan}[\cot[c/2]]])^{1/6} \cdot (\sin[c + dx] - \sin[2(c + dx)]) + 2 \cdot (10A + 7C) \cdot \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5}{6}, \frac{3}{2}, \cos\left[\frac{(dx)}{2} - \text{ArcTan}[\cot[c/2]]\right]^2\right] \cdot \sin[dx - 2 \text{ArcTan}[\cot[c/2]]]) / (20 \cdot d \cdot (a \cdot (1 + \cos[c + dx]))^{1/3} \cdot (\sin[(dx)/2 - \text{ArcTan}[\cot[c/2]]]^2)^{1/6})$

Maple [F] time = 0.28, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{a + \cos(dx + c)} a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3), x)

[Out] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a)^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(1/3), x)

3.202 $\int \frac{A+C \cos^2(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx$

Optimal. Leaf size=138

$$\frac{(4A + 7C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt[6]{2ad}(\cos(c + dx) + 1)^{5/6}} + \frac{3(A + C) \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}} + \frac{3C \sin(c + dx) \sqrt[3]{a}}{4a}$$

[Out] (3*(A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A + 7*C)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))

Rubi [A] time = 0.176594, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {3024, 2750, 2652, 2651}

$$\frac{(4A + 7C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt[6]{2ad}(\cos(c + dx) + 1)^{5/6}} + \frac{3(A + C) \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}} + \frac{3C \sin(c + dx) \sqrt[3]{a}}{4a}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]

[Out] (3*(A + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A + 7*C)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2750

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2652

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1

- (b*Sin[c + d*x])/a))/2)]/(d*sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx &= \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} + \frac{3 \int \frac{\frac{1}{3}a(4A+C) - aC \cos(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx}{4a} \\ &= \frac{3(A + C) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} + \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} - \frac{(4A + 7C) \int \sqrt[3]{a + a \cos(c + dx)}}{4a} \\ &= \frac{3(A + C) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} + \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} - \frac{((4A + 7C) \sqrt[3]{a + a \cos(c + dx)})}{4a \sqrt[3]{1 + \cos(c + dx)}} \\ &= \frac{3(A + C) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} + \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} - \frac{(4A + 7C) \sqrt[3]{a + a \cos(c + dx)}}{4a \sqrt[3]{1 + \cos(c + dx)}} \end{aligned}$$

Mathematica [F] time = 0.133988, size = 0, normalized size = 0.

$$\int \frac{A + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx$$

Verification is Not applicable to the result.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]

[Out] Integrate[(A + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) (a + \cos(dx + c) a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3), x)

[Out] int((A+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(a*cos(d*x + c) + a)^(2/3), x)

3.203 $\int (a + b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=277

$$\frac{(3a^2C + b^2(8A + 5C)) \sin(c + dx)(a + b \cos(c + dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) - 3aC(a + b)}{4\sqrt{2}b^2d\sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*a*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) + ((3*a^2*C + b^2*(8*A + 5*C))*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3)

Rubi [A] time = 0.356095, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3024, 2756, 2665, 139, 138}

$$\frac{(3a^2C + b^2(8A + 5C)) \sin(c + dx)(a + b \cos(c + dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) - 3aC(a + b)}{4\sqrt{2}b^2d\sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*a*(a + b)*C*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) + ((3*a^2*C + b^2*(8*A + 5*C))*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3)

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)

$\sqrt[n]{(\text{Sqrt}[1 + x] \cdot \text{Sqrt}[1 - x])}, x], x, \text{Sin}[c + d \cdot x]], x] /;$ FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -(d*(a + b*x))/(b*c - a*d), -(f*(a + b*x))/(b*e - a*f)])/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])

Rubi steps

$$\int (a + b \cos(c + dx))^{2/3} (A + C \cos^2(c + dx)) dx = \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{3 \int (a + b \cos(c + dx))^{2/3} \left(\frac{1}{3}b(8A - 3C) + C \cos^2(c + dx)\right) dx}{8bd}$$

$$= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} - \frac{(3aC) \int (a + b \cos(c + dx))^{5/3} dx}{8b^2}$$

$$= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{(3aC \sin(c + dx)) \text{Subst}\left(\int \frac{u}{\sqrt{1-u}} du, \frac{a + b \cos(c + dx)}{\sqrt{1 - \cos(c + dx)}}\right)}{8b^2 d \sqrt{1 - \cos(c + dx)}}$$

$$= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} - \frac{(3a(-a - b)C(a + b \cos(c + dx)))}{8b^2 d \sqrt{1 - \cos(c + dx)}}$$

$$= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} - \frac{3a(a + b)CF_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{5}{3}; \frac{3}{2}; \frac{1}{2}\right)(1 - \cos(c + dx))^{5/3}}{4\sqrt{2}b^2 d}$$

Mathematica [A] time = 2.59434, size = 279, normalized size = 1.01

$$3 \csc(c + dx)(a + b \cos(c + dx))^{2/3} \left(4(-6a^2C + 40Ab^2 + 25b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} (a + b \cos(c + dx)) F_1\left(\frac{5}{3}; \frac{1}{2}, \frac{1}{2}, \frac{5}{3}\right) + 4(40A*b^2 - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1/2, 8/3, ($$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + C*Cos[c + d*x]^2),x]

[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(60*a*(a^2 - b^2)*C*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)])*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + 4*(40*A*b^2 - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1/2, 8/3, (

$a + b \cdot \cos[c + d \cdot x] / (a - b), (a + b \cdot \cos[c + d \cdot x] / (a + b)) \cdot \sqrt{-((b \cdot (-1 + \cos[c + d \cdot x])) / (a + b))} \cdot \sqrt{(b \cdot (1 + \cos[c + d \cdot x])) / (-a + b)} \cdot (a + b \cdot \cos[c + d \cdot x]) - 20 \cdot b^2 \cdot C \cdot (2 \cdot a + 5 \cdot b \cdot \cos[c + d \cdot x]) \cdot \sin[c + d \cdot x]^2) / (800 \cdot b^3 \cdot d)$

Maple [F] time = 0.283, size = 0, normalized size = 0.

$$\int (a + b \cos(dx + c))^{\frac{2}{3}} (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)

[Out] int((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(b \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)(b \cos(dx + c) + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(b \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(2/3), x)
```

3.204 $\int \sqrt[3]{a + b \cos(c + dx)} (A + C \cos^2(c + dx)) dx$

Optimal. Leaf size=277

$$\frac{\sqrt{2}(3a^2C + b^2(7A + 4C)) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{7b^2d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{3\sqrt{2}aC(a + b \cos(c + dx))}{7b^2d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*Sqrt[2]*a*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3) + (Sqrt[2]*(3*a^2*C + b^2*(7*A + 4*C))*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3)

Rubi [A] time = 0.311047, antiderivative size = 277, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3024, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}(3a^2C + b^2(7A + 4C)) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{7b^2d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}} - \frac{3\sqrt{2}aC(a + b \cos(c + dx))}{7b^2d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) - (3*Sqrt[2]*a*(a + b)*C*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3) + (Sqrt[2]*(3*a^2*C + b^2*(7*A + 4*C))*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3)

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d

, n}], x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \sqrt[3]{a + b \cos(c + dx)} (A + C \cos^2(c + dx)) dx = \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{3 \int \sqrt[3]{a + b \cos(c + dx)} \left(\frac{1}{3}b(7A - 7b) - (3aC) \int (a + b \cos(c + dx))^{4/3} dx\right)}{7b^2}$$

$$= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} - \frac{(3aC \sin(c + dx)) \text{Subst}\left(\int \frac{(a + b \cos(c + dx))^{4/3}}{\sqrt{1 - \cos(c + dx)}} dx\right)}{7b^2 d \sqrt{1 - \cos(c + dx)}}$$

$$= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{(3a(-a - b)C \sqrt[3]{a + b \cos(c + dx)})}{7b^2 d \sqrt{1 - \cos(c + dx)}}$$

$$= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} - \frac{3\sqrt{2}a(a + b)CF_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{4}{3}; \frac{3}{2}; \frac{1}{2}\right)}{7b^2}$$

Mathematica [A] time = 2.51439, size = 276, normalized size = 1.

$$3 \csc(c + dx) \sqrt[3]{a + b \cos(c + dx)} \left((-3a^2C + 28Ab^2 + 16b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} (a + b \cos(c + dx)) F_1\left(\frac{4}{3}; \frac{1}{2}, \frac{1}{2}\right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + C*Cos[c + d*x]^2), x]

[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(12*a*(a^2 - b^2)*C*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)])*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + (28*A*b^2 - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Co

$s[c + d*x]))/(a + b)]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*C*(a + 4*b*Cos[c + d*x])*Sin[c + d*x]^2)/(112*b^3*d)$

Maple [F] time = 0.27, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + b \cos(dx + c)} (A + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)

[Out] int((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + A\right)(b \cos(dx + c) + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)*(b*cos(d*x + c) + a)^(1/3), x)
```


$$3.205 \quad \int \frac{A+C \cos^2(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$$

Optimal. Leaf size=274

$$\frac{\sqrt{2}(3a^2C + b^2(5A + 2C)) \sin(c + dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right) - 3\sqrt{2}aC \sin(c + dx)}{5b^2d\sqrt{\cos(c + dx) + 1} \sqrt[3]{a + b \cos(c + dx)}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) + (Sqrt[2]*(3*a^2*C + b^2*(5*A + 2*C))*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(1/3)

Rubi [A] time = 0.309737, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3024, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}(3a^2C + b^2(5A + 2C)) \sin(c + dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right) - 3\sqrt{2}aC \sin(c + dx)}{5b^2d\sqrt{\cos(c + dx) + 1} \sqrt[3]{a + b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*Sqrt[2]*a*C*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) + (Sqrt[2]*(3*a^2*C + b^2*(5*A + 2*C))*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(1/3)

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Sinp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)

$\sqrt[n]{\sqrt{1+x}\sqrt{1-x}}$, x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -(d*(a + b*x))/(b*c - a*d), -(f*(a + b*x))/(b*e - a*f)])/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplifierQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplifierQ[e + f*x, a + b*x]

Rubi steps

$$\int \frac{A + C \cos^2(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx = \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{3 \int \frac{\frac{1}{3}b(5A+2C)-aC \cos(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx}{5b}$$

$$= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{(3aC) \int (a + b \cos(c + dx))^{2/3} dx}{5b^2} + \frac{1}{5} \left(5A + \left(2 + \frac{3a^2}{b^2} \right) \right)$$

$$= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{(3aC \sin(c + dx)) \text{Subst} \left(\int \frac{(a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx) \right)}{5b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}}$$

$$= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{(3aC(a + b \cos(c + dx))^{2/3} \sin(c + dx)) \text{Subst} \left(\int \frac{(-a+bx)^{2/3}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx) \right)}{5b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}}$$

$$= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{3\sqrt{2}aCF_1 \left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)) \right), \frac{b(1-\cos(c+dx))}{a+b}}{5b^2 d \sqrt{1 + \cos(c + dx)} \left(\frac{a+b \cos(c + dx)}{a} \right)}$$

Mathematica [A] time = 1.69111, size = 256, normalized size = 0.93

$$3 \csc(c + dx)(a + b \cos(c + dx))^{2/3} \left(5(3a^2C + 5Ab^2 + 2b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} F_1 \left(\frac{2}{3}; \frac{1}{2}, \frac{1}{2}, \frac{5}{3}; \frac{a+b \cos(c+dx)}{a-b}, \frac{a-b \cos(c+dx)}{a+b} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]

[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 + 3*a^2*C + 2*b^2*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c

+ d*x]))/(-a + b)] - 6*a*C*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2)/(50*b^3*d)

Maple [F] time = 0.286, size = 0, normalized size = 0.

$$\int (A + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{a + b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3), x)

[Out] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(1/3), x)

$$3.206 \quad \int \frac{A+C \cos^2(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=272

$$\frac{(3a^2C + b^2(4A + C)) \sin(c + dx) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right) - 3aC \sin(c + dx) \sqrt[3]{\cos(c + dx)}}{2\sqrt{2}b^2d\sqrt{\cos(c + dx) + 1}(a + b \cos(c + dx))^{2/3}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*a*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3) + ((3*a^2*C + b^2*(4*A + C))*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(2/3)

Rubi [A] time = 0.314045, antiderivative size = 272, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {3024, 2756, 2665, 139, 138}

$$\frac{(3a^2C + b^2(4A + C)) \sin(c + dx) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right) - 3aC \sin(c + dx) \sqrt[3]{\cos(c + dx)}}{2\sqrt{2}b^2d\sqrt{\cos(c + dx) + 1}(a + b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) - (3*a*C*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3) + ((3*a^2*C + b^2*(4*A + C))*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*((a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(2/3)

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Sinp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)

$\int \frac{1}{\sqrt{1+x}\sqrt{1-x}} dx$, x, Sin[c + d*x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -(d*(a + b*x))/(b*c - a*d), -(f*(a + b*x))/(b*e - a*f)]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplerQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplerQ[e + f*x, a + b*x]

Rubi steps

$$\begin{aligned} \int \frac{A + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx &= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{3 \int \frac{1}{3} \frac{b(4A+C) - aC \cos(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx}{4b} \\ &= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} - \frac{(3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx}{4b^2} + \frac{1}{4} \left(4A + C + \frac{3a^2C}{b^2} \right) \\ &= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{(3aC \sin(c + dx)) \text{Subst} \left(\int \frac{\sqrt[3]{a+bx}}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cos(c + dx) \right)}{4b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\ &= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{(3aC \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)) \text{Subst} \left(\int \frac{\sqrt[3]{\frac{a}{-a-1}}}{\sqrt{1-x}} dx, x, \cos(c + dx) \right)}{4b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a}{-a-1}}} \\ &= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} - \frac{3aCF_1 \left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b} \right)}{2\sqrt{2}b^2 d \sqrt{1 + \cos(c + dx)} \sqrt[3]{\frac{a+b}{a-b}}} \end{aligned}$$

Mathematica [A] time = 1.82696, size = 256, normalized size = 0.94

$$3 \csc(c + dx) \sqrt[3]{a + b \cos(c + dx)} \left(4 \left(C(3a^2 + b^2) + 4Ab^2 \right) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} F_1 \left(\frac{1}{3}; \frac{1}{2}, \frac{1}{2}; \frac{4}{3}; \frac{a+b \cos(c+dx)}{a-b}, \frac{a+b \cos(c+dx)}{a-b} \right) \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[(A + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3), x]

[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 + (3*a^2 + b^2)*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(a + b)]

$d*x]))/(-a + b)] + C*(-3*a*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*\text{Cos}[c + d*x])]/(a - b), (a + b*\text{Cos}[c + d*x])/(a + b)]*\text{Sqrt}[-((b*(-1 + \text{Cos}[c + d*x]))/(a + b))]*\text{Sqrt}[(b*(1 + \text{Cos}[c + d*x]))/(-a + b)]*(a + b*\text{Cos}[c + d*x]) - 4*b^2*\text{Sin}[c + d*x]^2)))/(16*b^3*d)$

Maple [F] time = 0.271, size = 0, normalized size = 0.

$$\int (A + C(\cos(dx + c))^2)(a + b \cos(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x)

[Out] int((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + A)/(b*cos(d*x + c) + a)^(2/3), x)
```


3.207 $\int (a + b \cos(e + fx))^m (A - A \cos^2(e + fx)) dx$

Optimal. Leaf size=211

$$\frac{4\sqrt{2}A \sin(e + fx)(a + b \cos(e + fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e + fx) + 1}} - \frac{4\sqrt{2}A \sin(e + fx)(a + b \cos(e + fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e + fx) + 1}}$$

[Out] (-4*Sqrt[2]*A*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (4*Sqrt[2]*A*AppellF1[1/2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)

Rubi [A] time = 0.250905, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3018, 2755, 139, 138, 2784}

$$\frac{4\sqrt{2}A \sin(e + fx)(a + b \cos(e + fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e + fx) + 1}} - \frac{4\sqrt{2}A \sin(e + fx)(a + b \cos(e + fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e + fx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[e + f*x])^m*(A - A*Cos[e + f*x]^2), x]

[Out] (-4*Sqrt[2]*A*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (4*Sqrt[2]*A*AppellF1[1/2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)

Rule 3018

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] := Dist[A - C, Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && EqQ[A + C, 0] && !IntegerQ[2*m]

Rule 2755

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[(a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 - (d*x)/c], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,

m, n, p, x && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -(f*(a + b*x))/(b*e - a*f)]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])

Rule 2784

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*x)^n]/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m]

Rubi steps

$$\begin{aligned} \int (a + b \cos(e + fx))^m (A - A \cos^2(e + fx)) dx &= - \left(A \int (1 + \cos(e + fx))^2 (a + b \cos(e + fx))^m dx \right) + (2A) \int (1 + \cos(e + fx)) (a + b \cos(e + fx))^m dx \\ &= \frac{(A \sin(e + fx)) \operatorname{Subst} \left(\int \frac{(1+x)^{3/2} (a+bx)^m}{\sqrt{1-x}} dx, x, \cos(e + fx) \right) - (2A \sin(e + fx)) \operatorname{Subst} \left(\int \frac{(1+x)^{3/2} (a+bx)^m}{\sqrt{1-x}} dx, x, \cos(e + fx) \right)}{f \sqrt{1 - \cos(e + fx)} \sqrt{1 + \cos(e + fx)}} \\ &= \frac{\left(A (a + b \cos(e + fx))^m \left(-\frac{a+b \cos(e+fx)}{-a-b} \right)^{-m} \sin(e + fx) \right) \operatorname{Subst} \left(\int \frac{(1+x)^{3/2} (a+bx)^m}{\sqrt{1-x}} dx, x, \cos(e + fx) \right) - (2A \sin(e + fx)) \operatorname{Subst} \left(\int \frac{(1+x)^{3/2} (a+bx)^m}{\sqrt{1-x}} dx, x, \cos(e + fx) \right)}{f \sqrt{1 - \cos(e + fx)} \sqrt{1 + \cos(e + fx)}} \\ &= \frac{4\sqrt{2} A F_1 \left(\frac{1}{2}; -\frac{3}{2}, -m; \frac{3}{2}; \frac{1}{2} (1 - \cos(e + fx)), \frac{b(1 - \cos(e + fx))}{a+b} \right) (a + b \cos(e + fx))^m}{f \sqrt{1 + \cos(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.338669, size = 119, normalized size = 0.56

$$\frac{4A \sin(e + fx) \sqrt{\cos^2 \left(\frac{1}{2}(e + fx) \right) \tan^2 \left(\frac{1}{2}(e + fx) \right)} (a + b \cos(e + fx))^m \left(\frac{a+b \cos(e+fx)}{a+b} \right)^{-m} F_1 \left(\frac{3}{2}; -\frac{1}{2}, -m; \frac{5}{2}; \sin^2 \left(\frac{1}{2}(e + fx) \right) \right)}{3f}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*cos[e + f*x])^m*(A - A*cos[e + f*x]^2), x]

[Out] (4*A*AppellF1[3/2, -1/2, -m, 5/2, Sin[(e + f*x)/2]^2, (2*b*Sin[(e + f*x)/2]^2)/(a + b)]*Sqrt[Cos[(e + f*x)/2]^2]*(a + b*cos[e + f*x])^m*Sin[e + f*x]*Tan[(e + f*x)/2]^2)/(3*f*((a + b*cos[e + f*x])/(a + b))^m)

Maple [F] time = 1.404, size = 0, normalized size = 0.

$$\int (a + b \cos(fx + e))^m (A - A(\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)

[Out] int((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int (A \cos(fx + e)^2 - A)(b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="maxima")

[Out] -integrate((A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(A \cos(fx + e)^2 - A\right)(b \cos(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="fricas")

[Out] integral(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))**m*(A-A*cos(f*x+e)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\left(A \cos(fx + e)^2 - A\right)(b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(A-A*cos(f*x+e)^2),x, algorithm="giac")

[Out] integrate(-(A*cos(f*x + e)^2 - A)*(b*cos(f*x + e) + a)^m, x)

3.208 $\int (a + b \cos(e + fx))^m (A + C \cos^2(e + fx)) dx$

Optimal. Leaf size=285

$$\frac{\sqrt{2} \sin(e + fx) (a^2 C + b^2 (A(m + 2) + C(m + 1))) (a + b \cos(e + fx))^m \left(\frac{a + b \cos(e + fx)}{a + b} \right)^{-m} F_1 \left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2} (1 - \cos(e + fx)) \right)}{b^2 f(m + 2) \sqrt{\cos(e + fx) + 1}}$$

[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2*C + b^2*(C*(1 + m) + A*(2 + m)))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)

Rubi [A] time = 0.342942, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.2$, Rules used = {3024, 2756, 2665, 139, 138}

$$\frac{\sqrt{2} \sin(e + fx) (a^2 C + b^2 (A(m + 2) + C(m + 1))) (a + b \cos(e + fx))^m \left(\frac{a + b \cos(e + fx)}{a + b} \right)^{-m} F_1 \left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2} (1 - \cos(e + fx)) \right)}{b^2 f(m + 2) \sqrt{\cos(e + fx) + 1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2), x]

[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*a*(a + b)*C*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2*C + b^2*(C*(1 + m) + A*(2 + m)))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)

Rule 3024

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (C_)*sin[(e_) + (f_)*(x_)])^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) - a*C*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d}

, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -(d*(a + b*x))/(b*c - a*d), -(f*(a + b*x)/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])

Rubi steps

$$\begin{aligned} \int (a + b \cos(e + fx))^m (A + C \cos^2(e + fx)) dx &= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{\int (a + b \cos(e + fx))^m (b(C - A) \cos(e + fx) + A)}{b^2(2 + m)} \\ &= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{(aC) \int (a + b \cos(e + fx))^{1+m} \cos(e + fx) dx}{b^2(2 + m)} \\ &= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{(aC \sin(e + fx)) \operatorname{Subst}\left(\int (a + b \cos(x))^{1+m} \cos(x) dx, x, e + fx\right)}{b^2 f(2 + m) \sqrt{1 - \cos(e + fx)}} \\ &= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{\left(a(-a - b)C(a + b \cos(e + fx))^{1+m} \cos(e + fx)\right)}{b^2 f(2 + m) \sqrt{1 - \cos(e + fx)}} \\ &= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{\sqrt{2}a(a + b)CF_1\left(\frac{1}{2}; \frac{1}{2}, -1 - \frac{1}{2}\right)}{b^2 f(2 + m)} \end{aligned}$$

Mathematica [B] time = 26.1871, size = 10836, normalized size = 38.02

Result too large to show

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[e + f*x])^m*(A + C*Cos[e + f*x]^2), x]

[Out] Result too large to show

Maple [F] time = 1.402, size = 0, normalized size = 0.

$$\int (a + b \cos(fx + e))^m (A + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)`

[Out] `int((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + A \right) (b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="maxima")`

[Out] `integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos^2(fx + e) + A\right)(b \cos(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="fricas")`

[Out] `integral((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(f*x+e))**m*(A+C*cos(f*x+e)**2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + A \right) (b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+b*cos(f*x+e))^m*(A+C*cos(f*x+e)^2),x, algorithm="giac")`

[Out] `integrate((C*cos(f*x + e)^2 + A)*(b*cos(f*x + e) + a)^m, x)`

3.209 $\int (a \cos(e+fx))^m (B \cos(e+fx) + C \cos^2(e+fx)) dx$

Optimal. Leaf size=141

$$\frac{B \sin(e+fx)(a \cos(e+fx))^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \cos^2(e+fx)\right)}{a^2 f(m+2) \sqrt{\sin^2(e+fx)}} - \frac{C \sin(e+fx)(a \cos(e+fx))^{m+3} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; \cos^2(e+fx)\right)}{a^3 f(m+3) \sqrt{\sin^2(e+fx)}}$$

[Out] $-\left(\frac{B(a \cos[e+fx])^{2+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(2+m)}{2}, \frac{(4+m)}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a^{2m} f(m+2) \sqrt{\sin^2[e+fx]}}\right) - \left(\frac{C(a \cos[e+fx])^{3+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(3+m)}{2}, \frac{(5+m)}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a^{3m} f(m+3) \sqrt{\sin^2[e+fx]}}\right)$

Rubi [A] time = 0.13511, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {3010, 2748, 2643}

$$\frac{B \sin(e+fx)(a \cos(e+fx))^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \cos^2(e+fx)\right)}{a^2 f(m+2) \sqrt{\sin^2(e+fx)}} - \frac{C \sin(e+fx)(a \cos(e+fx))^{m+3} {}_2F_1\left(\frac{1}{2}, \frac{m+3}{2}; \frac{m+5}{2}; \cos^2(e+fx)\right)}{a^3 f(m+3) \sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(a \cos[e+fx])^m (B \cos[e+fx] + C \cos^2[e+fx]), x]$

[Out] $-\left(\frac{B(a \cos[e+fx])^{2+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(2+m)}{2}, \frac{(4+m)}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a^{2m} f(m+2) \sqrt{\sin^2[e+fx]}}\right) - \left(\frac{C(a \cos[e+fx])^{3+m} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(3+m)}{2}, \frac{(5+m)}{2}, \cos[e+fx]^2\right] \sin[e+fx]}{a^{3m} f(m+3) \sqrt{\sin^2[e+fx]}}\right)$

Rule 3010

$\text{Int}[(b \sin[e+fx] + (f \cos[e+fx]))^m (B \sin[e+fx] + C \cos^2[e+fx]), x] \text{ :> } \text{Dist}\left[\frac{1}{b}, \text{Int}[(b \sin[e+fx])^{m+1} (B + C \sin[e+fx]), x], x\right] \text{ ; FreeQ}\{b, e, f, B, C, m\}, x]$

Rule 2748

$\text{Int}[(b \sin[e+fx] + (f \cos[e+fx]))^m (c + d \sin[e+fx]), x] \text{ :> } \text{Dist}[c, \text{Int}[(b \sin[e+fx])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \sin[e+fx])^{m+1}, x], x] \text{ ; FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c+dx] + (d \cos[c+dx]))^n, x] \text{ :> } \text{Simp}[(\cos[c+dx])^{n+1} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(n+1)}{2}, \frac{(n+3)}{2}, \sin^2[c+dx]\right] (b \sin[c+dx])^{n+1} \sqrt{\cos^2[c+dx]}], x] \text{ ; FreeQ}\{b, c, d, n\}, x]$
&& !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (a \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx &= \frac{\int (a \cos(e + fx))^{1+m} (B + C \cos(e + fx)) dx}{a} \\ &= \frac{B \int (a \cos(e + fx))^{1+m} dx}{a} + \frac{C \int (a \cos(e + fx))^{2+m} dx}{a^2} \\ &= -\frac{B(a \cos(e + fx))^{2+m} {}_2F_1\left(\frac{1}{2}, \frac{2+m}{2}; \frac{4+m}{2}; \cos^2(e + fx)\right) \sin(e + fx)}{a^2 f(2+m) \sqrt{\sin^2(e + fx)}} \end{aligned}$$

Mathematica [A] time = 0.252161, size = 118, normalized size = 0.84

$$\frac{\sqrt{\sin^2(e + fx)} \cos(e + fx) \cot(e + fx) (a \cos(e + fx))^m \left(B(m+3) {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \cos^2(e + fx)\right) + C(m+2) \cos(e + fx) \right)}{f(m+2)(m+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]

[Out] -((Cos[e + f*x]*(a*Cos[e + f*x])^m*Cot[e + f*x]*(B*(3 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2] + C*(2 + m)*Cos[e + f*x]*Hypergeometric2F1[1/2, (3 + m)/2, (5 + m)/2, Cos[e + f*x]^2])*Sqrt[Sin[e + f*x]^2])/(f*(2 + m)*(3 + m))

Maple [F] time = 1.489, size = 0, normalized size = 0.

$$\int (a \cos(fx + e))^m (B \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)

[Out] int((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e)) (a \cos(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxima")

[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(fx + e)^2 + B \cos(fx + e)\right) (a \cos(fx + e))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + B \cos(fx + e) \right) \left(a \cos(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e))^m, x)
```

3.210 $\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=167

$$\frac{3B \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+13); \cos^2(c+dx)\right) - 3C \sin(c+dx) \sqrt[3]{b \cos(c+dx)}}{d(3m+7) \sqrt{\sin^2(c+dx)}}$$

[Out] (-3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2]))

Rubi [A] time = 0.12846, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{3B \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+13); \cos^2(c+dx)\right) - 3C \sin(c+dx) \sqrt[3]{b \cos(c+dx)}}{d(3m+7) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x]/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2]))

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m+1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

$+ d*x]^2) / (b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]$
 $\&\& !IntegerQ[2*n]$

Rubi steps

$$\begin{aligned} \int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\sqrt[3]{b \cos(c + dx)} \int \cos^{\frac{1}{3}+m}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= \frac{\sqrt[3]{b \cos(c + dx)} \int \cos^{\frac{4}{3}+m}(c + dx) (B + C \cos(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= \frac{(B \sqrt[3]{b \cos(c + dx)}) \int \cos^{\frac{4}{3}+m}(c + dx) dx}{\sqrt[3]{\cos(c + dx)}} + \frac{(C \sqrt[3]{b \cos(c + dx)}) \int \cos^{\frac{4}{3}+m}(c + dx) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= -\frac{3B \cos^{2+m}(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 7); \frac{1}{6}(3m + 13); \cos^2(c + dx)\right)}{d(7 + 3m)\sqrt[3]{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.387994, size = 140, normalized size = 0.84

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \sqrt[3]{b \cos(c + dx)} \cos^{m+2}(c + dx) \left(B(3m + 10) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 7); \frac{1}{6}(3m + 13); \cos^2(c + dx)\right) + C(7 + 3m) \right)}{d(3m + 7)(3m + 10)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(C*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + B*(10 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m))

Maple [F] time = 0.417, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m \sqrt[3]{b \cos(dx + c)} (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

3.211 $\int \cos^m(c+dx)(b \cos(c+dx))^{2/3} (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=167

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+8); \frac{1}{6}(3m+14); \cos^2(c+dx)\right) - 3C \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+2}(c+dx)}{d(3m+8)\sqrt{\sin^2(c+dx)}}$$

[Out] $(-3*B*\text{Cos}[c+d*x]^{(2+m)}*(b*\text{Cos}[c+d*x])^{(2/3)}*\text{Hypergeometric2F1}[1/2, (8+3*m)/6, (14+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(8+3*m)*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (3*C*\text{Cos}[c+d*x]^{(3+m)}*(b*\text{Cos}[c+d*x])^{(2/3)}*\text{Hypergeometric2F1}[1/2, (11+3*m)/6, (17+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(11+3*m)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rubi [A] time = 0.132205, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{3B \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+8); \frac{1}{6}(3m+14); \cos^2(c+dx)\right) - 3C \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+2}(c+dx)}{d(3m+8)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]^m*(b*\text{Cos}[c+d*x])^{(2/3)}*(B*\text{Cos}[c+d*x] + C*\text{Cos}[c+d*x]^2), x]$

[Out] $(-3*B*\text{Cos}[c+d*x]^{(2+m)}*(b*\text{Cos}[c+d*x])^{(2/3)}*\text{Hypergeometric2F1}[1/2, (8+3*m)/6, (14+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(8+3*m)*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (3*C*\text{Cos}[c+d*x]^{(3+m)}*(b*\text{Cos}[c+d*x])^{(2/3)}*\text{Hypergeometric2F1}[1/2, (11+3*m)/6, (17+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(11+3*m)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_*)}*((b_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

$\text{Int}[(b_*\sin[(e_*) + (f_*)(x_*)])^{(m_*)}((B_*)\sin[(e_*) + (f_*)(x_*)] + (C_*)\sin[(e_*) + (f_*)(x_*)]^2), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}*(B + C*\sin[e + f*x]), x], x] /;$ FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

$\text{Int}[(b_*\sin[(e_*) + (f_*)(x_*)])^{(m_*)}((c_*) + (d_*)\sin[(e_*) + (f_*)(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

$\text{Int}[(b_*\sin[(c_*) + (d_*)(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\sin[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2]), x]$

+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^m(c + dx)(b \cos(c + dx))^{2/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{(b \cos(c + dx))^{2/3} \int \cos^{2/3+m}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx}{\cos^{2/3}(c + dx)} \\ &= \frac{(b \cos(c + dx))^{2/3} \int \cos^{5/3+m}(c + dx) (B + C \cos(c + dx)) dx}{\cos^{2/3}(c + dx)} \\ &= \frac{(B(b \cos(c + dx))^{2/3}) \int \cos^{5/3+m}(c + dx) dx}{\cos^{2/3}(c + dx)} + \frac{C(b \cos(c + dx))^{2/3} \int \cos^{8/3+m}(c + dx) dx}{\cos^{2/3}(c + dx)} \\ &= -\frac{3B \cos^{2+m}(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 8); \frac{m}{2} + \frac{7}{3}; \cos^2(c + dx)\right) + C(3b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 8); \frac{m}{2} + \frac{7}{3}; \cos^2(c + dx)\right)}{d(8 + 3m)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.367617, size = 140, normalized size = 0.84

$$\frac{3\sqrt{\sin^2(c + dx)} \operatorname{csc}(c + dx)(b \cos(c + dx))^{2/3} \cos^{m+2}(c + dx) \left(B(3m + 11) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 8); \frac{m}{2} + \frac{7}{3}; \cos^2(c + dx)\right) + C(3b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 8); \frac{m}{2} + \frac{7}{3}; \cos^2(c + dx)\right) \right)}{d(3m + 8)(3m + 11)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(B*(11 + 3*m)*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2] + C*(8 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (11 + 3*m)/6, (17 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(8 + 3*m)*(11 + 3*m))

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (b \cos(dx + c))^{2/3} (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^{2/3} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giacc [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

3.212 $\int \cos^m(c+dx)(b \cos(c+dx))^{4/3} (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=169

$$\frac{3bB \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+3}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+10); \frac{1}{6}(3m+16); \cos^2(c+dx)\right) - 3bC \sin(c+dx) \sqrt[3]{b \cos(c+dx)}}{d(3m+10) \sqrt{\sin^2(c+dx)}}$$

[Out] $(-3*b*B*\text{Cos}[c + d*x]^{(3 + m)}*(b*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/2, (10 + 3*m)/6, (16 + 3*m)/6, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(10 + 3*m)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*b*C*\text{Cos}[c + d*x]^{(4 + m)}*(b*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/2, (13 + 3*m)/6, (19 + 3*m)/6, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(13 + 3*m)*\text{Sqrt}[\text{Sin}[c + d*x]^2]))$

Rubi [A] time = 0.132992, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{3bB \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+3}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+10); \frac{1}{6}(3m+16); \cos^2(c+dx)\right) - 3bC \sin(c+dx) \sqrt[3]{b \cos(c+dx)}}{d(3m+10) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^m*(b*\text{Cos}[c + d*x])^{(4/3)}*(B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2), x]$

[Out] $(-3*b*B*\text{Cos}[c + d*x]^{(3 + m)}*(b*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/2, (10 + 3*m)/6, (16 + 3*m)/6, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(10 + 3*m)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*b*C*\text{Cos}[c + d*x]^{(4 + m)}*(b*\text{Cos}[c + d*x])^{(1/3)}*\text{Hypergeometric2F1}[1/2, (13 + 3*m)/6, (19 + 3*m)/6, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(13 + 3*m)*\text{Sqrt}[\text{Sin}[c + d*x]^2]))$

Rule 20

$\text{Int}[(u_.)*((a_.)*(v_))^{(m_)}*((b_.)*(v_))^{(n_)}, x_Symbol] := \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})], \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3010

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((B_.)*\sin[(e_.) + (f_.)*(x_)] + (C_.)*\sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := \text{Dist}[1/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}*(B + C*\sin[e + f*x]), x], x] /;$ FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

$\text{Int}[(b_.)*\sin[(e_.) + (f_.)*(x_)]^{(m_)}*((c_.) + (d_.)*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] := \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

$\text{Int}[(b_.)*\sin[(c_.) + (d_.)*(x_)]^{(n_)}, x_Symbol] := \text{Simp}[(\text{Cos}[c + d*x]*(b*\sin[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c$

$+ d*x]^2) / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x\}$
 $\&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \cos^m(c + dx)(b \cos(c + dx))^{4/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{(b \sqrt[3]{b \cos(c + dx)}) \int \cos^{4/3+m}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= \frac{(b \sqrt[3]{b \cos(c + dx)}) \int \cos^{7/3+m}(c + dx) (B + C \cos(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= \frac{(b B \sqrt[3]{b \cos(c + dx)}) \int \cos^{7/3+m}(c + dx) dx}{\sqrt[3]{\cos(c + dx)}} + \frac{(b C \sqrt[3]{b \cos(c + dx)}) \int \cos^{10/3+m}(c + dx) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= -\frac{3bB \cos^{3+m}(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + m; \frac{3}{2} + m; \cos^2(c + dx)\right) + C(3bC \cos^{6+m}(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{2}, \frac{1}{2} + m; \frac{3}{2} + m; \cos^2(c + dx)\right))}{d(10 + 3m)} \end{aligned}$$

Mathematica [A] time = 0.541032, size = 140, normalized size = 0.83

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx)(b \cos(c + dx))^{4/3} \cos^{m+2}(c + dx) \left(B(3m + 13) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{5}{3}; \frac{m}{2} + \frac{8}{3}; \cos^2(c + dx)\right) + C(3m + 10) {}_2F_1\left(\frac{1}{2}, \frac{m}{2} + \frac{5}{3}; \frac{m}{2} + \frac{8}{3}; \cos^2(c + dx)\right) \right)}{d(3m + 10)(3m + 13)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(B*(13 + 3*m)*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + C*(10 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (13 + 3*m)/6, (19 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(10 + 3*m)*(13 + 3*m))

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (b \cos(dx + c))^{4/3} (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^{4/3} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2\right)(b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)

$$3.213 \quad \int \frac{\cos^m(c+dx)(B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=167

$$\frac{3B \sin(c+dx) \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+5); \frac{1}{6}(3m+11); \cos^2(c+dx)\right)}{d(3m+5) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} - \frac{3C \sin(c+dx) \cos^{m+3}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+8); \frac{1}{6}(3m+11); \cos^2(c+dx)\right)}{d(3m+8) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

[Out] $(-3*B*\text{Cos}[c+d*x]^{(2+m)}*\text{Hypergeometric2F1}[1/2, (5+3*m)/6, (11+3*m)/6, \text{Cos}[c+d*x]^2*\text{Sin}[c+d*x]]/(d*(5+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (3*C*\text{Cos}[c+d*x]^{(3+m)}*\text{Hypergeometric2F1}[1/2, (8+3*m)/6, (14+3*m)/6, \text{Cos}[c+d*x]^2*\text{Sin}[c+d*x]]/(d*(8+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]))$

Rubi [A] time = 0.128716, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{3B \sin(c+dx) \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+5); \frac{1}{6}(3m+11); \cos^2(c+dx)\right)}{d(3m+5) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} - \frac{3C \sin(c+dx) \cos^{m+3}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+8); \frac{1}{6}(3m+11); \cos^2(c+dx)\right)}{d(3m+8) \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]^m*(B*\text{Cos}[c+d*x]+C*\text{Cos}[c+d*x]^2))/(b*\text{Cos}[c+d*x])^{(1/3)}, x]$

[Out] $(-3*B*\text{Cos}[c+d*x]^{(2+m)}*\text{Hypergeometric2F1}[1/2, (5+3*m)/6, (11+3*m)/6, \text{Cos}[c+d*x]^2*\text{Sin}[c+d*x]]/(d*(5+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (3*C*\text{Cos}[c+d*x]^{(3+m)}*\text{Hypergeometric2F1}[1/2, (8+3*m)/6, (14+3*m)/6, \text{Cos}[c+d*x]^2*\text{Sin}[c+d*x]]/(d*(8+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]))$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_*))^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, m, n\}, x \ \&\& \ \text{IntegerQ}[m] \ \&\& \ \text{IntegerQ}[n] \ \&\& \ \text{IntegerQ}[m+n]$

Rule 3010

$\text{Int}[(b_* \sin[e_*] + f_*(x_*))^{(m_*)}((B_* \sin[e_*] + f_*(x_*)) + (C_* \sin[e_*] + f_*(x_*))^2), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(b*\text{Sin}[e+f*x])^{(m+1)}*(B+C*\text{Sin}[e+f*x]), x], x] /; \text{FreeQ}\{b, e, f, B, C, m\}, x]$

Rule 2748

$\text{Int}[(b_* \sin[e_*] + f_*(x_*))^{(m_*)}((c_*) + (d_*) \sin[e_*] + f_*(x_*)), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e+f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e+f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^m(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} dx &= \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{-\frac{1}{3}+m}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} \\ &= \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{\frac{2}{3}+m}(c + dx) (B + C \cos(c + dx)) dx}{\sqrt[3]{b \cos(c + dx)}} \\ &= \frac{(B \sqrt[3]{\cos(c + dx)}) \int \cos^{\frac{2}{3}+m}(c + dx) dx}{\sqrt[3]{b \cos(c + dx)}} + \frac{(C \sqrt[3]{\cos(c + dx)}) \int \cos^{\frac{5}{3}+m}(c + dx) dx}{\sqrt[3]{b \cos(c + dx)}} \\ &= -\frac{3B \cos^{2+m}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(5 + 3m); \frac{1}{6}(11 + 3m); \cos^2(c + dx)\right) + C \cos^{5+m}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(8 + 3m); \frac{1}{6}(14 + 3m); \cos^2(c + dx)\right)}{d(5 + 3m) \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.429072, size = 140, normalized size = 0.84

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \cos^{m+2}(c + dx) \left(B(3m + 8) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 5); \frac{1}{6}(3m + 11); \cos^2(c + dx)\right) + C(3m + 5) \cos(c + dx) \right)}{d(3m + 5)(3m + 8) \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]
```

```
[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(8 + 3*m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + C*(5 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(5 + 3*m)*(8 + 3*m)*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.336, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)
```

```
[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/b, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C \cos(c + dx)) \cos(c + dx) \cos^m(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3), x)
```

```
[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

$$3.214 \quad \int \frac{\cos^m(c+dx)(B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=167

$$\frac{3B \sin(c+dx) \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+4); \frac{1}{6}(3m+10); \cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{2/3}} - \frac{3C \sin(c+dx) \cos^{m+3}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+10); \cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$

[Out] (-3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.122904, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{3B \sin(c+dx) \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+4); \frac{1}{6}(3m+10); \cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}(b \cos(c+dx))^{2/3}} - \frac{3C \sin(c+dx) \cos^{m+3}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+10); \cos^2(c+dx)\right)}{d(3m+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3), x]

[Out] (-3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*C*Cos[c + d*x]^(3 + m)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m+1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

$+ d*x]^2) / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x]$
 $\&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^m(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{2/3}} dx &= \frac{\cos^{2/3}(c + dx) \int \cos^{-2/3+m}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{2/3}} \\ &= \frac{\cos^{2/3}(c + dx) \int \cos^{1/3+m}(c + dx) (B + C \cos(c + dx))}{(b \cos(c + dx))^{2/3}} \\ &= \frac{(B \cos^{2/3}(c + dx)) \int \cos^{1/3+m}(c + dx) dx}{(b \cos(c + dx))^{2/3}} + \frac{(C \cos^{2/3}(c + dx)) \int \cos^{1/3+m}(c + dx) dx}{(b \cos(c + dx))^{2/3}} \\ &= -\frac{3B \cos^{2+m}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(4 + 3m); \frac{1}{6}(10 + 3m); \cos^2(c + dx)\right)}{d(4 + 3m)(b \cos(c + dx))^{2/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.423931, size = 140, normalized size = 0.84

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \cos^{m+2}(c + dx) \left(B(3m + 7) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 4); \frac{m}{2} + \frac{5}{3}; \cos^2(c + dx)\right) + C(3m + 4) \cos(c + dx) \right)}{d(3m + 4)(3m + 7)(b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(2/3)), x]

[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(7 + 3*m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + C*(4 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(4 + 3*m)*(7 + 3*m)*(b*Cos[c + d*x]^(2/3)))

Maple [F] time = 0.352, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \cos(dx + c)^m}{(b \cos(dx + c))^{2/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/b, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(B + C \cos(c + dx)) \cos(c + dx) \cos^m(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3), x)

[Out] Integral((B + C*cos(c + d*x))*cos(c + d*x)*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

$$3.215 \quad \int \frac{\cos^m(c+dx)(B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=173

$$\frac{3B \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \cos^2(c+dx)\right)}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}} - \frac{3C \sin(c+dx) \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}\right)}{bd(3m+5)\sqrt{\sin^2(c+dx)}}$$

[Out] $(-3*B*\text{Cos}[c+d*x]^{(1+m)}*\text{Hypergeometric2F1}[1/2, (2+3*m)/6, (8+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b*d*(2+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (3*C*\text{Cos}[c+d*x]^{(2+m)}*\text{Hypergeometric2F1}[1/2, (5+3*m)/6, (11+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b*d*(5+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rubi [A] time = 0.131254, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{3B \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \cos^2(c+dx)\right)}{bd(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}} - \frac{3C \sin(c+dx) \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}\right)}{bd(3m+5)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(\text{Cos}[c+d*x]^m*(B*\text{Cos}[c+d*x]+C*\text{Cos}[c+d*x]^2))/(b*\text{Cos}[c+d*x])^{(4/3)}, x]$

[Out] $(-3*B*\text{Cos}[c+d*x]^{(1+m)}*\text{Hypergeometric2F1}[1/2, (2+3*m)/6, (8+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b*d*(2+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (3*C*\text{Cos}[c+d*x]^{(2+m)}*\text{Hypergeometric2F1}[1/2, (5+3*m)/6, (11+3*m)/6, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b*d*(5+3*m)*(b*\text{Cos}[c+d*x])^{(1/3)}*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_*)}*((b_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

$\text{Int}[(b_*\sin[(e_*)+(f_*)(x_*)])^{(m_*)}((B_*)\sin[(e_*)+(f_*)(x_*)]+(C_*)\sin[(e_*)+(f_*)(x_*)]^2), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(b*\sin[e+f*x])^{(m+1)}*(B+C*\sin[e+f*x]), x], x] /;$ FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

$\text{Int}[(b_*\sin[(e_*)+(f_*)(x_*)])^{(m_*)}((c_*)+(d_*)\sin[(e_*)+(f_*)(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e+f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e+f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

$\text{Int}[(b_*\sin[(c_*)+(d_*)(x_*)])^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\sin[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c$

$+ d*x]^2] / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x]$
 $\&\& \text{!IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^m(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{4/3}} dx &= \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{-\frac{4}{3}+m}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx))}{b \sqrt[3]{b \cos(c + dx)}} \\ &= \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{-\frac{1}{3}+m}(c + dx) (B + C \cos(c + dx)) dx}{b \sqrt[3]{b \cos(c + dx)}} \\ &= \frac{(B \sqrt[3]{\cos(c + dx)}) \int \cos^{-\frac{1}{3}+m}(c + dx) dx}{b \sqrt[3]{b \cos(c + dx)}} + \frac{(C \sqrt[3]{\cos(c + dx)}) \int \cos^{-\frac{1}{3}+m}(c + dx) dx}{b \sqrt[3]{b \cos(c + dx)}} \\ &= \frac{3B \cos^{1+m}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(2 + 3m); \frac{1}{6}(8 + 3m); \cos^2(c + dx)\right) + 3C \cos^{2+m}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(2 + 3m); \frac{1}{6}(8 + 3m); \cos^2(c + dx)\right)}{bd(2 + 3m) \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.350667, size = 140, normalized size = 0.81

$$\frac{3 \sqrt{\sin^2(c + dx)} \csc(c + dx) \cos^{m+2}(c + dx) \left(B(3m + 5) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 2); \frac{1}{6}(3m + 8); \cos^2(c + dx)\right) + C(3m + 2) \cos(c + dx) \right)}{d(3m + 2)(3m + 5)(b \cos(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (-3*Cos[c + d*x]^(2 + m)*Csc[c + d*x]*(B*(5 + 3*m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + C*(2 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(4/3))

Maple [F] time = 0.382, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m}{b^2 \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

3.216 $\int (a \cos(c+dx))^m (b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=167

$$\frac{B \sin(c+dx)(a \cos(c+dx))^{m+2}(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m+n+2); \frac{1}{2}(m+n+4); \cos^2(c+dx)\right) + C \sin(c+dx)(a \cos(c+dx))^{m+1}(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m+n+2); \frac{1}{2}(m+n+4); \cos^2(c+dx)\right)}{a^2 d(m+n+2) \sqrt{\sin^2(c+dx)}}$$

[Out] -((B*(a*Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)*Sqrt[Sin[c + d*x]^2])) - (C*(a*Cos[c + d*x])^(3 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^3*d*(3 + m + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.163732, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{B \sin(c+dx)(a \cos(c+dx))^{m+2}(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m+n+2); \frac{1}{2}(m+n+4); \cos^2(c+dx)\right) + C \sin(c+dx)(a \cos(c+dx))^{m+1}(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m+n+2); \frac{1}{2}(m+n+4); \cos^2(c+dx)\right)}{a^2 d(m+n+2) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] -((B*(a*Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)*Sqrt[Sin[c + d*x]^2])) - (C*(a*Cos[c + d*x])^(3 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^3*d*(3 + m + n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m+1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2]), x]

$+ d*x]^2] / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x]$
 $\&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx))^m (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx &= ((a \cos(c + dx))^{-n} (b \cos(c + dx))^n) \int (a \cos(c + dx))^m (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= \frac{((a \cos(c + dx))^{-n} (b \cos(c + dx))^n) \int (a \cos(c + dx))^m (B \cos(c + dx) + C \cos^2(c + dx)) dx}{a} \\ &= \frac{(B(a \cos(c + dx))^{-n} (b \cos(c + dx))^n) \int (a \cos(c + dx))^m (B \cos(c + dx) + C \cos^2(c + dx)) dx}{a} \\ &= -\frac{B(a \cos(c + dx))^{2+m} (b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m + n + 2); \frac{1}{2}(m + n + 2); \sin^2(c + dx)\right)}{a^2 d(2 + m + n)} \end{aligned}$$

Mathematica [A] time = 0.337966, size = 136, normalized size = 0.81

$$\frac{\sqrt{\sin^2(c + dx)} \cos(c + dx) \cot(c + dx) (a \cos(c + dx))^m (b \cos(c + dx))^n \left(B(m + n + 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m + n + 2); \frac{1}{2}(m + n + 2); \sin^2(c + dx)\right) + C(m + n + 2) \right)}{d(m + n + 2)(m + n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] -((Cos[c + d*x]*(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 + m + n)*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2] + C*(2 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m + n)/2, (5 + m + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + m + n)*(3 + m + n))

Maple [F] time = 2.434, size = 0, normalized size = 0.

$$\int (\cos(dx + c) a)^m (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(d*x+c)*a)^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int((cos(d*x+c)*a)^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (a \cos(dx + c))^m (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (a \cos(dx + c))^m (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (a \cos(dx + c))^m (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)

3.217 $\int \cos^2(c+dx)(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=141

$$\frac{B \sin(c+dx)(b \cos(c+dx))^{n+4} {}_2F_1\left(\frac{1}{2}, \frac{n+4}{2}; \frac{n+6}{2}; \cos^2(c+dx)\right)}{b^4 d(n+4) \sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+5} {}_2F_1\left(\frac{1}{2}, \frac{n+5}{2}; \frac{n+7}{2}; \cos^2(c+dx)\right)}{b^5 d(n+5) \sqrt{\sin^2(c+dx)}}$$

[Out] $-\left(\frac{B(b \cos[c+dx])^{4+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^4 d(4+n) \sqrt{\sin^2[c+dx]}}\right) - \left(\frac{C(b \cos[c+dx])^{5+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5+n}{2}, \frac{7+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^5 d(5+n) \sqrt{\sin^2[c+dx]}}\right)$

Rubi [A] time = 0.161929, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3010, 2748, 2643}

$$\frac{B \sin(c+dx)(b \cos(c+dx))^{n+4} {}_2F_1\left(\frac{1}{2}, \frac{n+4}{2}; \frac{n+6}{2}; \cos^2(c+dx)\right)}{b^4 d(n+4) \sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+5} {}_2F_1\left(\frac{1}{2}, \frac{n+5}{2}; \frac{n+7}{2}; \cos^2(c+dx)\right)}{b^5 d(n+5) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] $-\left(\frac{B(b \cos[c+dx])^{4+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{4+n}{2}, \frac{6+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^4 d(4+n) \sqrt{\sin^2[c+dx]}}\right) - \left(\frac{C(b \cos[c+dx])^{5+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{5+n}{2}, \frac{7+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^5 d(5+n) \sqrt{\sin^2[c+dx]}}\right)$

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3010

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((B_)*sin[(e_)+(f_)*(x_)] + (C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e+f*x])^(m+1)*(B+C*Sin[e+f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((c_)+(d_)*sin[(e_)+(f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e+f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e+f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[(b_)*sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] :> Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c+d*x]^2]/(b*d*(n+1)*Sqrt[Cos[c+d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned}
\int \cos^2(c + dx)(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{2+n} (B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^2} \\
&= \frac{\int (b \cos(c + dx))^{3+n} (B + C \cos(c + dx)) dx}{b^3} \\
&= \frac{B \int (b \cos(c + dx))^{3+n} dx}{b^3} + \frac{C \int (b \cos(c + dx))^{4+n} dx}{b^4} \\
&= -\frac{B(b \cos(c + dx))^{4+n} {}_2F_1\left(\frac{1}{2}, \frac{4+n}{2}; \frac{6+n}{2}; \cos^2(c + dx)\right) + C(n+4) \cos(c + dx)}{b^4 d(4+n) \sqrt{\sin^2(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.464566, size = 120, normalized size = 0.85

$$\frac{\sqrt{\sin^2(c + dx)} \cos^3(c + dx) \cot(c + dx) (b \cos(c + dx))^n \left(B(n+5) {}_2F_1\left(\frac{1}{2}, \frac{n+4}{2}; \frac{n+6}{2}; \cos^2(c + dx)\right) + C(n+4) \cos(c + dx) \right)}{d(n+4)(n+5)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] -((Cos[c + d*x]^3*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(5 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2] + C*(4 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + n)/2, (7 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(4 + n)*(5 + n))

Maple [F] time = 2.171, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx+c)^4 + B \cos(dx+c)^3\right) (b \cos(dx+c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx+c)^2 + B \cos(dx+c)\right) (b \cos(dx+c))^n \cos(dx+c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c)^2, x)

3.218 $\int \cos(c+dx)(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))$

Optimal. Leaf size=141

$$\frac{B \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+4} {}_2F_1\left(\frac{1}{2}, \frac{n+4}{2}; \frac{n+6}{2}; \cos^2(c+dx)\right)}{b^4 d(n+4) \sqrt{\sin^2(c+dx)}}$$

[Out] $-(B*(b*\text{Cos}[c+d*x])^{(3+n)}*\text{Hypergeometric2F1}[1/2, (3+n)/2, (5+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b^3*d*(3+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (C*(b*\text{Cos}[c+d*x])^{(4+n)}*\text{Hypergeometric2F1}[1/2, (4+n)/2, (6+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b^4*d*(4+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rubi [A] time = 0.155058, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {16, 3010, 2748, 2643}

$$\frac{B \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+4} {}_2F_1\left(\frac{1}{2}, \frac{n+4}{2}; \frac{n+6}{2}; \cos^2(c+dx)\right)}{b^4 d(n+4) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c+d*x]*(b*\text{Cos}[c+d*x])^n*(B*\text{Cos}[c+d*x] + C*\text{Cos}[c+d*x]^2), x]$

[Out] $-(B*(b*\text{Cos}[c+d*x])^{(3+n)}*\text{Hypergeometric2F1}[1/2, (3+n)/2, (5+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b^3*d*(3+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (C*(b*\text{Cos}[c+d*x])^{(4+n)}*\text{Hypergeometric2F1}[1/2, (4+n)/2, (6+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b^4*d*(4+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_)^{(m_*)}*((b_*)*(v_))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3010

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_*)}*((B_*)*\text{sin}[(e_*) + (f_*)*(x_)] + (C_*)*\text{sin}[(e_*) + (f_*)*(x_)]^2), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}*(B + C*\text{Sin}[e + f*x]), x], x] /;$ FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_)]^{(m_*)}*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\text{Sin}[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2]), x] /;$ FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{1+n} (B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{\int (b \cos(c + dx))^{2+n} (B + C \cos(c + dx)) dx}{b^2} \\ &= \frac{B \int (b \cos(c + dx))^{2+n} dx}{b^2} + \frac{C \int (b \cos(c + dx))^3 dx}{b^3} \\ &= -\frac{B(b \cos(c + dx))^{3+n} {}_2F_1\left(\frac{1}{2}, \frac{3+n}{2}; \frac{5+n}{2}; \cos^2(c + dx)\right) + C \int (b \cos(c + dx))^3 dx}{b^3 d(3+n) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.276531, size = 120, normalized size = 0.85

$$\frac{\sqrt{\sin^2(c + dx)} \cos^2(c + dx) \cot(c + dx) (b \cos(c + dx))^n \left(B(n+4) {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c + dx)\right) + C(n+3) \cos(c + dx) \right)}{d(n+3)(n+4)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(4 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + C*(3 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(3 + n)*(4 + n))

Maple [F] time = 1.759, size = 0, normalized size = 0.

$$\int \cos(dx + c) (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2\right) (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c), x)

3.219 $\int (b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=141

$$\frac{B \sin(c+dx)(b \cos(c+dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right)}{b^2 d(n+2) \sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c+dx)}}$$

[Out] $-\left(\frac{B(b \cos[c+dx])^{2+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^2 d(2+n) \sqrt{\sin^2[c+dx]}}\right) - \left(\frac{C(b \cos[c+dx])^{3+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^3 d(3+n) \sqrt{\sin^2[c+dx]}}\right)$

Rubi [A] time = 0.13834, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {3010, 2748, 2643}

$$\frac{B \sin(c+dx)(b \cos(c+dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right)}{b^2 d(n+2) \sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*cos[c + d*x])^n*(B*cos[c + d*x] + C*cos[c + d*x]^2),x]

[Out] $-\left(\frac{B(b \cos[c+dx])^{2+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{2+n}{2}, \frac{4+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^2 d(2+n) \sqrt{\sin^2[c+dx]}}\right) - \left(\frac{C(b \cos[c+dx])^{3+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, \frac{3+n}{2}, \frac{5+n}{2}, \cos^2[c+dx]\right] \sin[c+dx]}{b^3 d(3+n) \sqrt{\sin^2[c+dx]}}\right)$

Rule 3010

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_) + (C_.)*sin[(e_.) + (f_.)*(x_)^2]), x_Symbol] :> Dist[1/b, Int[(b*Sin[e + f*x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{\int (b \cos(c + dx))^{1+n} (B + C \cos(c + dx)) dx}{b}$$

$$= \frac{B \int (b \cos(c + dx))^{1+n} dx}{b} + \frac{C \int (b \cos(c + dx))^{2+n} dx}{b^2}$$

$$= -\frac{B(b \cos(c + dx))^{2+n} {}_2F_1\left(\frac{1}{2}, \frac{2+n}{2}; \frac{4+n}{2}; \cos^2(c + dx)\right) \sin(c + dx)}{b^2 d(2+n) \sqrt{\sin^2(c + dx)}}$$

Mathematica [A] time = 0.226185, size = 118, normalized size = 0.84

$$\frac{\sqrt{\sin^2(c + dx)} \cos(c + dx) \cot(c + dx) (b \cos(c + dx))^n \left(B(n+3) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c + dx)\right) + C(n+2) \cos(c + dx) \right)}{d(n+2)(n+3)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(3 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + C*(2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(2 + n)*(3 + n))

Maple [F] time = 1.421, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n, x)

3.220 $\int (b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) \sec(c+dx) dx$

Optimal. Leaf size=141

$$\frac{C \sin(c+dx)(b \cos(c+dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right)}{b^2 d(n+2) \sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c+dx)\right)}{bd(n+1) \sqrt{\sin^2(c+dx)}}$$

[Out] $-(B*(b*\text{Cos}[c+d*x])^{(1+n)}*\text{Hypergeometric2F1}[1/2, (1+n)/2, (3+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b*d*(1+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (C*(b*\text{Cos}[c+d*x])^{(2+n)}*\text{Hypergeometric2F1}[1/2, (2+n)/2, (4+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b^2*d*(2+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rubi [A] time = 0.157493, antiderivative size = 141, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 36, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {16, 3010, 2748, 2643}

$$\frac{C \sin(c+dx)(b \cos(c+dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right)}{b^2 d(n+2) \sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c+dx)\right)}{bd(n+1) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c+d*x])^n*(B*\text{Cos}[c+d*x] + C*\text{Cos}[c+d*x]^2)*\text{Sec}[c+d*x], x]$

[Out] $-(B*(b*\text{Cos}[c+d*x])^{(1+n)}*\text{Hypergeometric2F1}[1/2, (1+n)/2, (3+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b*d*(1+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2]) - (C*(b*\text{Cos}[c+d*x])^{(2+n)}*\text{Hypergeometric2F1}[1/2, (2+n)/2, (4+n)/2, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(b^2*d*(2+n)*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x] \ \&\& \ \text{IntegerQ}[m]$

Rule 3010

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((B_*)*\sin[(e_*) + (f_*)*(x_*)] + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}*(B + C*\sin[e + f*x]), x], x] /; \text{FreeQ}\{b, e, f, B, C, m\}, x]$

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]^{(n_*)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c+d*x]*(b*\sin[c+d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c+d*x]^2])/(b*d*(n+1)*\text{Sqrt}[\text{Cos}[c+d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x] \ \&\& \ !\text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int (b \cos(c + dx))^{-1+n} (B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \\
&= \int (b \cos(c + dx))^n (B + C \cos(c + dx)) dx \\
&= B \int (b \cos(c + dx))^n dx + \frac{C \int (b \cos(c + dx))^{1+n} dx}{b} \\
&= -\frac{B(b \cos(c + dx))^{1+n} {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \cos^2(c + dx)\right) + C(b \cos(c + dx))^{1+n} {}_2F_1\left(\frac{1}{2}, \frac{1+n}{2}; \frac{3+n}{2}; \cos^2(c + dx)\right)}{bd(1+n)\sqrt{\sin^2(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.157823, size = 112, normalized size = 0.79

$$\frac{\sqrt{\sin^2(c + dx)} \cot(c + dx) (b \cos(c + dx))^n \left(B(n+2) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) + C(n+1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) \right)}{d(n+1)(n+2)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] -(((b*Cos[c + d*x])^n*Cot[c + d*x]*(B*(2 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] + C*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(1 + n)*(2 + n)))

Maple [F] time = 1.625, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)

[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c), x)

$$3.221 \quad \int (b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) \sec^2(c+dx) dx$$

Optimal. Leaf size=132

$$\frac{B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

[Out] -((B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (C*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.166147, antiderivative size = 132, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3010, 2748, 2643}

$$\frac{B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] -((B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])) - (C*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3010

Int[((b_)*sin[(e_)] + (f_)*(x_))]^(m_)*((B_)*sin[(e_)] + (f_)*(x_)] + (C_)*sin[(e_)] + (f_)*(x_)]^2, x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m + 1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)] + (f_)*(x_))]^(m_)*((c_) + (d_)*sin[(e_)] + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_)*sin[(c_)] + (d_)*(x_))]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]

&& !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int (b \cos(c + dx))^{-2+n} (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= b \int (b \cos(c + dx))^{-1+n} (B + C \cos(c + dx)) dx \\ &= (bB) \int (b \cos(c + dx))^{-1+n} dx + C \int (b \cos(c + dx))^{-1+n} \cos(c + dx) dx \\ &= -\frac{B(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{2+n}{2}; \cos^2(c + dx)\right) \sin(c + dx)}{dn \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.18466, size = 109, normalized size = 0.83

$$\frac{b \sqrt{\sin^2(c + dx)} \cot(c + dx) (b \cos(c + dx))^{n-1} \left(B(n+1) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right) + Cn \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) \right)}{dn(n+1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Cot[c + d*x]*(B*(1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + C*n*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*n*(1 + n))

Maple [F] time = 1.261, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

3.222 $\int (b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) \sec^3(c+dx) dx$

Optimal. Leaf size=131

$$\frac{bB \sin(c+dx)(b \cos(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

[Out] (b*B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.171403, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3010, 2748, 2643}

$$\frac{bB \sin(c+dx)(b \cos(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}} - \frac{C \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3, x]

[Out] (b*B*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2]) - (C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3010

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((B_)*sin[(e_.) + (f_)*(x_)] + (C_)*sin[(e_.) + (f_)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m+1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_.) + (f_)*(x_)]^(m_))*((c_) + (d_)*sin[(e_.) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_)*sin[(c_.) + (d_)*(x_)]^(n_)), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]

&& !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int (b \cos(c + dx))^{-3+n} (B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx \\ &= b^2 \int (b \cos(c + dx))^{-2+n} (B + C \cos(c + dx)) dx \\ &= (b^2 B) \int (b \cos(c + dx))^{-2+n} dx + (bC) \int (b \cos(c + dx))^{-1+n} dx \\ &= \frac{bB(b \cos(c + dx))^{-1+n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-1+n); \frac{1+n}{2}; \cos^2(c + dx)\right) + bC(b \cos(c + dx))^{-1+n} {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right)}{d(1-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.176918, size = 109, normalized size = 0.83

$$\frac{b\sqrt{\sin^2(c + dx)} \csc(c + dx)(b \cos(c + dx))^{n-1} \left(B n {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right) + C(n-1) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right) \right)}{d(n-1)n}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*Csc[c + d*x]*(B*n*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] + C*(-1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-1 + n)*n))

Maple [F] time = 1.461, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

3.223 $\int (b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) \sec^4(c+dx) dx$

Optimal. Leaf size=139

$$\frac{b^2 B \sin(c+dx)(b \cos(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}} + \frac{bC \sin(c+dx)(b \cos(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

[Out] (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*C*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.182595, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 38, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {16, 3010, 2748, 2643}

$$\frac{b^2 B \sin(c+dx)(b \cos(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}} + \frac{bC \sin(c+dx)(b \cos(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (b^2*B*(b*Cos[c + d*x])^(-2 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2]) + (b*C*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3010

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((B_)*sin[(e_)+(f_)*(x_)] + (C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m+1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_)*((c_)+(d_)*sin[(e_)+(f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c + d*x]^2])/(b*d*(n+1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]

&& !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int (b \cos(c + dx))^{-4+n} (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= b^3 \int (b \cos(c + dx))^{-3+n} (B + C \cos(c + dx)) dx \\ &= (b^3 B) \int (b \cos(c + dx))^{-3+n} dx + (b^2 C) \int (b \cos(c + dx))^{-3+n} \cos(c + dx) dx \\ &= \frac{b^2 B (b \cos(c + dx))^{-2+n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-2+n); \frac{n}{2}; \cos^2(c + dx)\right) + b^2 C (b \cos(c + dx))^{-2+n} {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(-2+n); \frac{n}{2}; \cos^2(c + dx)\right)}{d(2-n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.158109, size = 118, normalized size = 0.85

$$\frac{\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^2(c + dx) (b \cos(c + dx))^n \left(B(n-1) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c + dx)\right) + C(n-2) \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c + dx)\right) \right)}{d(n-2)(n-1)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] -(((b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + n)*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] + C*(-2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2])*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(d*(-2 + n)*(-1 + n)))

Maple [F] time = 1.709, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

3.224 $\int \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx) \cos^{\frac{9}{2}}(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+9); \frac{1}{4}(2n+13); \cos^2(c+dx)\right) - 2C \sin(c+dx) \cos^{\frac{11}{2}}(c+dx)}{d(2n+9)\sqrt{\sin^2(c+dx)}}$$

[Out] $(-2*B*\text{Cos}[c + d*x]^{(9/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (9 + 2*n)/4, (13 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(9 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (2*C*\text{Cos}[c + d*x]^{(11/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (11 + 2*n)/4, (15 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(11 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rubi [A] time = 0.134688, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx) \cos^{\frac{9}{2}}(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+9); \frac{1}{4}(2n+13); \cos^2(c+dx)\right) - 2C \sin(c+dx) \cos^{\frac{11}{2}}(c+dx)}{d(2n+9)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Cos}[c + d*x]^{(5/2)}*(b*\text{Cos}[c + d*x])^n*(B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2), x]$

[Out] $(-2*B*\text{Cos}[c + d*x]^{(9/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (9 + 2*n)/4, (13 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(9 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (2*C*\text{Cos}[c + d*x]^{(11/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (11 + 2*n)/4, (15 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(11 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_))^{(m_)}*((b_*)*(v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})], \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

$\text{Int}[(b_*)\sin[(e_*) + (f_*)*(x_)]^{(m_)}*((B_*)\sin[(e_*) + (f_*)*(x_)] + (C_*)\sin[(e_*) + (f_*)*(x_)]^2), x_Symbol] \rightarrow \text{Dist}[1/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}*(B + C*\sin[e + f*x]), x], x] /;$ FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

$\text{Int}[(b_*)\sin[(e_*) + (f_*)*(x_)]^{(m_)}*((c_*) + (d_*)\sin[(e_*) + (f_*)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

$\text{Int}[(b_*)\sin[(c_*) + (d_*)*(x_)]^{(n_)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\sin[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c$

$+ d*x]^2) / (b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x\}$
 $\&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{5}{2}+n}(c + dx) dx \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{7}{2}+n}(c + dx) dx \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{7}{2}+n}(c + dx) dx \\ &= \frac{2B \cos^{\frac{9}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(9 + 2n); \frac{1}{4}(9 + 2n) + 1; \cos^2(c + dx)\right)}{d(9 + 2n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.252827, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)} \cos^{\frac{9}{2}}(c + dx) \csc(c + dx)(b \cos(c + dx))^n \left(B(2n + 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 9); \frac{1}{4}(2n + 13); \cos^2(c + dx)\right) + C \right)}{d(2n + 9)(2n + 11)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-2*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(11 + 2*n)*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2] + C*(9 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (11 + 2*n)/4, (15 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(9 + 2*n)*(11 + 2*n))

Maple [F] time = 0.691, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^{\frac{5}{2}} (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^4 + B \cos(dx + c)^3\right) (b \cos(dx + c))^n \sqrt{\cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \cos(dx + c)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c)^(5/2), x)

$$3.225 \quad \int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n \left(B \cos(c+dx) + C \cos^2(c+dx) \right) dx$$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+7); \frac{1}{4}(2n+11); \cos^2(c+dx)\right) - 2C \sin(c+dx) \cos^{\frac{9}{2}}(c+dx)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

[Out] (-2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.128157, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx) \cos^{\frac{7}{2}}(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+7); \frac{1}{4}(2n+11); \cos^2(c+dx)\right) - 2C \sin(c+dx) \cos^{\frac{9}{2}}(c+dx)}{d(2n+7)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(9/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(9 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((B_)*sin[(e_)+(f_)*(x_)]+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e+f*x])^(m+1)*(B+C*Sin[e+f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((c_)+(d_)*sin[(e_)+(f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e+f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e+f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_)*sin[(c_)+(d_)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c+d*x]*(b*Sin[c+d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

+ d*x]^2))/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx) \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{5}{2}+n}(c + dx) \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{5}{2}+n}(c + dx) \\ &= \frac{2B \cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(7 + 2n); \frac{1}{4}(7 + 2n); \cos^2(c + dx)\right) + C(2n + 7) \cos^{\frac{5}{2}+n}(c + dx)}{d(7 + 2n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.392705, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)} \cos^{\frac{7}{2}}(c + dx) \csc(c + dx)(b \cos(c + dx))^n \left(B(2n + 9) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 7); \frac{1}{4}(2n + 11); \cos^2(c + dx)\right) + C(2n + 7) \cos^{\frac{5}{2}+n}(c + dx) \right)}{d(2n + 7)(2n + 9)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-2*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(9 + 2*n)*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] + C*(7 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (9 + 2*n)/4, (13 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(7 + 2*n)*(9 + 2*n))

Maple [F] time = 0.653, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^{\frac{3}{2}} (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2\right) (b \cos(dx + c))^n \sqrt{\cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)

3.226 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx) \cos^{\frac{5}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+5); \frac{1}{4}(2n+9); \cos^2(c+dx)\right) + 2C \sin(c+dx) \cos^{\frac{7}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+7); \frac{1}{4}(2n+9); \cos^2(c+dx)\right)}{d(2n+5) \sqrt{\sin^2(c+dx)}}$$

[Out] $(-2*B*\text{Cos}[c + d*x]^{(5/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (5 + 2*n)/4, (9 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(5 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (2*C*\text{Cos}[c + d*x]^{(7/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (7 + 2*n)/4, (11 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(7 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rubi [A] time = 0.124363, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx) \cos^{\frac{5}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+5); \frac{1}{4}(2n+9); \cos^2(c+dx)\right) + 2C \sin(c+dx) \cos^{\frac{7}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+7); \frac{1}{4}(2n+9); \cos^2(c+dx)\right)}{d(2n+5) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[\text{Cos}[c + d*x]]*(b*\text{Cos}[c + d*x])^n*(B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2), x]$

[Out] $(-2*B*\text{Cos}[c + d*x]^{(5/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (5 + 2*n)/4, (9 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(5 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (2*C*\text{Cos}[c + d*x]^{(7/2)}*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (7 + 2*n)/4, (11 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x]]/(d*(7 + 2*n)*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_.*((a_.*(v_))^{(m_)}*((b_.*(v_))^{(n_)}), x_Symbol] := \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})], \text{Int}[u*(a*v)^{(m+n)}, x], x] /;$ $\text{FreeQ}\{a, b, m, n, x\} \ \&\& \ !\text{IntegerQ}[m] \ \&\& \ !\text{IntegerQ}[n] \ \&\& \ !\text{IntegerQ}[m+n]$

Rule 3010

$\text{Int}[(b_.*\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((B_.*\sin[(e_.) + (f_.)*(x_)] + (C_.*\sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := \text{Dist}[1/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}*(B + C*\sin[e + f*x]), x], x] /;$ $\text{FreeQ}\{b, e, f, B, C, m\}, x]$

Rule 2748

$\text{Int}[(b_.*\sin[(e_.) + (f_.)*(x_)])^{(m_)}*((c_.) + (d_.*\sin[(e_.) + (f_.)*(x_)]), x_Symbol] := \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^{(m+1)}, x], x] /;$ $\text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b_.*\sin[(c_.) + (d_.)*(x_)])^{(n_)}, x_Symbol] := \text{Simp}[(\text{Cos}[c + d*x]*(b*\sin[c + d*x])^{(n+1)}*\text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d*x]^2]), x]$

+ d*x]^2))/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c + dx)} (b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx)) dx &= (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) dx \\ &= (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx) dx \\ &= (B \cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx) dx \\ &= -\frac{2B \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(5 + 2n); \frac{1}{4}(5 + 2n) + 1; \cos^2(c + dx)\right) + C \int \cos^{\frac{3}{2}+n}(c + dx) dx}{d(5 + 2n)\sqrt{\sin(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.317083, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)} \cos^{\frac{5}{2}}(c + dx) \csc(c + dx) (b \cos(c + dx))^n \left(B(2n + 7) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 5); \frac{1}{4}(2n + 9); \cos^2(c + dx)\right) + C \int \cos^{\frac{3}{2}+n}(c + dx) dx \right)}{d(2n + 5)(2n + 7)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-2*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(7 + 2*n)*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + C*(5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(5 + 2*n)*(7 + 2*n))

Maple [F] time = 0.646, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right) (b \cos(dx + c))^n \sqrt{\cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) \right) (b \cos(dx + c))^n \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

$$3.227 \quad \int \frac{(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \cos^2(c+dx)\right) + 2C \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{d(2n+3) \sqrt{\sin^2(c+dx)}}$$

[Out] (-2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.128319, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \cos^2(c+dx)\right) + 2C \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{d(2n+3) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]

[Out] (-2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((B_)*sin[(e_)+(f_)*(x_)]+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] :> Dist[1/b, Int[(b*Sin[e+f*x])^(m+1)*(B+C*Sin[e+f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)+(f_)*(x_)]^(m_))*((c_)+(d_)*sin[(e_)+(f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e+f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e+f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) (B + C \cos(c + dx)) dx \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) dx + (C \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n}(c + dx) dx \\ &= \frac{2B \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(3 + 2n); \frac{1}{4}(7 + 2n); \cos^2(c + dx)\right) + C(2n + 5) \cos^{\frac{1}{2}}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 3); \frac{1}{4}(2n + 7); \cos^2(c + dx)\right)}{d(3 + 2n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.23707, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)} \cos^{\frac{3}{2}}(c + dx) \csc(c + dx) (b \cos(c + dx))^n \left(B(2n + 5) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 3); \frac{1}{4}(2n + 7); \cos^2(c + dx)\right) + C(2n + 5) \cos^{\frac{1}{2}}(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 3); \frac{1}{4}(2n + 7); \cos^2(c + dx)\right) \right)}{d(2n + 3)(2n + 5)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]
```

```
[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(5 + 2*n)*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + C*(3 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2])/Sqrt[Sin[c + d*x]^2]/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F] time = 0.788, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c) + B)(b \cos(dx + c))^n \sqrt{\cos(dx + c)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

$$3.228 \quad \int \frac{(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+1); \frac{1}{4}(2n+5); \cos^2(c+dx)\right)}{d(2n+1) \sqrt{\sin^2(c+dx)}} - \frac{2C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^{n-1}}{d(2n+1) \sqrt{\sin^2(c+dx)}}$$

[Out] (-2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x]]/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x]]/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]))

Rubi [A] time = 0.130123, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+1); \frac{1}{4}(2n+5); \cos^2(c+dx)\right)}{d(2n+1) \sqrt{\sin^2(c+dx)}} - \frac{2C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^{n-1}}{d(2n+1) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (-2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x]]/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x]]/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]))

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((B_)*sin[(e_)+(f_)*(x_)]+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Ssin[e+f*x])^(m+1)*(B+C*Ssin[e+f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((c_)+(d_)*sin[(e_)+(f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e+f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e+f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643


```
Int[(b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) (B + C \cos(c + dx)) dx \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) dx + (C \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) dx \\ &= -\frac{2B\sqrt{\cos(c + dx)}(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(1 + 2n); \frac{1}{4}(5 + 2n); \cos^2(c + dx)\right) + C \cos^{-n}(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 1); \frac{1}{4}(2n + 5); \cos^2(c + dx)\right)}{d(1 + 2n)\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.219555, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)}\sqrt{\cos(c + dx)}\csc(c + dx)(b \cos(c + dx))^n \left(B(2n + 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 1); \frac{1}{4}(2n + 5); \cos^2(c + dx)\right) + C(2n + 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 3); \frac{1}{4}(2n + 5); \cos^2(c + dx)\right) \right)}{d(2n + 1)(2n + 3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]
```

```
[Out] (-2*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(3 + 2*n)*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] + C*(1 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2])*(b*Cos[c + d*x])^n*Sqrt[Sin[c + d*x]^2])/(d*(1 + 2*n)*(3 + 2*n))
```

Maple [F] time = 0.779, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

$$3.229 \quad \int \frac{(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-1); \frac{1}{4}(2n+3); \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2C \sin(c+dx)\sqrt{\cos(c+dx)}(b \cos(c+dx))^{n+1}}{d(2n+1)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.133048, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-1); \frac{1}{4}(2n+3); \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}} - \frac{2C \sin(c+dx)\sqrt{\cos(c+dx)}(b \cos(c+dx))^{n+1}}{d(2n+1)\sqrt{\sin^2(c+dx)}\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e + f*x])^(m+1)*(B + C*Sin[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) (B + C \cos(c + dx)) dx \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) dx + (C \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{1}{2}+n}(c + dx) dx \\ &= \frac{2B(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-1 + 2n); \frac{1}{4}(3 + 2n); \cos^2(c + dx)\right) + C(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right)}{d(1 - 2n)\sqrt{\cos(c + dx)}\sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.246398, size = 133, normalized size = 0.82

$$\frac{2\sqrt{\sin^2(c + dx)} \csc(c + dx)(b \cos(c + dx))^n \left(B(2n + 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right) + C(2n - 1) \cos(c + dx) \right)}{d(4n^2 - 1)\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(1 + 2*n)*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + C*(-1 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2]/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]])
```

Maple [F] time = 0.765, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

$$3.230 \quad \int \frac{(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-3); \frac{1}{4}(2n+1); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{2C \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-1); \frac{1}{4}(2n+1); \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)} \sqrt{\cos(c+dx)}}$$

[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.127389, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-3); \frac{1}{4}(2n+1); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)} \cos^{\frac{3}{2}}(c+dx)} + \frac{2C \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-1); \frac{1}{4}(2n+1); \cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)} \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*SIN[e + f*x])^(m+1)*(B + C*SIN[e + f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) (B + C \cos(c + dx)) dx \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) dx + (C \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) dx \\ &= \frac{2B(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-3 + 2n); \frac{1}{4}(1 + 2n); \cos^2(c + dx)\right) + C(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-1 + 2n); \frac{1}{4}(1 + 2n); \cos^2(c + dx)\right)}{d(3 - 2n) \cos^{\frac{3}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.212731, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)} \csc(c + dx)(b \cos(c + dx))^n \left(B(2n - 1) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right) + C(2n - 3) \cos(c + dx) \right)}{d(2n - 3)(2n - 1) \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-1 + 2*n)*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] + C*(-3 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2))
```

Maple [F] time = 0.729, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

$$3.231 \quad \int \frac{(b \cos(c+dx))^n (B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=163

$$\frac{2B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx)} + \frac{2C \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-3); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}$$

[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.124042, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 40, $\frac{\text{number of rules}}{\text{integrand size}} = 0.1$, Rules used = {20, 3010, 2748, 2643}

$$\frac{2B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)} \cos^{\frac{5}{2}}(c+dx)} + \frac{2C \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-3); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (2*B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 - 2*n)*Cos[c + d*x]^(5/2)*Sqrt[Sin[c + d*x]^2]) + (2*C*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 - 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3010

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((B_)*sin[(e_)+(f_)*(x_)]+(C_)*sin[(e_)+(f_)*(x_)]^2), x_Symbol] := Dist[1/b, Int[(b*Sin[e+f*x])^(m+1)*(B+C*Sin[e+f*x]), x], x] /; FreeQ[{b, e, f, B, C, m}, x]

Rule 2748

Int[((b_)*sin[(e_)+(f_)*(x_)])^(m_)*((c_)+(d_)*sin[(e_)+(f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e+f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e+f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{9}{2}+n}(c + dx) (B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) (B + C \cos(c + dx)) dx \\ &= (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) dx + (C \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) dx \\ &= \frac{2B(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-5 + 2n); \frac{1}{4}(-1 + 2n); \cos^2(c + dx)\right) + C(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(-3 + 2n); \frac{1}{4}(-1 + 2n); \cos^2(c + dx)\right)}{d(5 - 2n) \cos^{\frac{5}{2}}(c + dx) \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.213749, size = 138, normalized size = 0.85

$$\frac{2\sqrt{\sin^2(c + dx)} \csc(c + dx) (b \cos(c + dx))^n \left(B(2n - 3) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 5); \frac{1}{4}(2n - 1); \cos^2(c + dx)\right) + C(2n - 5) \cos(c + dx) \right)}{d(2n - 5)(2n - 3) \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^n*(B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]
```

```
[Out] (-2*(b*Cos[c + d*x])^n*Csc[c + d*x]*(B*(-3 + 2*n)*Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d*x]^2] + C*(-5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]^2])/(d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d*x]^(5/2))
```

Maple [F] time = 0.731, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{9}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x)
```

```
[Out] int((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c)) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c) + B)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c) + B)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c))^n/cos(d*x + c)^(9/2), x)

3.232 $\int (a + a \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx$

Optimal. Leaf size=173

$$\frac{2^{m+\frac{1}{2}} (Bm(m+2) + C(m^2 + m + 1)) \sin(e + fx) (\cos(e + fx) + 1)^{-m-\frac{1}{2}} (a \cos(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx))\right)}{f(m+1)(m+2)}$$

[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m)) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(B*m*(2 + m) + C*(1 + m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rubi [A] time = 0.209168, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3023, 2751, 2652, 2651}

$$\frac{2^{m+\frac{1}{2}} (Bm(m+2) + C(m^2 + m + 1)) \sin(e + fx) (\cos(e + fx) + 1)^{-m-\frac{1}{2}} (a \cos(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2} - m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx))\right)}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2), x]

[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m)) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(B*m*(2 + m) + C*(1 + m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2751

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 2652

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n +
1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
- (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int (a + a \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx &= \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} + \frac{\int (a + a \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx}{af(2 + m)} \\ &= -\frac{(C - B(2 + m))(a + a \cos(e + fx))^m \sin(e + fx)}{f(1 + m)(2 + m)} + \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} \\ &= -\frac{(C - B(2 + m))(a + a \cos(e + fx))^m \sin(e + fx)}{f(1 + m)(2 + m)} + \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} \\ &= -\frac{(C - B(2 + m))(a + a \cos(e + fx))^m \sin(e + fx)}{f(1 + m)(2 + m)} + \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} \end{aligned}$$

Mathematica [C] time = 45.8027, size = 356, normalized size = 2.06

$$i4^{-m-1} e^{-2i(e+fx)} (1 + e^{i(e+fx)})^{-2m} \left(e^{-\frac{1}{2}i(e+fx)} (1 + e^{i(e+fx)}) \right)^{2m} \cos^{-2m} \left(\frac{1}{2}(e + fx) \right) (a \cos(e + fx) + 1)^m ((m + 2)e^{i(e+fx)})$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] (I*4^(-1 - m)*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*(a*(1 + Cos
[e + f*x]))^m*(C*m*(2 - m - 2*m^2 + m^3)*Hypergeometric2F1[-2 - m, -2*m, -1
- m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(2 + m)*(2*B*m*(2 - 3*m + m^2)*Hy
pergeometric2F1[-1 - m, -2*m, -m, -E^(I*(e + f*x))] + E^(I*(e + f*x))*(1 +
m)*(2*B*E^(I*(e + f*x))*(-2 + m)*m*Hypergeometric2F1[1 - m, -2*m, 2 - m, -E
^(I*(e + f*x))] + C*(-1 + m)*(E^((2*I)*(e + f*x))*m*Hypergeometric2F1[2 - m
, -2*m, 3 - m, -E^(I*(e + f*x))] + 2*(-2 + m)*Hypergeometric2F1[-2*m, -m, 1
- m, -E^(I*(e + f*x))])))))/E^((2*I)*(e + f*x))*(1 + E^(I*(e + f*x)))^(2*
m)*f*(-2 + m)*(-1 + m)*m*(1 + m)*(2 + m)*Cos[(e + f*x)/2]^(2*m))
```

Maple [F] time = 1.662, size = 0, normalized size = 0.

$$\int (a + a \cos(fx + e))^m (B \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e))(a \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxima")

[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(fx + e)^2 + B \cos(fx + e)\right)\left(a \cos(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")

[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a (\cos(e + fx) + 1)\right)^m (B + C \cos(e + fx)) \cos(e + fx) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)

[Out] Integral((a*(cos(e + f*x) + 1))**m*(B + C*cos(e + f*x))*cos(e + f*x), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(fx + e)^2 + B \cos(fx + e)\right)\left(a \cos(fx + e) + a\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")

[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(a*cos(f*x + e) + a)^m, x)

3.233 $\int (a+b \cos(e+fx))^m (B \cos(e+fx) + C \cos^2(e+fx)) dx$

Optimal. Leaf size=295

$$\frac{\sqrt{2} \sin(e+fx) (a^2 C - abB(m+2) + b^2 C(m+1)) (a+b \cos(e+fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e+fx))\right)}{b^2 f(m+2) \sqrt{\cos(e+fx)+1}}$$

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a +
b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/
2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Ssin[e + f*x])/(b^
2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqr
t[2]*(a^2*C + b^2*C*(1 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1
- Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*
Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(
a + b))^m)
```

Rubi [A] time = 0.362854, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2} \sin(e+fx) (a^2 C - abB(m+2) + b^2 C(m+1)) (a+b \cos(e+fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e+fx))\right)}{b^2 f(m+2) \sqrt{\cos(e+fx)+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2), x]
```

```
[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a +
b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/
2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Ssin[e + f*x])/(b^
2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqr
t[2]*(a^2*C + b^2*C*(1 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1
- Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*
Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(
a + b))^m)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rule 2756

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_)
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_)*(x_)^(m_))*((c_) + (d_)*(x_)^(n_))*((e_) + (f_)*(x_)
^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -(f*(a + b*x)/(b*e - a*f))]/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned}
\int (a + b \cos(e + fx))^m (B \cos(e + fx) + C \cos^2(e + fx)) dx &= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{\int (a + b \cos(e + fx))^{1+m} \sin(e + fx) dx}{bf(2 + m)} \\
&= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{(-aC + bB(2 + m))}{bf(2 + m)} \\
&= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{((-aC + bB(2 + m))}{b^2 f(2 + m)} \\
&= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{\left((-a - b)(-aC + bB(2 + m)) \right)}{b^2 f(2 + m)} \\
&= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{\sqrt{2}(a + b)(aC - bB)}{b^2 f(2 + m)}
\end{aligned}$$

Mathematica [B] time = 26.3456, size = 13480, normalized size = 45.69

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] Result too large to show
```


Maple [F] time = 1.539, size = 0, normalized size = 0.

$$\int (a + b \cos(fx + e))^m (B \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)

[Out] int((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e))(b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxima")

[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(fx + e)^2 + B \cos(fx + e)\right)(b \cos(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")

[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))**m*(B*cos(f*x+e)+C*cos(f*x+e)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e))(b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e))*(b*cos(f*x + e) + a)^m, x)
```

3.234 $\int (a+b \cos(c+dx))^{2/3} (B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=284

$$\frac{(-3a^2C + 8abB - 5b^2C) \sin(c+dx)(a+b \cos(c+dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right)}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1} \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3}} + \frac{(a+b) \dots}{\dots}$$

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - ((8*a*b*B - 3*
a^2*C - 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[
2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rubi [A] time = 0.343938, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{(-3a^2C + 8abB - 5b^2C) \sin(c+dx)(a+b \cos(c+dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right)}{4\sqrt{2}b^2d\sqrt{\cos(c+dx)+1} \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3}} + \frac{(a+b) \dots}{\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3
*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d
*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sq
rt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) - ((8*a*b*B - 3*
a^2*C - 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1
- Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[
2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rule 2756

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m,
x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b,
c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 2665

```
Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
```

$\sqrt[n]{(\sqrt{1+x}\sqrt{1-x})}$, x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -(d*(a + b*x))/(b*c - a*d), -(f*(a + b*x))/(b*e - a*f)])/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rubi steps

$$\int (a + b \cos(c + dx))^{2/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{3 \int (a + b \cos(c + dx))^{2/3} (B \cos(c + dx) + C \cos^2(c + dx)) dx}{8bd} + \frac{(8bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx}{8bd} - \frac{((8bB - 3aC) \sin(c + dx))^{5/3}}{8b^2 d \sqrt{a + b \cos(c + dx)}} + \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{((-a - b)(8bB - 3aC))^{5/3}}{8bd} = \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{(a + b)(8bB - 3aC)}{8bd}$$

Mathematica [A] time = 2.79931, size = 290, normalized size = 1.02

$$\frac{3 \csc(c + dx)(a + b \cos(c + dx))^{2/3} \left((-6a^2C + 16abB + 25b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}} (a + b \cos(c + dx)) F_1\left(\frac{5}{3}, \frac{1}{2}, \frac{1}{2}, \frac{5}{3}, \frac{a + b \cos(c + dx)}{a - b}, \frac{a + b \cos(c + dx)}{a + b}\right) \sqrt{-\frac{b(-1 + \cos(c + dx))}{a + b}} \sqrt{-\frac{b(1 + \cos(c + dx))}{a - b}} + (16*a*b*B - 6*a^2*C + 25*b^2*C) \text{AppellF1}\left[\frac{5}{3}, \frac{1}{2}, 1\right] \right)}{8bd}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] (-3*(a + b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-a^2 + b^2)*(8*b*B - 3*a*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + (16*a*b*B - 6*a^2*C + 25*b^2*C)*AppellF1[5/3, 1/2, 1

/2, 8/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] * Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] * (a + b*cos[c + d*x]) - 5*b^2*(8*b*B + 2*a*C + 5*b*C*cos[c + d*x])*Sin[c + d*x]^2)/(200*b^3*d)

Maple [F] time = 0.293, size = 0, normalized size = 0.

$$\int (a + b \cos(dx + c))^{\frac{2}{3}} (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] int((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right)(b \cos(dx + c) + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(2/3), x)
```

3.235 $\int \sqrt[3]{a + b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=284

$$\frac{\sqrt{2}(-3a^2C + 7abB - 4b^2C) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) + \sqrt{2}(a + b) \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}{7b^2d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(7*a*b*B - 3*a^2*C - 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))

Rubi [A] time = 0.334546, antiderivative size = 284, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}(-3a^2C + 7abB - 4b^2C) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right) + \sqrt{2}(a + b) \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}{7b^2d \sqrt{\cos(c + dx) + 1} \sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) - (Sqrt[2]*(7*a*b*B - 3*a^2*C - 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)

$\sqrt[n]{\sqrt{1+x}\sqrt{1-x}}$, x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*(b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -(d*(a + b*x))/(b*c - a*d), -(f*(a + b*x))/(b*e - a*f)])/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])

Rubi steps

$$\int \sqrt[3]{a + b \cos(c + dx)} (B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{3 \int \sqrt[3]{a + b \cos(c + dx)} dx}{7bd} \\ = \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{(7bB - 3aC) \int (a + b \cos(c + dx))^{1/3} dx}{7bd} \\ = \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} - \frac{((7bB - 3aC) \sin(c + dx) \sqrt[3]{a + b \cos(c + dx)})}{7b^2 d \sqrt{1 - \cos^2(c + dx)}} \\ = \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{((-a - b)(7bB - 3aC) \sqrt[3]{a + b \cos(c + dx)})}{7bd} \\ = \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{\sqrt{2}(a + b)(7bB - 3aC) \sqrt[3]{a + b \cos(c + dx)}}{7bd}$$

Mathematica [A] time = 2.75448, size = 289, normalized size = 1.02

$$\frac{3 \csc(c + dx) \sqrt[3]{a + b \cos(c + dx)} \left((-3a^2C + 7abB + 16b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{-\frac{b(\cos(c+dx)+1)}{a-b}} (a + b \cos(c + dx)) F_1\left(\frac{4}{3}; \frac{1}{3}, \frac{1}{2}\right) \right)}{7bd}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + (7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))])

2, 7/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a + b)]*Sqrt[-(b*(-1 + Cos[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))]*(a + b*cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*cos[c + d*x])*Sin[c + d*x]^2)/(112*b^3*d)

Maple [F] time = 0.379, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + b \cos(dx + c)} (B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] int((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c)\right)(b \cos(dx + c) + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c))(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))*(b*cos(d*x + c) + a)^(1/3), x)
```

$$3.236 \quad \int \frac{B \cos(c+dx) + C \cos^2(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$$

Optimal. Leaf size=281

$$\frac{\sqrt{2}(-3a^2C + 5abB - 2b^2C) \sin(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) + \sqrt{2}(5bB - 3a^2C)}{5b^2d\sqrt{\cos(c+dx)} + 1\sqrt[3]{a+b \cos(c+dx)}} + \frac{\sqrt{2}(5bB - 3a^2C)}{5b^2d\sqrt{\cos(c+dx)} + 1\sqrt[3]{a+b \cos(c+dx)}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) - (Sqrt[2]*(5*a*b*B - 3*a^2*C - 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(1/3)

Rubi [A] time = 0.323062, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2}(-3a^2C + 5abB - 2b^2C) \sin(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) + \sqrt{2}(5bB - 3a^2C)}{5b^2d\sqrt{\cos(c+dx)} + 1\sqrt[3]{a+b \cos(c+dx)}} + \frac{\sqrt{2}(5bB - 3a^2C)}{5b^2d\sqrt{\cos(c+dx)} + 1\sqrt[3]{a+b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3),x]

[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) - (Sqrt[2]*(5*a*b*B - 3*a^2*C - 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(1/3)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_)^(m_))*((c_) + (d_.)*(x_)^(n_))*((e_) + (f_.)*(x_)
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_)^(m_))*((c_) + (d_.)*(x_)^(n_))*((e_) + (f_.)*(x_)
^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{3 \int \frac{\frac{2bC}{3} + \frac{1}{3}(5bB - 3aC) \cos(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx}{5b} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{(5bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx}{5b^2} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{((5bB - 3aC) \sin(c + dx)) \operatorname{Subst}\left(\int \frac{(a + bx)}{\sqrt{1 - xv}}\right)}{5b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{((5bB - 3aC)(a + b \cos(c + dx))^{2/3} \sin(c + dx))}{5b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{\sqrt{2}(5bB - 3aC) F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{5b^2 d \sqrt{1 + \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 2.0313, size = 263, normalized size = 0.94

$$\frac{3 \csc(c + dx)(a + b \cos(c + dx))^{2/3} \left(5(3a^2C - 5abB + 2b^2C) \sqrt{-\frac{b(\cos(c + dx) - 1)}{a + b}} \sqrt{-\frac{b(\cos(c + dx) + 1)}{a - b}} F_1\left(\frac{2}{3}; \frac{1}{2}, \frac{1}{2}, \frac{5}{3}; \frac{a + b \cos(c + dx)}{a - b}\right)\right)}{5b^2 d \sqrt{1 + \cos(c + dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3), x]
```

```
[Out] (-3*(a + b*cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(-5*a*b*B + 3*a^2*C + 2*b^2*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*cos[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F] time = 0.335, size = 0, normalized size = 0.

$$\int (B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{a + b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c)}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c)}{(b \cos(dx + c) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c)}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm
="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(1/3), x
)
```

$$3.237 \quad \int \frac{B \cos(c+dx) + C \cos^2(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=281

$$\frac{(-3a^2C + 4abB - b^2C) \sin(c+dx) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}; \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a+b \cos(c+dx))^{2/3}} + \frac{(4bB - 3aC) \sin(c+dx)}{(a+b \cos(c+dx))^{2/3}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3) - ((4*a*b*B - 3*a^2*C - b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(2/3)

Rubi [A] time = 0.327218, antiderivative size = 281, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{(-3a^2C + 4abB - b^2C) \sin(c+dx) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}; \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a+b \cos(c+dx))^{2/3}} + \frac{(4bB - 3aC) \sin(c+dx)}{(a+b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(1/3) - ((4*a*b*B - 3*a^2*C - b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(2/3)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^(n)/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{B \cos(c + dx) + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx = \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{3 \int \frac{\frac{bc}{3} + \frac{1}{3}(4bB - 3aC) \cos(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx}{4b}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{(4bB - 3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx}{4b^2} - \frac{(4a^2C - 3a^2B)}{4b^2}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} - \frac{((4bB - 3aC) \sin(c + dx)) \text{Subst}\left(\int \frac{\sqrt[3]{a + bx}}{\sqrt{1-x}\sqrt{1+x}} dx\right)}{4b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} - \frac{((4bB - 3aC) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx))}{4b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{(4bB - 3aC) F_1\left(\frac{1}{2}, \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt{2}b^2 d \sqrt{1 + \cos(c + dx)}}$$

Mathematica [A] time = 2.1283, size = 261, normalized size = 0.93

$$3 \csc(c + dx) \sqrt[3]{a + b \cos(c + dx)} \left(4(3a^2C - 4abB + b^2C) \sqrt{-\frac{b(\cos(c + dx) - 1)}{a + b}} \sqrt{\frac{b(\cos(c + dx) + 1)}{a - b}} F_1\left(\frac{1}{3}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}; \frac{a + b \cos(c + dx)}{a - b}, \frac{a + b \cos(c + dx)}{a - b}\right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3), x]
```

```
[Out] (-3*(a + b*Cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-4*a*b*B + 3*a^2*C + b^2*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a - b)] + (4*a*b*B - 3*a^2*C - b^2*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a - b)])/(2*b^2*d*Sqrt[1 + Cos[c + d*x]]*Sqrt[1 - Cos[c + d*x]])
```


$d*x]/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] * Sqrt[-((b*(1 + Cos[c + d*x]))/(a - b))] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3, (a + b*Cos[c + d*x])/(a - b), (a + b*Cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] * Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*Cos[c + d*x]) - 4*b^2*C*Sin[c + d*x]^2)/(16*b^3*d)$

Maple [F] time = 0.303, size = 0, normalized size = 0.

$$\int (B \cos(dx + c) + C (\cos(dx + c))^2) (a + b \cos(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x)

[Out] int((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c)}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c)}{(b \cos(dx + c) + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c)}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c))/(b*cos(d*x + c) + a)^(2/3), x)

3.238 $\int (a \cos(e+fx))^m (A + B \cos(e+fx) + C \cos^2(e+fx)) dx$

Optimal. Leaf size=187

$$\frac{B \sin(e+fx)(a \cos(e+fx))^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \cos^2(e+fx)\right)}{a^2 f(m+2) \sqrt{\sin^2(e+fx)}} - \frac{(A(m+2) + C(m+1)) \sin(e+fx)(a \cos(e+fx))}{af(m+1)(m+2) \sqrt{\sin^2(e+fx)}}$$

[Out] (C*(a*cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) - ((C*(1 + m) + A*(2 + m))*(a*cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]) - (B*(a*cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a^2*f*(2 + m)*Sqrt[Sin[e + f*x]^2])

Rubi [A] time = 0.161499, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3023, 2748, 2643}

$$\frac{B \sin(e+fx)(a \cos(e+fx))^{m+2} {}_2F_1\left(\frac{1}{2}, \frac{m+2}{2}; \frac{m+4}{2}; \cos^2(e+fx)\right)}{a^2 f(m+2) \sqrt{\sin^2(e+fx)}} - \frac{(A(m+2) + C(m+1)) \sin(e+fx)(a \cos(e+fx))}{af(m+1)(m+2) \sqrt{\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

[In] Int[(a*cos[e + f*x])^m*(A + B*cos[e + f*x] + C*cos[e + f*x]^2),x]

[Out] (C*(a*cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) - ((C*(1 + m) + A*(2 + m))*(a*cos[e + f*x])^(1 + m)*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a*f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2]) - (B*(a*cos[e + f*x])^(2 + m)*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2]*Sin[e + f*x])/(a^2*f*(2 + m)*Sqrt[Sin[e + f*x]^2])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*cos[e + f*x]*(a + b*sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[(b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (a \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx)) dx &= \frac{C(a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} + \frac{\int (a \cos(e + fx))^m (a(C \\ &= \frac{C(a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} + \frac{B \int (a \cos(e + fx))^{1+m}}{a} \\ &= \frac{C(a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} - \frac{\left(A + \frac{C(1+m)}{2+m}\right) (a \cos(e + fx))^{1+m}}{af(2 + m)} \end{aligned}$$

Mathematica [A] time = 0.277456, size = 142, normalized size = 0.76

$$\frac{\sin(e + fx) \cos(e + fx) (a \cos(e + fx))^m \left((A(m + 2) + C(m + 1)) {}_2F_1\left(\frac{1}{2}, \frac{m+1}{2}; \frac{m+3}{2}; \cos^2(e + fx)\right) + (m + 1) \left(B \cos(e + fx) \right) \right)}{f(m + 1)(m + 2) \sqrt{\sin^2(e + fx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(a*cos[e + f*x])^m*(A + B*cos[e + f*x] + C*cos[e + f*x]^2), x]

[Out] -((Cos[e + f*x]*(a*cos[e + f*x])^m*Sin[e + f*x]*((C*(1 + m) + A*(2 + m))*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Cos[e + f*x]^2] + (1 + m)*(B*cos[e + f*x]*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Cos[e + f*x]^2] - C*Sqrt[Sin[e + f*x]^2])))/(f*(1 + m)*(2 + m)*Sqrt[Sin[e + f*x]^2])

Maple [F] time = 1.463, size = 0, normalized size = 0.

$$\int (a \cos(fx + e))^m (A + B \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2), x)

[Out] int((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e) + A) (a \cos(fx + e))^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2), x, algorithm="maxima")

[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(fx + e)^2 + B \cos(fx + e) + A\right) (a \cos(fx + e))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + B \cos(fx + e) + A \right) \left(a \cos(fx + e) \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e))^m, x)
```

3.239 $\int \cos^2(c+dx) \sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=209

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45bd} + \frac{2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2B \sin(c + dx)(b \cos(c + dx))^{5/2}}{7b^2d}$$

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^3*d)

Rubi [A] time = 0.242092, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45bd} + \frac{2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2B \sin(c + dx)(b \cos(c + dx))^{5/2}}{7b^2d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (10*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^2} \\
&= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^3 d} + \frac{2 \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{9b^3 d} \\
&= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^3 d} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{9b^3 d} + \frac{2A \int (b \cos(c + dx))^{3/2} dx}{9b^3 d} \\
&= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45bd} + \frac{2(9A + 7B)(b \cos(c + dx))^{3/2}}{45bd} \\
&= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2(9A + 7B)(b \cos(c + dx))^{3/2}}{45bd} \\
&= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}} \\
&= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 1.00047, size = 125, normalized size = 0.6

$$\frac{\sqrt{b \cos(c + dx)} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (7(36A + 43C) \cos(c + dx) + 5(18B \cos(2(c + dx)) + 78B + 7C \cos(3(c + dx)))) \right)}{630d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/(6*30*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 4.052, size = 382, normalized size = 1.8

$$-\frac{2b}{315d} \sqrt{b(2(\cos(1/2 dx + c/2))^2 - 1) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(-1120 C \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^{10} + (720 B + 2240 C) \sin(1/2 dx + c/2) (\sin(1/2 dx + c/2))^{10} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(-1120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```


[Out] `integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c)^2, x)`

3.240 $\int \cos(c+dx)\sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=180

$$\frac{2(7A + 5C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2b(7A + 5C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx) (b \cos(c + dx))^{5/2}}{7b^2 d}$$

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)

Rubi [A] time = 0.203627, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3023, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2(7A + 5C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2b(7A + 5C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx) (b \cos(c + dx))^{5/2}}{7b^2 d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^2*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c

+ d*x]^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Ssin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \cos(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d} + \frac{2 \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{7b^2d} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^2d} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{7b^2d} + \frac{2A \int (b \cos(c + dx))^{3/2} dx}{7b^2d} \\ &= \frac{2(7A + 5C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2A \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\ &= \frac{2(7A + 5C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2A \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\ &= \frac{6B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d \sqrt{\cos(c + dx)}} + \frac{2b(7A + 5C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \end{aligned}$$

Mathematica [A] time = 0.753828, size = 111, normalized size = 0.62

$$\frac{(b \cos(c + dx))^{3/2} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (70A + 42B \cos(c + dx) + 15C \cos(2(c + dx)) + 65C) + 10(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{105bd \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d

$*x] + 15*C*\text{Cos}[2*(c + d*x)]*\text{Sin}[c + d*x]))/(105*b*d*\text{Cos}[c + d*x]^(3/2))$

Maple [A] time = 3.873, size = 351, normalized size = 2.

$$-\frac{2b}{105d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(240 C (\sin(1/2 dx + c/2))^8 \cos(1/2 dx + c/2) + (-168 B - 360 C) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x)`

[Out] $-2/105*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(240*C*\sin(1/2*d*x+1/2*c)^8*\cos(1/2*d*x+1/2*c)+(-168*B-360*C)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+35*A*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^(1/2))-63*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^(1/2))*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)+25*C*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^(1/2)/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*cos(d*x + c), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)
```

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

[Out] Timed out

3.241 $\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=145

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2B \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d} + \frac{2bB\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx) \sqrt{b \cos(c + dx)}}{5b^2 d}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rubi [A] time = 0.152183, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3023, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2B \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d} + \frac{2bB\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2C \sin(c + dx) \sqrt{b \cos(c + dx)}}{5b^2 d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b*d)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{2 \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b} \\ &= \frac{2B\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5bd} \\ &= \frac{2(5A + 3C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} \\ &= \frac{2(5A + 3C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2bB\sqrt{\cos(c + dx)} \sin(c + dx)}{3d} \end{aligned}$$

Mathematica [A] time = 0.329081, size = 94, normalized size = 0.65

$$\frac{2\sqrt{b \cos(c + dx)} \left(3(5A + 3C) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + \sin(c + dx) \sqrt{\cos(c + dx)} (5B + 3C \cos(c + dx)) + 5BF\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{15d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d*x]))/(15*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 3.611, size = 317, normalized size = 2.2

$$\frac{2b}{15d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(24C (\sin(1/2 dx + c/2))^6 \cos(1/2 dx + c/2) + (-20B - 24C) \sin(1/2 dx + c/2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x)`

[Out]
$$\frac{2}{15} \cdot (b \cdot (2 \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^{1/2} \cdot b \cdot (24 \cdot C \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^6 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) + (-20 \cdot B - 24 \cdot C) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) + (10 \cdot B + 6 \cdot C) \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) + 15 \cdot A \cdot (\sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^{1/2} \cdot (2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^{1/2} \cdot \text{EllipticE}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c), 2^{1/2}) - 5 \cdot B \cdot (\sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^{1/2} \cdot (2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^{1/2} \cdot \text{EllipticF}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c), 2^{1/2}) + 9 \cdot C \cdot (\sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2)^{1/2} \cdot (2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1)^{1/2} \cdot \text{EllipticE}(\cos(1/2 \cdot d \cdot x + 1/2 \cdot c), 2^{1/2})) / (-b \cdot (2 \cdot \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^4 - \sin(1/2 \cdot d \cdot x + 1/2 \cdot c)^2))^{1/2} / \sin(1/2 \cdot d \cdot x + 1/2 \cdot c) / (b \cdot (2 \cdot \cos(1/2 \cdot d \cdot x + 1/2 \cdot c) - 1))^{1/2} / d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c)), x)
```

$$3.242 \quad \int \sqrt{b \cos(c + dx)} \left(A + B \cos(c + dx) + C \cos^2(c + dx) \right) \sec(c + dx) dx$$

Optimal. Leaf size=112

$$\frac{2b(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2BE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rubi [A] time = 0.153898, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.18$, Rules used = {16, 3023, 2748, 2642, 2641, 2640, 2639}

$$\frac{2b(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2BE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_)^(n_)), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\ &= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2}{3} \int \frac{1}{2} b (3A + C) \sec(c + dx) dx \\ &= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + B \int \sqrt{b \cos(c + dx)} dx \\ &= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b(3A + C)) \int \sec(c + dx) dx}{3d} \\ &= \frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{\cos(c + dx)}} + \frac{2b(3A + C) \int \sec(c + dx) dx}{3d} \end{aligned}$$

Mathematica [A] time = 0.205672, size = 83, normalized size = 0.74

$$\frac{b \left(2(3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx)) \right)}{3d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]
```

```
[Out] (b*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.71, size = 283, normalized size = 2.5

$$-\frac{2b}{3d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(4C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) + 3A \sqrt{(\sin(1/2 dx + c/2))^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x)`

[Out]
$$-2/3*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b*(4*C*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c), x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)*(b*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)*(b*cos(d*x+c))^(1/2), x  
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(  
d*x + c), x)
```

3.243 $\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx$

Optimal. Leaf size=109

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.186631, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3021, 2748, 2642, 2641, 2640, 2639}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2Ab\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :=> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx)) \sec^2(c+dx) dx &= b^2 \int \frac{A + B \cos(c+dx) + C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx \\ &= \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{2 \int \frac{\frac{b^2 B}{2} - \frac{1}{2} b^2 (A-C) \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx}{b} \\ &= \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + (bB) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx \\ &= \frac{2Ab \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{(bB\sqrt{\cos(c+dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{\sqrt{b \cos(c+dx)}} \\ &= \frac{2(A-C)\sqrt{b \cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right)}{d\sqrt{\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.274768, size = 78, normalized size = 0.72

$$\frac{2b \left(-(A-C)\sqrt{\cos(c+dx)} E\left(\frac{1}{2}(c+dx) \middle| 2\right) + A \sin(c+dx) + B\sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right) \right)}{d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^2,x]
```

```
[Out] (2*b*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*
x]])
```

Maple [A] time = 3.895, size = 259, normalized size = 2.4

$$\frac{b\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2 b} \left(A\sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \right)}{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x)`

[Out]
$$-2*b*(-2*b*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2))-2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+B*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^(1/2))-C*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^(1/2)/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2*(b*cos(d*x+c))**(1/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)
```

3.244 $\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(dx) dx$

Optimal. Leaf size=140

$$\frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bB \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b*(A + 3*C)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^2*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b*B*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.214617, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bB \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2BE\left(\frac{1}{2}(c + dx) \middle| 2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[\text{Sqrt}[b*\text{Cos}[c + d*x]]*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^3, x]$

[Out] $(-2*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b*(A + 3*C)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^2*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b*B*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_)^(m_*)*((b_*)*(v_))^(n_), x_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m + n), x], x] /;$ $\text{FreeQ}\{b, n\}, x\} \&\& \text{IntegerQ}[m]$

Rule 3021

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]])^(m_*)*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_)] + (C_*)*\text{sin}[(e_*) + (f_*)*(x_)]^2), x_Symbol] := -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^(m + 1)]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /;$ $\text{FreeQ}\{a, b, e, f, A, B, C\}, x\} \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2748

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_)]])^(m_*)*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]), x_Symbol] := \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^(m + 1), x], x] /;$ $\text{FreeQ}\{b, c, d, e, f, m\}, x\}$

Rule 2636

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]])^(n_), x_Symbol] := \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{In}$

$\text{t}[(b*\text{Sin}[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[\text{Sin}[c + d*x]]/\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[1/\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2}{3} \int \frac{\frac{3b^2B}{2} + \frac{1}{2}b^2(A + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + (b^2B) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab^2 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2bB \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - E \\ &= \frac{2b(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2b(A + 3C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}} - \frac{2b(A + 3C)\sqrt{\cos(c + dx)}}{d\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.361735, size = 90, normalized size = 0.64

$$\frac{2b \left(\tan(c + dx)(A + 3B \cos(c + dx)) + (A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 3B\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (2*b*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x])

))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 8.23, size = 505, normalized size = 3.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2), x)

[Out] $\frac{2}{3} * (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} / \sin(1/2 * d * x + 1/2 * c) ^ 3 / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) * (2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 12 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) + 2 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 3 * B * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} + 6 * B * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) - 3 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (-2 * b * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2 * b) ^ {1/2} / (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2}) / d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)

$$3.245 \quad \int \sqrt{b \cos(c + dx)} \left(A + B \cos(c + dx) + C \cos^2(c + dx) \right) \sec^4(dx) dx$$

Optimal. Leaf size=181

$$\frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \dots$$

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.248213, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \dots$$

Antiderivative was successfully verified.

[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4, x]

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5}(2b) \int \frac{\frac{5b^2B}{2} + \frac{1}{2}b^2C}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + (b^3B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\ &= \frac{2Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} \\ &= \frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) - 9A \sin(2(c + dx)) - 6A \tan(c + dx) - 10B \sin^2(c + dx)}{15d\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.552948, size = 122, normalized size = 0.67

$$\frac{\sec^2(c + dx)\sqrt{b \cos(c + dx)}\left(6(3A + 5C) \cos^{\frac{3}{2}}(c + dx)E\left(\frac{1}{2}(c + dx) \middle| 2\right) - 9A \sin(2(c + dx)) - 6A \tan(c + dx) - 10B \sin^2(c + dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[
c + d*x]^4,x]
```

```
[Out] -(Sqrt[b*cos[c + d*x]]*Sec[c + d*x]^2*(6*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)] - 6*A*Tan[c + d*x]))/(15*d)
```

Maple [B] time = 10.465, size = 804, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2), x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2), x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)
```

3.246 $\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(dx) dx$

Optimal. Leaf size=210

$$\frac{2b^2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b^3B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \dots$$

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^3*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.269163, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2b^2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b^3B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5, x]
```

```
[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^3*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3021

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx &= b^5 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7} (2b^2) \int \frac{\frac{7b^2B}{2} + \frac{1}{2}b^2}{(b \cos(c + dx))^{7/2}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + (b^4B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx \\
&= \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^3B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
&= \frac{2Ab^4 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^3B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
&= \frac{2b(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} \\
&= -\frac{6B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2b(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.916425, size = 143, normalized size = 0.68

$$2 \sec^3(c + dx) \sqrt{b \cos(c + dx)} \left(5(5A + 7C) \cos^{\frac{5}{2}}(c + dx) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + \frac{25}{2} A \sin(2(c + dx)) + 15A \tan(c + dx) + 21 \right)$$

105d

Antiderivative was successfully verified.

[In] Integrate[Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*Sec[c + d*x]^3*(-63*B*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*B*Sin[c + d*x] + 63*B*Cos[c + d*x]^2*Sin[c + d*x] + (25*A*Sin[2*(c + d*x)]))/2 + (35*C*Sin[2*(c + d*x)])/2 + 15*A*Tan[c + d*x]))/(105*d)

Maple [B] time = 12.198, size = 725, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b*(A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5*(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

3.247 $\int \cos(c+dx)(b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=210

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2b(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{10b^2B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}}$$

[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^2*d)

Rubi [A] time = 0.232338, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3023, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{2b(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{10b^2B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (2*b*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (10*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^2*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]
)*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^2d} + \frac{2 \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx)) dx}{9b^2d} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^2d} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{9b^2d} \\ &= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} + \frac{2B \int (b \cos(c + dx))^{3/2} dx}{9b^2d} \\ &= \frac{10bB\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2(9A + 7C) \int (b \cos(c + dx))^{3/2} dx}{9b^2d} \\ &= \frac{2b(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \cos(c + dx)\right)}{15d\sqrt{\cos(c + dx)}} \\ &= \frac{2b(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| \cos(c + dx)\right)}{15d\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.875592, size = 128, normalized size = 0.61

$$\frac{(b \cos(c + dx))^{5/2} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (7(36A + 43C) \cos(c + dx) + 5(18B \cos(2(c + dx)) + 78B + 7C \cos(3(c + dx)))) \right)}{630bd \cos^2(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/(630*b*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 3.683, size = 384, normalized size = 1.8

$$-\frac{2b^2}{315d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(-1120 C \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{10} + (720 B + 2240 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^8 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-504 A - 1080 B - 2072 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (504 A + 840 B + 952 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-126 A - 240 B - 168 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) - 189 A \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{2-1} \right)^{1/2} \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) + 75 B \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{2-1} \right)^{1/2} \text{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) - 147 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{2-1} \right)^{1/2} \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) \right) / \left(-b \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 - \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} / \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right) / \left(b \left(2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(-1120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A \right) (b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(C b \cos(dx + c)^4 + B b \cos(dx + c)^3 + A b \cos(dx + c)^2 \right) \sqrt{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```


[Out] integral((C*b*cos(d*x + c)^4 + B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*cos(d*x + c), x)

3.248 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=181

$$\frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b(7A + 5C) \sin(c + dx)\sqrt{b \cos(c + dx)}}{21d} + \frac{2B \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d}$$

[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)

Rubi [A] time = 0.182872, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3023, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2b^2(7A + 5C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2b(7A + 5C) \sin(c + dx)\sqrt{b \cos(c + dx)}}{21d} + \frac{2B \sin(c + dx)(b \cos(c + dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b*d)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,

d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} + \frac{2 \int (b \cos(c + dx))^5 dx}{7bd} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7bd} + \frac{B \int (b \cos(c + dx))^5 dx}{b} \\ &= \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2B(b \cos(c + dx))^5}{21d} \\ &= \frac{2b(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2B(b \cos(c + dx))^5}{21d} \\ &= \frac{6bB\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \end{aligned}$$

Mathematica [A] time = 0.0935151, size = 108, normalized size = 0.6

$$\frac{(b \cos(c + dx))^{3/2} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (70A + 42B \cos(c + dx) + 15C \cos(2(c + dx)) + 65C) + 10(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{105d \cos^3(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x] + 15*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*d*Cos[c + d*x]^(3/2))

Maple [A] time = 3.533, size = 353, normalized size = 2.

$$-\frac{2b^2}{105d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(240C (\sin(1/2 dx + c/2))^8 \cos(1/2 dx + c/2) + (-168B - 360C) \sin(1/2 dx + c/2) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)`

[Out]
$$-2/105*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b^2*(240*C*\sin(1/2*d*x+1/2*c)^8*\cos(1/2*d*x+1/2*c)+(-168*B-360*C)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+35*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+25*C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c))\sqrt{b \cos(dx + c)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

[Out] `integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)`

[Out] Timed out

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.249 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=146

$$\frac{2b(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bB \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d} + \frac{2b^2C \sin^2(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rubi [A] time = 0.174741, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3023, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2b(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bB \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d} + \frac{2b^2C \sin^2(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (2*b*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},

x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[SIN[c + d*x]]/Sqrt[b*SIN[c + d*x]], Int[1/Sqrt[SIN[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \\
 &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2}{5} \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \\
 &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + B \int (b \cos(c + dx))^{3/2} dx + \frac{2A}{5} \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{2bB\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{2A}{5} \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{2b(5A + 3C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2B}{5} \int \sqrt{b \cos(c + dx)} dx \\
 &= \frac{2b(5A + 3C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2B}{5} \int \sqrt{b \cos(c + dx)} dx
 \end{aligned}$$

Mathematica [A] time = 0.232387, size = 95, normalized size = 0.65

$$\frac{2b\sqrt{b \cos(c + dx)} \left(3(5A + 3C) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + \sin(c + dx) \sqrt{\cos(c + dx)} (5B + 3C \cos(c + dx)) + 5BF\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{15d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]
```

```
[Out] (2*b*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c
```

+ d*x]))/(15*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 3.819, size = 319, normalized size = 2.2

$$\frac{2b^2}{15d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(24C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-20B - 24C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (10B + 6C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + 15A \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{\frac{1}{2}} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{\frac{1}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{\frac{1}{2}} \right) - 5B \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{\frac{1}{2}} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{\frac{1}{2}} \operatorname{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{\frac{1}{2}} \right) + 9C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{\frac{1}{2}} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{\frac{1}{2}} \operatorname{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{\frac{1}{2}} \right) \right) / \left(-b \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 - \sin \left(\frac{1}{2} dx + \frac{c}{2} \right)^2 \right)^{\frac{1}{2}} / \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) / \left(b \left(2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right)^{\frac{1}{2}} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)

[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(24*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left((Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)) \sqrt{b \cos(dx + c)} \sec(dx + c), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c), x)

3.250 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=116

$$\frac{2b^2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bBE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2bC \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rubi [A] time = 0.178475, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3023, 2748, 2642, 2641, 2640, 2639}

$$\frac{2b^2(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2bBE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2bC \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (2*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\ &= \frac{2bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3}(2b) \int \frac{A + B \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\ &= \frac{2bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + (bB) \int \frac{A + \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\ &= \frac{2bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b^2(3A + C)) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d} \\ &= \frac{2bB\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 2b^2 \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}} + \dots \end{aligned}$$

Mathematica [A] time = 0.165396, size = 85, normalized size = 0.73

$$\frac{b^2 \left(2(3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx)) \right)}{3d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (b^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.479, size = 285, normalized size = 2.5

$$-\frac{2b^2}{3d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2} dx + \frac{c}{2}\right) \right)^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(4C \left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right) \right)^4 \cos\left(\frac{1}{2} dx + \frac{c}{2}\right) + 3A \sqrt{\left(\sin\left(\frac{1}{2} dx + \frac{c}{2}\right)\right)^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)`

[Out]
$$-2/3*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b^2*(4*C*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c))\sqrt{b \cos(dx + c)} \sec(dx + c)^2, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")`

[Out] `integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^2, x)
```

3.251 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=114

$$\frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2b^2 B \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{b \cos(c + dx)}}$$

[Out] $(-2*b*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^2*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.185184, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3021, 2748, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b(A - C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2b^2 B \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{3/2}*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^3, x]$

[Out] $(-2*b*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^2*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x] \&\& \text{IntegerQ}[m]$

Rule 3021

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)] + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)}]/(b*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[\text{Sin}[c + d*x]]/\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[1/\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} \\ &= \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + 2 \int \frac{\frac{b^2 B}{2} - \frac{1}{2}b^2(A - C)}{\sqrt{b \cos(c + dx)}} \\ &= \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + (b^2 B) \int \frac{1}{\sqrt{b \cos(c + dx)}} \\ &= \frac{2Ab^2 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + \frac{(b^2 B \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}}}{\sqrt{b \cos(c + dx)}} \\ &= \frac{2b(A - C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(c + dx) + B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.251277, size = 80, normalized size = 0.7

$$\frac{2b^2 \left(-(A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(c + dx) + B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] (2*b^2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.801, size = 261, normalized size = 2.3

$$\frac{b^2 \sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE}\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(c + dx) + B \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d \sqrt{b \cos(c + dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)`

[Out]
$$-2*b^2*(-2*b*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2*b)^{(1/2)}*(A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*A*\cos(1/2*d*x+1/2*c)*\sin(1/2*d*x+1/2*c)^2+B*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)}))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c))\sqrt{b \cos(dx + c)} \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")`

[Out] `integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^3, x)
```

3.252 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=145

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2bBE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*b*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^2*(A + 3*C)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b^2*B*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.211059, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2bBE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^(3/2)*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^4, x]$

[Out] $(-2*b*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^2*(A + 3*C)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b^2*B*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_)^(m_*)*((b_*)*(v_))^(n_), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m + n), x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]])^(m_*)*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_)] + (C_*)*\text{sin}[(e_*) + (f_*)*(x_)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^(m + 1)]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_)]])^(m_*)*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^(m + 1), x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

$\text{Int}[(b_*)*\text{sin}[(c_*) + (d_*)*(x_)]])^(n_), x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{In}$

$t[(b*\sin[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[b*\sin[c + d*x]]/\text{Sqrt}[\sin[c + d*x]], \text{Int}[\text{Sqrt}[\sin[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_*)\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[\sin[c + d*x]]/\text{Sqrt}[b*\sin[c + d*x]], \text{Int}[1/\text{Sqrt}[\sin[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{1}{3}(2b) \int \frac{\frac{3b^2B}{2} + \frac{1}{2}b^2}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + (b^3B) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab^3 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} \\ &= \frac{2b^2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} \\ &= -\frac{2bB\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}} + \dots \end{aligned}$$

Mathematica [A] time = 0.295315, size = 92, normalized size = 0.63

$$\frac{2b^2 \left(\tan(c + dx)(A + 3B \cos(c + dx)) + (A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 3B\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (2*b^2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*

x]))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 8.06, size = 506, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] $\frac{2}{3} * (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * b / \sin(1/2 * d * x + 1/2 * c) ^ 3 / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) * (2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 12 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) + 2 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 3 * B * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} + 6 * B * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) - 3 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (-2 * b * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2 * b) ^ {1/2} / (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2}) / d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)) \sqrt{b \cos(dx + c)} \sec(dx + c)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^4, x)

3.253 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=186

$$\frac{2b^2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} - \frac{2b(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^3B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \dots$$

[Out] $(-2*b*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^4*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*b^3*B*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b^2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.241045, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2b^2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} - \frac{2b(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^3B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^(3/2)*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^5, x]$

[Out] $(-2*b*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^4*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*b^3*B*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b^2*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_)^(m_*)*((b_)*(v_))^(n_), x_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m + n), x], x] /; \text{FreeQ}\{b, n\}, x] \&\& \text{IntegerQ}[m]$

Rule 3021

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_)]])^(m_*)*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_)] + (C_*)*\text{sin}[(e_*) + (f_*)*(x_)]^2), x_Symbol] := -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^(m + 1)]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2748

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_)]])^(m_*)*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_)]), x_Symbol] := \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^(m + 1), x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2636

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx &= b^5 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5} (2b^2) \int \frac{5b^2B}{2} + \dots \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + (b^4B) \int \frac{1}{(b \cos(c + dx))^{3/2}} \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3B \sin(c + dx)}{3d(b \cos(c + dx))^3} \\ &= \frac{2Ab^4 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^3B \sin(c + dx)}{3d(b \cos(c + dx))^3} \\ &= -\frac{2b(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx)\right)}{5d\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.483896, size = 122, normalized size = 0.66

$$\frac{\sec^3(c + dx)(b \cos(c + dx))^{3/2} \left(6(3A + 5C) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) - 9A \sin(2(c + dx)) - 6A \tan(c + dx) - 10B \right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^5,x]
```

```
[Out] -((b*cos[c + d*x])^(3/2)*Sec[c + d*x]^3*(6*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 10*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 10*B*Sin[c + d*x] - 9*A*Sin[2*(c + d*x)] - 15*C*Sin[2*(c + d*x)] - 6*A*Tan[c + d*x]))/(15*d)
```

Maple [B] time = 9.945, size = 805, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right)\sqrt{b \cos(dx + c)} \sec(dx + c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5, x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^5, x)
```

3.254 $\int (b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=215

$$\frac{2b^3(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^4B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \dots$$

[Out] (-6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^4*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^3*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^2*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.269374, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2b^3(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^4B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \dots$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^6, x]

[Out] (-6*b*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^5*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^4*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^3*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^2*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
 \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^6(c + dx) dx &= b^6 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} \\
 &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7} (2b^3) \int \frac{\frac{7b^2B}{2} + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} \\
 &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + (b^5B) \int \frac{1}{(b \cos(c + dx))^{5/2}} \\
 &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^4B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
 &= \frac{2Ab^5 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^4B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\
 &= \frac{2b^2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} \\
 &= -\frac{6bB\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \dots
 \end{aligned}$$

Mathematica [A] time = 1.61624, size = 134, normalized size = 0.62

$$\frac{\sec^5(c + dx)(b \cos(c + dx))^{3/2} \left(2 \sin(c + dx)(10(5A + 7C) \cos(2(c + dx)) + 110A + 273B \cos(c + dx) + 63B \cos(3(c + dx))) \right)}{420d}$$

420d

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(3/2)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2)*Sec[c + d*x]^6,x]

[Out] ((b*cos[c + d*x])^(3/2)*Sec[c + d*x]^5*(-504*B*cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(110*A + 70*C + 273*B*cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d*x)] + 63*B*cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)

Maple [B] time = 11.725, size = 727, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2*(A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c))\sqrt{b \cos(dx + c)} \sec(dx + c)^6, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6, x, algorithm="fricas")
```

```
[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}} \sec(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)*sec(d*x + c)^6, x)
```

3.255 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=212

$$\frac{2b^2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{10b^2B \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d}$$

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rubi [A] time = 0.201388, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3023, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2b^2(9A + 7C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{15d\sqrt{\cos(c + dx)}} + \frac{2b(9A + 7C) \sin(c + dx)(b \cos(c + dx))^{3/2}}{45d} + \frac{10b^2B \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (2*b^2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*d*Sqrt[Cos[c + d*x]]) + (10*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b*d)
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} + \frac{2 \int (b \cos(c + dx))^5 dx}{9bd} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9bd} + \frac{B \int (b \cos(c + dx))^4 dx}{b} \\ &= \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} + \frac{2B(b \cos(c + dx))^4}{b} \\ &= \frac{10b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2b(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45d} \\ &= \frac{2b^2(9A + 7C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}} + \frac{10b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\ &= \frac{2b^2(9A + 7C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15d\sqrt{\cos(c + dx)}} + \frac{10b^3 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \end{aligned}$$

Mathematica [A] time = 0.24348, size = 125, normalized size = 0.59

$$\frac{(b \cos(c + dx))^{5/2} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (7(36A + 43C) \cos(c + dx) + 5(18B \cos(2(c + dx)) + 78B + 7C \cos(3(c + dx)))) \right)}{630d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(84*(9*A + 7*C)*EllipticE[(c + d*x)/2, 2] + 300*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(7*(36*A + 43*C)*Cos[c + d*x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[c + d*x]))/(630*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 3.772, size = 384, normalized size = 1.8

$$-\frac{2b^3}{315d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(-1120 C \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{10} + (720 B + 2240 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^8 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-504 A - 1080 B - 2072 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (504 A + 840 B + 952 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-126 A - 240 B - 168 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) - 189 A \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{2-1} \right)^{1/2} \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) + 75 B \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{2-1} \right)^{1/2} \text{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) - 147 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{2-1} \right)^{1/2} \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) \right) / \left(-b \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 - \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} / \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right) / \left(b \left(2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(-1120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(C b^2 \cos(dx + c)^4 + B b^2 \cos(dx + c)^3 + A b^2 \cos(dx + c)^2 \right) \sqrt{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2), x)

3.256 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=183

$$\frac{2b^2(7A + 5C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2b^3(7A + 5C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{6b^2 B E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}}$$

[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rubi [A] time = 0.20982, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3023, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2b^2(7A + 5C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{21d} + \frac{2b^3(7A + 5C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d \sqrt{b \cos(c + dx)}} + \frac{6b^2 B E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*b*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \\
&= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + \frac{2}{7} \int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \\
&= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7d} + B \int (b \cos(c + dx))^{3/2} \sec(c + dx) dx \\
&= \frac{2b^2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} + \frac{2b^2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21d} \\
&= \frac{6b^2B\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \frac{2b^3}{5d\sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.100298, size = 109, normalized size = 0.6

$$\frac{b(b \cos(c + dx))^{3/2} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (70A + 42B \cos(c + dx) + 15C \cos(2(c + dx)) + 65C) + 10(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{105d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x], x]
```

```
[Out] (b*(b*Cos[c + d*x])^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)
*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c +
```

$$d*x] + 15*C*\text{Cos}[2*(c + d*x)]*\text{Sin}[c + d*x]))/(105*d*\text{Cos}[c + d*x]^(3/2))$$

Maple [A] time = 3.893, size = 353, normalized size = 1.9

$$-\frac{2b^3}{105d} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(240C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^8 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + (-168B - 360C) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)

[Out]
$$-2/105*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(240*C*\sin(1/2*d*x+1/2*c)^8*\cos(1/2*d*x+1/2*c)+(-168*B-360*C)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+35*A*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^(1/2))-63*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^(1/2))*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)+25*C*(\sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*\sin(1/2*d*x+1/2*c)^2-1)^(1/2)*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^(1/2)/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2) \sqrt{b \cos(dx + c)} \sec(dx + c), x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c), x)

$$3.257 \quad \int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal. Leaf size=151

$$\frac{2b^2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2B \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d} + \frac{2b^3B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} +$$

[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rubi [A] time = 0.192517, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2b^2(5A + 3C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2b^2B \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d} + \frac{2b^3B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} +$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (2*b^2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},

x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[SIN[c + d*x]]/Sqrt[b*SIN[c + d*x]], Int[1/Sqrt[SIN[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
 \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx \\
 &= \frac{2bC(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + \frac{1}{5}(2b) \int (b \cos(c + dx))^{3/2} \sec^2(c + dx) dx \\
 &= \frac{2bC(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} + (bB) \int (b \cos(c + dx))^{3/2} \sec^2(c + dx) dx \\
 &= \frac{2b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{2bC(b \cos(c + dx))^{3/2} \sin(c + dx)}{5d} \\
 &= \frac{2b^2(5A + 3C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{5d \sqrt{\cos(c + dx)}} \\
 &= \frac{2b^2(5A + 3C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx)\right)}{5d \sqrt{\cos(c + dx)}}
 \end{aligned}$$

Mathematica [A] time = 0.19601, size = 97, normalized size = 0.64

$$\frac{2b^2 \sqrt{b \cos(c + dx)} \left(3(5A + 3C) E\left(\frac{1}{2}(c + dx)\right) + \sin(c + dx) \sqrt{\cos(c + dx)} (5B + 3C \cos(c + dx)) + 5BF\left(\frac{1}{2}(c + dx)\right) \right)}{15d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (2*b^2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin
```

$[c + d*x]))/(15*d*\text{Sqrt}[\text{Cos}[c + d*x]])$

Maple [A] time = 3.619, size = 319, normalized size = 2.1

$$\frac{2b^3}{15d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(24C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-20B - 24C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (10B + 6C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + 15A \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) - 5B \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \text{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) + 9C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) \right) / \left(-b \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 - \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} / \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) / \left(b \left(2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(24*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\left(Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2 \right) \sqrt{b \cos(dx + c)} \sec(dx + c)^2, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x, algorithm="fricas")

[Out] integral((C*b^2*cos(d*x + c)^4 + B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^2, x)
```

3.258 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=120

$$\frac{2b^3(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2b^2BE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2b^2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

[Out] $(2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)$

Rubi [A] time = 0.170232, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3023, 2748, 2642, 2641, 2640, 2639}

$$\frac{2b^3(3A + C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2b^2BE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}} + \frac{2b^2C \sin(c + dx)\sqrt{b \cos(c + dx)}}{3d}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{5/2}*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^3, x]$

[Out] $(2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^3*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)] + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\text{Sin}[e + f*x], x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[\text{Sin}[c + d*x]]/\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[1/\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /;$ FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx \\ &= \frac{2b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{1}{3} (2b^2) \\ &= \frac{2b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + (b^2 B) \\ &= \frac{2b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b^3 (3A + C))}{3d} \\ &= \frac{2b^2 B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(c + dx) \sqrt{\cos(c + dx)}}{d \sqrt{\cos(c + dx)}} + \frac{b^3 (3A + C)}{3d} \end{aligned}$$

Mathematica [A] time = 0.193615, size = 79, normalized size = 0.66

$$\frac{2(b \cos(c + dx))^{5/2} \left((3A + C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 3BE\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(c + dx) \sqrt{\cos(c + dx)} \right)}{3d \cos^{5/2}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3, x]
```

```
[Out] (2*(b*Cos[c + d*x])^(5/2)*(3*B*EllipticE[(c + d*x)/2, 2] + (3*A + C)*EllipticF[(c + d*x)/2, 2] + C*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 3.121, size = 285, normalized size = 2.4

$$-\frac{2b^3}{3d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(4C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) + 3A \sqrt{(\sin(1/2 dx + c/2))^2} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)`

[Out]
$$-2/3*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*b^3*(4*C*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((Cb^2 \cos(dx + c)^4 + Bb^2 \cos(dx + c)^3 + Ab^2 \cos(dx + c)^2) \sqrt{b \cos(dx + c)} \sec(dx + c)^3, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x, algorithm="fricas")`

[Out] `integral((C*b^2*cos(d*x + c)^4 + B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^3, x)
```

3.259 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=116

$$-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

[Out] $(-2*b^2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^3*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.190206, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3021, 2748, 2642, 2641, 2640, 2639}

$$-\frac{2b^2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{2Ab^3\sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{2b^3B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{5/2}*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^4, x]$

[Out] $(-2*b^2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^3*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^3*\text{Sin}[c + d*x])/(d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^{(m+n)}, x], x] /; \text{FreeQ}\{b, n\}, x] \&\& \text{IntegerQ}[m]$

Rule 3021

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)] + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)}]/(b*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1)*\text{Sin}[e + f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_*)*\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[\text{Sin}[c + d*x]]/\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[1/\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} \\ &= \frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + (2b) \int \frac{\frac{b^2 B}{2} - \frac{1}{2}b^2(A - C)}{\sqrt{b \cos(c + dx)}} \\ &= \frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + (b^3 B) \int \frac{1}{\sqrt{b \cos(c + dx)}} \\ &= \frac{2Ab^3 \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} + \frac{(b^3 B \sqrt{\cos(c + dx)})}{\sqrt{b \cos(c + dx)}} \int \\ &= \frac{2b^2(A - C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(c + dx) + B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.269255, size = 80, normalized size = 0.69

$$\frac{2b^3 \left(-(A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(c + dx) + B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4, x]
```

```
[Out] (2*b^3*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.858, size = 261, normalized size = 2.3

$$\frac{b^3 \sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \right)}{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)`

[Out] $-2b^3(-2b\sin(1/2dx+1/2c)^4+\sin(1/2dx+1/2c)^2b)^{1/2}(A(\sin(1/2dx+1/2c)^2)^{1/2}(2\sin(1/2dx+1/2c)^2-1)^{1/2}\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2})-2A\cos(1/2dx+1/2c)\sin(1/2dx+1/2c)^2+B(\sin(1/2dx+1/2c)^2)^{1/2}(2\sin(1/2dx+1/2c)^2-1)^{1/2}\text{EllipticF}(\cos(1/2dx+1/2c),2^{1/2})-C(\sin(1/2dx+1/2c)^2)^{1/2}(2\sin(1/2dx+1/2c)^2-1)^{1/2}\text{EllipticE}(\cos(1/2dx+1/2c),2^{1/2}))/(-b(2\sin(1/2dx+1/2c)^4-\sin(1/2dx+1/2c)^2)^{1/2}/\sin(1/2dx+1/2c)/(b(2\cos(1/2dx+1/2c)^2-1))^{1/2}/d$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="maxima")`

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

`integral((Cb^2 cos(dx + c)^4 + Bb^2 cos(dx + c)^3 + Ab^2 cos(dx + c)^2)*sqrt(b cos(dx + c) sec(dx + c)^4), x)`

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x, algorithm="fricas")`

[Out] `integral((C*b^2*cos(dx + c)^4 + B*b^2*cos(dx + c)^3 + A*b^2*cos(dx + c)^2)*sqrt(b*cos(dx + c))*sec(dx + c)^4, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^4, x)
```

3.260 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=147

$$\frac{2b^3(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^3B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b^2BE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

[Out] $(-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])$

Rubi [A] time = 0.223392, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2b^3(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx)\middle|2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^3B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{2b^2BE\left(\frac{1}{2}(c + dx)\middle|2\right)\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^(5/2)*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^5, x]$

[Out] $(-2*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]) + (2*b^3*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^4*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*b^3*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_)^(m_*)*((b_*)*(v_))^(n_), x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m + n), x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_)]])^(m_*)*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_)] + (C_*)*\sin[(e_*) + (f_*)*(x_)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^(m + 1)]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_)]])^(m_*)*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^(m + 1), x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

$\text{Int}[(b_*)*\sin[(c_*) + (d_*)*(x_)]])^(n_), x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{In}$

$t[(b*\sin[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_*)\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[b*\sin[c + d*x]]/\text{Sqrt}[\sin[c + d*x]], \text{Int}[\text{Sqrt}[\sin[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticE}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_*)\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[\sin[c + d*x]]/\text{Sqrt}[b*\sin[c + d*x]], \text{Int}[1/\text{Sqrt}[\sin[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_*) + (d_*)*(x_*)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx &= b^5 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx \\ &= \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{1}{3} (2b^2) \int \frac{3b^2B}{2} + \dots \\ &= \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + (b^4B) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx \\ &= \frac{2Ab^4 \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2b^3B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} \\ &= \frac{2b^3(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} \\ &= -\frac{2b^2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d\sqrt{\cos(c + dx)}} + \dots \end{aligned}$$

Mathematica [A] time = 0.276697, size = 92, normalized size = 0.63

$$\frac{2b^3 \left(\tan(c + dx)(A + 3B \cos(c + dx)) + (A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 3B\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^5,x]

[Out] (2*b^3*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*

x)))/(3*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 9.055, size = 508, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x)

[Out] $\frac{2}{3} * (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * b ^ 2 / \sin(1/2 * d * x + 1/2 * c) ^ 3 / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) * (2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 12 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) + 2 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 3 * B * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} + 6 * B * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) - 3 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (-2 * b * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2 * b) ^ {1/2} / (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2}) / d$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

integral(((Cb² cos(dx + c)⁴ + Bb² cos(dx + c)³ + Ab² cos(dx + c)²)sqrt(b cos(dx + c) sec(dx + c)⁵), x)

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="fricas")

[Out] integral((C*b²*cos(d*x + c)⁴ + B*b²*cos(d*x + c)³ + A*b²*cos(d*x + c)²)*sqrt(b*cos(d*x + c))*sec(d*x + c)⁵, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**5,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^5 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^5,x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^5, x)

3.261 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=188

$$\frac{2b^3(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2b^2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

[Out] $(-2*b^2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^3*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^5*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*b^4*B*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b^3*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.249823, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2b^3(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}} - \frac{2b^2(3A + 5C)E\left(\frac{1}{2}(c + dx) \middle| 2\right) \sqrt{b \cos(c + dx)}}{5d\sqrt{\cos(c + dx)}} + \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^(5/2)*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)*\text{Sec}[c + d*x]^6, x]$

[Out] $(-2*b^2*(3*A + 5*C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(5*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*b^3*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*b^5*\text{Sin}[c + d*x])/(5*d*(b*\text{Cos}[c + d*x])^(5/2)) + (2*b^4*B*\text{Sin}[c + d*x])/(3*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*b^3*(3*A + 5*C)*\text{Sin}[c + d*x])/(5*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 16

$\text{Int}[(u_*)*(v_*)^(m_*)*((b_*)*(v_*))^(n_), x_Symbol] := \text{Dist}[1/b^m, \text{Int}[u*(b*v)^(m + n), x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_*)]^(m_*)*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_*)] + (C_*)*\text{sin}[(e_*) + (f_*)*(x_*)]^2), x_Symbol] := -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^(m + 1)]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_*)]^(m_*)*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_*)]), x_Symbol] := \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^(m + 1), x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^6(c + dx) dx &= b^6 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{1}{5} (2b^3) \int \frac{5b^2B}{2} + \dots \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + (b^5B) \int \frac{1}{(b \cos(c + dx))^{3/2}} \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4B \sin(c + dx)}{3d(b \cos(c + dx))^3} \\ &= \frac{2Ab^5 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b^4B \sin(c + dx)}{3d(b \cos(c + dx))^3} \\ &= -\frac{2b^2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx)\right)}{5d\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.370061, size = 121, normalized size = 0.64

$$\frac{2b^4 \left(3(3A + 5C) \cos^{\frac{3}{2}}(c + dx) E\left(\frac{1}{2}(c + dx) \middle| 2\right) - \frac{9}{2} A \sin(2(c + dx)) - 3A \tan(c + dx) - 5B \sin(c + dx) - 5B \cos^{\frac{3}{2}}(c + dx) \right)}{15d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^6,x]
```

```
[Out] (-2*b^4*(3*(3*A + 5*C)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] - 5*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] - 5*B*Sin[c + d*x] - (9*A*Sin[2*(c + d*x)])/2 - (15*C*Sin[2*(c + d*x)])/2 - 3*A*Tan[c + d*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

Maple [B] time = 11.293, size = 807, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^2/sin(1/2*d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb^2 \cos(dx+c)^4 + Bb^2 \cos(dx+c)^3 + Ab^2 \cos(dx+c)^2\right)\sqrt{b \cos(dx+c)} \sec(dx+c)^6, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^
2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^6, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)
**6,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^6 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^6
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*se
c(d*x + c)^6, x)
```

3.262 $\int (b \cos(c+dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=217

$$\frac{2b^4(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^3(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^5B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \dots$$

```
[Out] (-6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^5*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^4*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^3*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.267899, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2b^4(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} + \frac{2b^3(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^5B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \dots$$

Antiderivative was successfully verified.

```
[In] Int[(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^7, x]
```

```
[Out] (-6*b^2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]) + (2*b^3*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^6*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^5*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b^4*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*b^3*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3021

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^7(c + dx) dx &= b^7 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7} (2b^4) \int \frac{\frac{7b^2B}{2} + \dots}{(b \cos(c + dx))^{7/2}} \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + (b^6B) \int \frac{1}{(b \cos(c + dx))^{7/2}} \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^5B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\ &= \frac{2Ab^6 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^5B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} \\ &= \frac{2b^3(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} \\ &= -\frac{6b^2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5d\sqrt{\cos(c + dx)}} + \dots \end{aligned}$$

Mathematica [A] time = 0.911582, size = 134, normalized size = 0.62

$$\frac{\sec^6(c + dx)(b \cos(c + dx))^{5/2} \left(2 \sin(c + dx)(10(5A + 7C) \cos(2(c + dx)) + 110A + 273B \cos(c + dx) + 63B \cos(3(c + dx))) \right)}{420d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2)*Sec[c + d*x]^7,x]

[Out] ((b*cos[c + d*x])^(5/2)*Sec[c + d*x]^6*(-504*B*cos[c + d*x]^(7/2)*EllipticE[(c + d*x)/2, 2] + 40*(5*A + 7*C)*Cos[c + d*x]^(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(110*A + 70*C + 273*B*cos[c + d*x] + 10*(5*A + 7*C)*Cos[2*(c + d*x)] + 63*B*cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)

Maple [B] time = 11.339, size = 727, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x)

[Out] -2*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*b^3*(A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))-1/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2))/d

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7,x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

integral((Cb^2 cos(dx + c)^4 + Bb^2 cos(dx + c)^3 + Ab^2 cos(dx + c)^2)√b cos(dx + c) sec(dx + c)^7, x)

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7, x, algorithm="fricas")
```

```
[Out] integral((C*b^2*cos(d*x + c)^4 + B*b^2*cos(d*x + c)^3 + A*b^2*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))*sec(d*x + c)^7, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**7, x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}} \sec(dx + c)^7 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^7, x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)*sec(d*x + c)^7, x)
```

$$3.263 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=214

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^2d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{5/2}}{7b^3d}$$

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)

Rubi [A] time = 0.221136, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^2d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15bd\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{5/2}}{7b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^2*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^4*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{b^3} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^4 d} + \frac{2 \int (b \cos(c + dx))^{5/2} \left(\frac{1}{2} + \frac{1}{2} \cos^2(c + dx)\right)}{b^3} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^4 d} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^4} \\ &= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45b^2 d} + \frac{2B(b \cos(c + dx))^{5/2}}{7b^3} \\ &= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} + \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45b^2 d} \\ &= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15bd\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} \\ &= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15bd\sqrt{\cos(c + dx)}} + \frac{10B\sqrt{\cos(c + dx)} \sin(c + dx)}{21d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.675739, size = 127, normalized size = 0.59

$$\frac{\sin(2(c + dx))(7(36A + 43C) \cos(c + dx) + 5(18B \cos(2(c + dx)) + 78B + 7C \cos(3(c + dx)))) + 168(9A + 7C)\sqrt{\cos(c + dx)}}{1260d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]/(1260*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 3.726, size = 381, normalized size = 1.8

$$-\frac{2}{315d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(-1120 C \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{10} + (720 B + 2240 C) \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)

[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```

$$3.264 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=185

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b \cos(c+dx)}}{21bd} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^2d} +$$

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*1*b*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)

Rubi [A] time = 0.193974, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2(7A+5C)\sin(c+dx)\sqrt{b \cos(c+dx)}}{21bd} + \frac{2(7A+5C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^2d} +$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*1*b*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^3*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{b^2} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^3 d} + \frac{2 \int (b \cos(c + dx))^{3/2} \left(\frac{1}{2}\right)}{b^2} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^3 d} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^3} \\ &= \frac{2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} + \frac{2B(b \cos(c + dx))^{3/2}}{5b^2} \\ &= \frac{2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21bd} + \frac{2B(b \cos(c + dx))^{3/2}}{5b^2} \\ &= \frac{6B\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2(7A + 5C)\sqrt{\cos(c + dx)}}{21d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.623247, size = 108, normalized size = 0.58

$$\frac{\sqrt{\cos(c + dx)} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (70A + 42B \cos(c + dx) + 15C \cos(2(c + dx)) + 65C) + 10(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{105d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]))/105d

+ 15*C*cos[2*(c + d*x)]*sin[c + d*x]))/(105*d*Sqrt[b*cos[c + d*x]])

Maple [A] time = 3.917, size = 350, normalized size = 1.9

$$-\frac{2}{105d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(240 C (\sin(1/2 dx + c/2))^8 \cos(1/2 dx + c/2) + (-168 B - 360 C) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)

[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-63*B*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*cos(d*x + c)), x)

$$3.265 \quad \int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

Optimal. Leaf size=150

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d} + \frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} + \frac{2B\sqrt{\cos(c+dx)}}{3bd}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)

Rubi [A] time = 0.156328, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3023, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)(b\cos(c+dx))^{3/2}}{5b^2d} + \frac{2B\sin(c+dx)\sqrt{b\cos(c+dx)}}{3bd} + \frac{2B\sqrt{\cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^2*d)

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :=> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] :=> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[SIN[c + d*x]]/Sqrt[b*SIN[c + d*x]], Int[1/Sqrt[SIN[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx &= \frac{\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2d} + \frac{2 \int \sqrt{b \cos(c + dx)} \left(\frac{1}{2}b\right)}{b^2} \\ &= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2d} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b^2} \\ &= \frac{2B\sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^2d} \\ &= \frac{2(5A + 3C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{b \cos(c + dx)}}{3d\sqrt{b}} \\ &= \frac{2(5A + 3C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)}}{3d\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.182133, size = 97, normalized size = 0.65

$$\frac{2\sqrt{b \cos(c + dx)} \left(3(5A + 3C) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + \sin(c + dx) \sqrt{\cos(c + dx)} (5B + 3C \cos(c + dx)) + 5BF\left(\frac{1}{2}(c + dx) \middle| 2\right)\right)}{15bd\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (2*Sqrt[b*Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d*x]))/(15*b*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 3.245, size = 316, normalized size = 2.1

$$\frac{2}{15d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(24 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-20 B - 24 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (10 B + 6 C) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + 15 A \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \operatorname{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) - 5 B \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \operatorname{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) + 9 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \operatorname{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) \right) / \left(-b \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 - \sin \left(\frac{1}{2} dx + \frac{c}{2} \right)^2 \right)^{1/2} / \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) / \left(b \left(2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)`

[Out] `2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/sqrt(b*cos(d*x + c)), x)

$$3.266 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=117

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd}$$

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rubi [A] time = 0.124221, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3023, 2748, 2642, 2641, 2640, 2639}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2C \sin(c+dx)\sqrt{b \cos(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]], x]

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b*d)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx &= \frac{2C\sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{2 \int \frac{\frac{1}{2}b(3A+C) + \frac{3}{2}bB \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx}{3b} \\ &= \frac{2C\sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{B \int \sqrt{b \cos(c + dx)} dx}{b} + \frac{1}{3}(3A + C) \int \frac{1}{\sqrt{\cos(c+dx)}} dx \\ &= \frac{2C\sqrt{b \cos(c + dx)} \sin(c + dx)}{3bd} + \frac{((3A + C)\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{3\sqrt{b \cos(c + dx)}} \\ &= \frac{2B\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2(3A + C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.0856365, size = 82, normalized size = 0.7

$$\frac{2(3A + C)\sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6B\sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx))}{3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.313, size = 282, normalized size = 2.4

$$-\frac{2}{3d} \sqrt{b \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^2 - 1} \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2 \left(4C \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right) \right)^4 \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 3A \sqrt{\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)
```

```
[Out] -2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*C*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-3*B*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
```

$1/2) * \text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}) - 2*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c) / (-b*(2*\sin(1/2*d*x+1/2*c)^4 - \sin(1/2*d*x+1/2*c)^2))^{(1/2)} / \sin(1/2*d*x+1/2*c) / (b*(2*\cos(1/2*d*x+1/2*c)^2 - 1))^{(1/2)} / d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/sqrt(b*cos(d*x + c)), x)

$$3.267 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=110

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b \cos(c+dx)}}$$

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.159898, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.18$, Rules used = {16, 3021, 2748, 2642, 2641, 2640, 2639}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]], x]

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_)^(n_)), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt{b \cos(c + dx)}} dx = b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx$$

$$= \frac{2A \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + \frac{2 \int \frac{\frac{b^2 B}{2} - \frac{1}{2} b^2 (A - C) \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx}{b^2}$$

$$= \frac{2A \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx - \frac{(A - C) \int \sqrt{b \cos(c + dx)}}{b}$$

$$= \frac{2A \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{\sqrt{b \cos(c + dx)}} - \frac{((A - C) \int \sqrt{b \cos(c + dx)}}{b}$$

$$= -\frac{2(A - C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd \sqrt{\cos(c + dx)}} + \frac{2B \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{d \sqrt{b \cos(c + dx)}}$$

Mathematica [C] time = 6.2823, size = 803, normalized size = 7.3

$$\frac{(B + C \cos(c + dx) + A \sec(c + dx)) \left(\frac{4A \sec(c) \sec(c + dx) \sin(dx)}{d} - \frac{2(-2A + C + C \cos(2c)) \csc(c) \sec(c)}{d} \right) \cos^2(c + dx)}{\sqrt{b \cos(c + dx)} (2A + C + 2B \cos(c + dx) + C \cos(2c + 2dx))} + \frac{2A \csc(c) (B + C \cos(c + dx) + A \sec(c + dx))}{d \sqrt{b \cos(c + dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (Cos[c + d*x]^2*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-2*(-2*A + C + C*Cos[2*c])*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*B*Cos[c + d*x]^(3/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2*A*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])
```

```

*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1
+ Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(C
os[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^
2]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*
*x])) - (2*C*Cos[c + d*x]^(3/2)*Csc[c]*(B + C*Cos[c + d*x] + A*Sec[c + d*x]
)*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin
[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1
+ Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 +
Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[
1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(
Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*
d*x]))

```

Maple [A] time = 3.992, size = 258, normalized size = 2.4

$$-2 \frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2} b \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE} \right)}{\dots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] -2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*
x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/
d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d
*x + c)/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/sqrt(b*cos(d
*x + c)), x)
```


$$3.268 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=139

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.202712, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{d\sqrt{b \cos(c+dx)}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]], x]

[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_) + (C_)*sin[(e_) + (f_)*(x_)^2]), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

Int[((b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1))/(b*d*(n+1)), x] + Dist[(n+2)/(b^2*(n+1)), In

$t[(b*\sin[c + d*x])^{(n + 2)}, x], x] /; \text{FreeQ}\{b, c, d\}, x] \ \&\& \ \text{LtQ}[n, -1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_)*\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \ :> \ \text{Dist}[\text{Sqrt}[b*\sin[c + d*x]]/\text{Sqrt}[\sin[c + d*x]], \text{Int}[\text{Sqrt}[\sin[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2639

$\text{Int}[\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \ :> \ \text{Simp}[(2*\text{EllipticE}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_)*\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \ :> \ \text{Dist}[\text{Sqrt}[\sin[c + d*x]]/\text{Sqrt}[b*\sin[c + d*x]], \text{Int}[1/\text{Sqrt}[\sin[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\sin[(c_.) + (d_.)*(x_)]], x_Symbol] \ :> \ \text{Simp}[(2*\text{EllipticF}[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}\{c, d\}, x]$

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx = b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

$$= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2 \int \frac{\frac{3b^2B}{2} + \frac{1}{2}b^2(A+3C) \cos(c+dx)}{(b \cos(c+dx))^{3/2}} dx}{3b}$$

$$= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + (bB) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{1}{3}(A + 3C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{d\sqrt{b \cos(c + dx)}} - \frac{B \int \sqrt{b \cos(c + dx)}}{b}$$

$$= \frac{2(A + 3C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3d\sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

$$= -\frac{2B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd\sqrt{\cos(c + dx)}} + \frac{2(A + 3C)\sqrt{\cos(c + dx)}}{3d\sqrt{b \cos(c + dx)}}$$

Mathematica [C] time = 6.29658, size = 757, normalized size = 5.45

$$2B \csc(c) \cos^{\frac{5}{2}}(c + dx) (A \sec^2(c + dx) + B \sec(c + dx) + C) \left(\frac{\tan(c) \sin(\tan^{-1}(\tan(c)) + dx) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2(dx + t)\right)}{\sqrt{\tan^2(c) + 1} \sqrt{1 - \cos(\tan^{-1}(\tan(c)) + dx)} \sqrt{\cos(\tan^{-1}(\tan(c)) + dx) + 1} \sqrt{\cos(c)}}}{d\sqrt{b \cos(c + dx)}(2A + 2B \cos(c + dx) + C \cos(2c + 2dx))} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/Sqrt[b*Cos[c + d*x]],x]

[Out] (Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*A*Cos[c + d*x]^(5/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*Cos[c + d*x]^(5/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2*B*Cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*(HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))

Maple [B] time = 8.132, size = 508, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2),x)

[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d
*x + c)^2/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1
/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/2)
,x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*cos
(d*x + c)), x)
```

$$3.269 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=180

$$\frac{2Ab^2 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2(3A+5C) \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2bB \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} +$$

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.230786, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2Ab^2 \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2(3A+5C) \sin(c+dx)}{5d\sqrt{b \cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5bd\sqrt{\cos(c+dx)}} + \frac{2bB \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} +$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*Cos[c + d*x]], x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx = b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

$$= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2}{5} \int \frac{\frac{5b^2B}{2} + \frac{1}{2}b^2(3A + 5C) \cos(c + dx)}{(b \cos(c + dx))^{5/2}}$$

$$= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + (b^2B) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + \frac{1}{5}(b^2(3A + 5C)) \int \frac{\cos(c + dx)}{(b \cos(c + dx))^{5/2}}$$

$$= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}$$

$$= \frac{2Ab^2 \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2bB \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5d\sqrt{b \cos(c + dx)}}$$

$$= -\frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)}}{3d\sqrt{b}}$$

Mathematica [A] time = 0.426265, size = 116, normalized size = 0.64

$$\frac{2\left(-3(3A + 5C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 9A \sin(c + dx) + 3A \tan(c + dx) \sec(c + dx) + 5B \tan(c + dx) + 5B\sqrt{\cos(c + dx)}\right)}{15d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/Sqrt[b*C
os[c + d*x]], x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*d*Sqrt[b*Cos[c
+ d*x]])
```

Maple [B] time = 10.779, size = 807, normalized size = 4.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b/sin(1/2*d*
x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/
2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/2*
d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1
/2*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*d*
x+1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2
*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-60*
C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^4*
cos(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/
(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2)
,x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos
(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^3}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*cos(d*x + c)), x)
```


$$3.270 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^4(c+dx)}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=209

$$\frac{2Ab^3 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2b(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2b^2B \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} +$$

[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.258565, antiderivative size = 209, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^3 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2b(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d\sqrt{b \cos(c+dx)}} + \frac{2b^2B \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} +$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]], x]

[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b*d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b^2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*b*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx)}{\sqrt{b \cos(c + dx)}} dx = b^4 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx$$

$$= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{1}{7}(2b) \int \frac{\frac{7b^2B}{2} + \frac{1}{2}b^2(5A + 7C) \cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

$$= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + (b^3B) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + \frac{1}{7}(b^2(5A + 7C) \int \frac{\cos(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

$$= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^2B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}}$$

$$= \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2b^2B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2b(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}}$$

$$= \frac{2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

$$= -\frac{6B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5bd\sqrt{\cos(c + dx)}} + \frac{2(5A + 7C)\sqrt{\cos(c + dx)}}{21d\sqrt{b \cos(c + dx)}} + \frac{2Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

Mathematica [A] time = 0.656483, size = 133, normalized size = 0.64

$$\frac{2\left(5(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 25A \tan(c + dx) + 15A \tan(c + dx) \sec^2(c + dx) + 63B \sin(c + dx) - 63B \cos(c + dx)\right)}{105d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4)/Sqrt[b*Cos[c + d*x]],x]

[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c + d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 11.517, size = 726, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x)

[Out]
$$-(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*A*(-1/56*\cos(1/2*d*x+1/2*c)/b*(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*\cos(1/2*d*x+1/2*c)/b*(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)}))-2/5*B/b/\sin(1/2*d*x+1/2*c)^2/(8*\sin(1/2*d*x+1/2*c)^6-12*\sin(1/2*d*x+1/2*c)^4+6*\sin(1/2*d*x+1/2*c)^2-1)*(12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^4-24*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)-12*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*\sin(1/2*d*x+1/2*c)^2+24*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticE}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})-8*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))*(-2*b*\sin(1/2*d*x+1/2*c)^4+\sin(1/2*d*x+1/2*c)^2*b)^{(1/2)}+2*C*(-1/6*\cos(1/2*d*x+1/2*c)/b*(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/(\cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(-2*\cos(1/2*d*x+1/2*c)^2+1)^{(1/2)}/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c), 2^{(1/2)})))/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Timed out

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^4}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^4/(b*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^4}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^4/sqrt(b*cos(d*x + c)), x)
```

$$3.271 \quad \int \frac{\cos^4(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=217

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^3d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{7b^4d}$$

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rubi [A] time = 0.220706, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^3d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{7b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^2*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^3*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^5*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]
)*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx &= \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^4} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^5 d} + \frac{2 \int (b \cos(c + dx))^{5/2} \left(\frac{1}{2}b(A + B \cos(c + dx) + C \cos^2(c + dx))\right) dx}{b^5} \\ &= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^5 d} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^5} + \frac{2A \int (b \cos(c + dx))^{5/2} dx}{b^5} \\ &= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45b^3 d} + \frac{2B(b \cos(c + dx))^{7/2}}{7b^4} \\ &= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^2 d} + \frac{2(9A + 7C)(b \cos(c + dx))^{3/2}}{45b^3 d} \\ &= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^2 d \sqrt{\cos(c + dx)}} + \frac{10B\sqrt{b \cos(c + dx)}}{21bd} \\ &= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^2 d \sqrt{\cos(c + dx)}} + \frac{10B\sqrt{\cos(c + dx)}}{21bd\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.713885, size = 130, normalized size = 0.6

$$\frac{\sin(2(c + dx))(7(36A + 43C) \cos(c + dx) + 5(18B \cos(2(c + dx)) + 78B + 7C \cos(3(c + dx)))) + 168(9A + 7C)\sqrt{\cos(c + dx)}}{1260bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]/(1260*b*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 3.568, size = 384, normalized size = 1.8

$$-\frac{2}{315bd} \sqrt{b \left(2 \left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} \left(-1120C \cos\left(\frac{1}{2}dx + \frac{c}{2}\right) \left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^{10} + (720B + \dots)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(-1120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(3/2), x)
```


$$3.272 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=188

$$\frac{2(7A+5C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{21b^2d} + \frac{2(7A+5C) \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21bd \sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx) (b \cos(c+dx))^{3/2}}{5b^3d}$$

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)

Rubi [A] time = 0.201367, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2(7A+5C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{21b^2d} + \frac{2(7A+5C) \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21bd \sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx) (b \cos(c+dx))^{3/2}}{5b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^2*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^4*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^3} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^4d} + \frac{2 \int (b \cos(c + dx))^{3/2} \left(\frac{1}{2}b(7A + 5C) + B \cos(c + dx)\right) dx}{b^4} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^4d} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^4} + \frac{2(7A + 5C) \int (b \cos(c + dx))^{3/2} dx}{5b^3d} \\ &= \frac{2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^2d} + \frac{2B(b \cos(c + dx))^{3/2}}{5b^3d} \\ &= \frac{2(7A + 5C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^2d} + \frac{2B(b \cos(c + dx))^{3/2}}{5b^3d} \\ &= \frac{6B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2d\sqrt{\cos(c + dx)}} + \frac{2(7A + 5C)\sqrt{\cos(c + dx)}}{21bd\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.623786, size = 108, normalized size = 0.57

$$\frac{\cos^{\frac{3}{2}}(c + dx) \left(\sin(c + dx) \sqrt{\cos(c + dx)} (70A + 42B \cos(c + dx) + 15C \cos(2(c + dx))) + 65C \right) + 10(7A + 5C) F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{105d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2),x]

[Out] (Cos[c + d*x]^(3/2)*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]

$$+ 15*C*\text{Cos}[2*(c + d*x)]*\text{Sin}[c + d*x])/((105*d*(b*\text{Cos}[c + d*x])^(3/2))$$

Maple [A] time = 3.393, size = 353, normalized size = 1.9

$$-\frac{2}{105bd} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(240 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^8 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-168 B - 360 C) \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)

[Out]
$$-\frac{2}{105*b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2}^{(1/2)}/b*(240*C*\sin(1/2*d*x+1/2*c)^8*\cos(1/2*d*x+1/2*c)+(-168*B-360*C)*\sin(1/2*d*x+1/2*c)^6*\cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c)+35*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-63*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+25*C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}}{b^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)

$$3.273 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=153

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^2d} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^3d} + \frac{2B \sin(c+dx)}{5b^3d}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)

Rubi [A] time = 0.161292, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^2d} + \frac{2C \sin(c+dx)(b \cos(c+dx))^{3/2}}{5b^3d} + \frac{2B \sin(c+dx)}{5b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[SIN[c + d*
x]]/Sqrt[b*SIN[c + d*x]], Int[1/Sqrt[SIN[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{\int \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^2}$$

$$= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} + \frac{2 \int \sqrt{b \cos(c + dx)} \left(\frac{1}{2}b(5A + 3C)\right) dx}{b^3}$$

$$= \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} + \frac{B \int (b \cos(c + dx))^{3/2} dx}{b^3} + \frac{2B \int \sqrt{b \cos(c + dx)} dx}{b^3}$$

$$= \frac{2B \sqrt{b \cos(c + dx)} \sin(c + dx)}{3b^2d} + \frac{2C(b \cos(c + dx))^{3/2} \sin(c + dx)}{5b^3d} + \frac{2(5A + 3C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2d \sqrt{\cos(c + dx)}} + \frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd \sqrt{\cos(c + dx)}}$$

$$= \frac{2(5A + 3C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2d \sqrt{\cos(c + dx)}} + \frac{2B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.340657, size = 94, normalized size = 0.61

$$\frac{2 \cos^{\frac{3}{2}}(c + dx) \left(3(5A + 3C) E\left(\frac{1}{2}(c + dx) \middle| 2\right) + \sin(c + dx) \sqrt{\cos(c + dx)} (5B + 3C \cos(c + dx)) + 5BF\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{15d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(3/2), x]
```

```
[Out] (2*Cos[c + d*x]^(3/2)*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellipt
icF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d
*x]))/(15*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 3.703, size = 319, normalized size = 2.1

$$\frac{2}{15bd} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(24C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-20B - 24C) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)

[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(24*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)

$$3.274 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=120

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}$$

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)

Rubi [A] time = 0.141233, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.18$, Rules used = {16, 3023, 2748, 2642, 2641, 2640, 2639}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{b \cos(c + dx)}} dx}{b}$$

$$= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3b^2 d} + \frac{2 \int \frac{\frac{1}{2}b(3A + C) + \frac{3}{2}bB \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx}{3b^2}$$

$$= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3b^2 d} + \frac{B \int \sqrt{b \cos(c + dx)} dx}{b^2} + \frac{(3A + C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b}$$

$$= \frac{2C \sqrt{b \cos(c + dx)} \sin(c + dx)}{3b^2 d} + \frac{((3A + C) \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{3b \sqrt{b \cos(c + dx)}}$$

$$= \frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c + dx)}} + \frac{2(3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.165143, size = 85, normalized size = 0.71

$$\frac{2(3A + C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 6B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) + C \sin(2(c + dx))}{3bd \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + C*Ssin[2*(c + d*x)])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.579, size = 285, normalized size = 2.4

$$-\frac{2}{3bd} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1\right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2} \left(4C (\sin(1/2 dx + c/2))^4 \cos(1/2 dx + c/2) + 3A \sqrt{(\sin(1/2 dx + c/2))^2 - 1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x)`

[Out]
$$-2/3*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/b*(4*C*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \cos(dx+c)}{(b \cos(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x + c))^(3/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A)\sqrt{b \cos(dx+c)}}{b^2 \cos(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
c))^(3/2), x)
```

$$3.275 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=116

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{bd \sqrt{b \cos(c+dx)}} + \frac{2B \sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd \sqrt{b \cos(c+dx)}}$$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.141465, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3021, 2748, 2642, 2641, 2640, 2639}

$$-\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{b^2 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{bd \sqrt{b \cos(c+dx)}} + \frac{2B \sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)/(b*\text{Cos}[c + d*x])^{3/2}, x]$

[Out] $(-2*(A - C)*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^2*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*B*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 3021

$\text{Int}[(a_. + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m + 1)}]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m + 1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}[\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2748

$\text{Int}[(b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^{(m_.)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] :> \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2642

$\text{Int}[1/\text{Sqrt}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] :> \text{Dist}[\text{Sqrt}[\text{Sin}[c + d*x]]/\text{Sqrt}[b*\text{Sin}[c + d*x]], \text{Int}[1/\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}[\{b, c, d\}, x]$

Rule 2641

$\text{Int}[1/\text{Sqrt}[\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] :> \text{Simp}[(2*\text{EllipticF}[(1*(c - \text{Pi}/2 + d*x))/2, 2])/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = \frac{2A \sin(c + dx)}{bd\sqrt{b \cos(c + dx)}} + \frac{2 \int \frac{\frac{b^2 B}{2} - \frac{1}{2} b^2 (A - C) \cos(c + dx)}{\sqrt{b \cos(c + dx)}} dx}{b^3}$$

$$= \frac{2A \sin(c + dx)}{bd\sqrt{b \cos(c + dx)}} + \frac{B \int \frac{1}{\sqrt{b \cos(c + dx)}} dx}{b} - \frac{(A - C) \int \sqrt{b \cos(c + dx)} dx}{b^2}$$

$$= \frac{2A \sin(c + dx)}{bd\sqrt{b \cos(c + dx)}} + \frac{(B\sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c + dx)}} dx}{b\sqrt{b \cos(c + dx)}} - \frac{((A - C)\sqrt{b \cos(c + dx)}) \int \sqrt{\cos(c + dx)} dx}{b^2\sqrt{b \cos(c + dx)}}$$

$$= -\frac{2(A - C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{bd\sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.177476, size = 80, normalized size = 0.69

$$\frac{2\left(- (A - C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + A \sin(c + dx) + B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)\right)}{bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (2*(-((A - C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]) + B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[c + d*x]))/(b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.946, size = 261, normalized size = 2.3

$$-2 \frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2 b} \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \text{EllipticE}(\cos(1/2 dx + c/2), 2^{1/2}) - 2A \cos(1/2 dx + c/2) \sin(1/2 dx + c/2) + B(\sin(1/2 dx + c/2))^2 \right)}{b^2 \sqrt{b \cos(c + dx)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)
```

```
[Out] -2/b*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
```

EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(3/2), x)
```


$$3.276 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=144

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}}$$

[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.192406, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd\sqrt{b \cos(c+dx)}} + \frac{2A \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{b^2d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)}{bd\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(3/2), x]

[Out] (-2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^2*d*Sqrt[Cos[c + d*x]]) + (2*(A + 3*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*B*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In

t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]

Rule 2640

Int[Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{3/2}} dx = b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx$$

$$= \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2 \int \frac{\frac{3b^2B}{2} + \frac{1}{2}b^2(A+3C) \cos(c+dx)}{(b \cos(c+dx))^{3/2}} dx}{3b^2}$$

$$= \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + B \int \frac{1}{(b \cos(c + dx))^{3/2}} dx + \frac{(A + 3C) \int}{b^2}$$

$$= \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{bd \sqrt{b \cos(c + dx)}} - \frac{B \int \sqrt{b \cos(c + dx)}}{b^2}$$

$$= \frac{2(A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3bd \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}}$$

$$= -\frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d \sqrt{\cos(c + dx)}} + \frac{2(A + 3C) \sqrt{\cos(c + dx)}}{3bd \sqrt{b \cos(c + dx)}}$$

Mathematica [C] time = 6.26369, size = 761, normalized size = 5.28

$$\frac{2B \csc(c) \cos^{\frac{5}{2}}(c+dx) (A \sec^2(c+dx) + B \sec(c+dx) + C)}{d \sqrt{b \cos(c+dx)} (2A + 2B \cos(c+dx) + C \cos(2c+2dx) + C)} \left(\frac{\tan(c) \sin(\tan^{-1}(\tan(c))+dx) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{4}; \frac{3}{4}; \cos^2(dx + \tan^{-1}(\tan(c)))\right)}{\sqrt{\tan^2(c)+1} \sqrt{1-\cos(\tan^{-1}(\tan(c))+dx)} \sqrt{\cos(\tan^{-1}(\tan(c))+dx)+1} \sqrt{\cos(c) \sqrt{\tan^2(c)+1} \cos(\tan^{-1}(\tan(c))+dx)}} \right)$$

Warning: Unable to verify antiderivative.

[In] Integrate[((A + B*cos[c + d*x] + C*cos[c + d*x]^2)*sec[c + d*x])/(b*cos[c + d*x])^(3/2), x]

[Out] ((Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((4*B*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(A*Sin[c] + 3*B*Sin[d*x]))/(3*d)))/(Sqrt[b*cos[c + d*x]]*(2*A + C + 2*B*cos[c + d*x] + C*cos[2*c + 2*d*x])) - (4*A*cos[c + d*x]^(5/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[b*cos[c + d*x]]*(2*A + C + 2*B*cos[c + d*x] + C*cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) - (4*C*cos[c + d*x]^(5/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*cos[c + d*x]]*(2*A + C + 2*B*cos[c + d*x] + C*cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]^2]) + (2*B*cos[c + d*x]^(5/2)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((HypergeometricPFQ[-1/2, -1/4], {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*cos[c]^2*cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]))/(d*Sqrt[b*cos[c + d*x]]*(2*A + C + 2*B*cos[c + d*x] + C*cos[2*c + 2*d*x]))/b

Maple [B] time = 7.868, size = 508, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2), x)

[Out] 2/3*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*d*x+1/2*c)^3/(4*sin(1/2*d*x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+6*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-12*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+6*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-3*B*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+6*B*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d
*x + c)/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(3/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(3/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(3/2), x)
```

$$3.277 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=183

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2B \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.241463, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}} + \frac{2(3A+5C)\sin(c+dx)}{5bd\sqrt{b \cos(c+dx)}} + \frac{2Ab \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2B \sin(c+dx)}{3d(b \cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(3/2), x]

[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{3/2}} dx = b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx$$

$$= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2 \int \frac{\frac{5b^2B}{2} + \frac{1}{2}b^2(3A+5C) \cos(c+dx)}{(b \cos(c+dx))^{5/2}} dx}{5b}$$

$$= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + (bB) \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + \frac{1}{5}(3A + 5C) \int \frac{1}{(b \cos(c + dx))^{3/2}} dx$$

$$= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}}$$

$$= \frac{2Ab \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3d(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5bd\sqrt{b \cos(c + dx)}}$$

$$= -\frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2d\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)}}{3bd\sqrt{b}}$$

Mathematica [A] time = 0.432935, size = 119, normalized size = 0.65

$$\frac{2\left(-3(3A + 5C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 9A \sin(c + dx) + 3A \tan(c + dx) \sec(c + dx) + 5B \tan(c + dx) + 5B\sqrt{\cos(c + dx)}\right)}{15bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
+ d*x])^(3/2), x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [B] time = 10.656, size = 807, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6
0*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2)
,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^2}{b^2 \cos(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(3/2), x)
```


$$3.278 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx)}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=212

$$\frac{2Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b \cos(c+dx)}} - \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.266425, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{2(5A+7C) \sin(c+dx)}{21d(b \cos(c+dx))^{3/2}} + \frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21bd\sqrt{b \cos(c+dx)}} - \frac{6BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2), x]

[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*b*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{(b \cos(c + dx))^{3/2}} dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx \\
&= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2}{7} \int \frac{\frac{7b^2B}{2} + \frac{1}{2}b^2(5A + 7C) \cos(c + dx)}{(b \cos(c + dx))^{7/2}} \\
&= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + (b^2B) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + \frac{1}{7}(b(5 \\
&= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2bB \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} \\
&= \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2bB \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21d(b \cos(c + dx))^{3/2}} \\
&= \frac{2(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21bd\sqrt{b \cos(c + dx)}} + \frac{2Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{3/2}} \\
&= -\frac{6B\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^2d\sqrt{\cos(c + dx)}} + \frac{2(5A + 7C)\sqrt{\cos(c + dx)}}{21bd\sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.692371, size = 136, normalized size = 0.64

$$\frac{2\left(5(5A + 7C)\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 25A \tan(c + dx) + 15A \tan(c + dx) \sec^2(c + dx) + 63B \sin(c + dx) - 63B \cos(c + dx)\right)}{105bd\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c + d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*b*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 11.339, size = 729, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2), x)
```

```
[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b*(2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))-2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+2*C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)} \sec(dx + c)^3}{b^2 \cos(dx + c)^2}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^3/(b^2*cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(3/2), x)
```

$$3.279 \quad \int \frac{\cos^5(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=217

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^4d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{7b^5d}$$

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rubi [A] time = 0.225294, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2640, 2639, 2642, 2641}

$$\frac{2(9A+7C) \sin(c+dx)(b \cos(c+dx))^{3/2}}{45b^4d} + \frac{2(9A+7C)E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{15b^3d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^{3/2}}{7b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (2*(9*A + 7*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(15*b^3*d*Sqrt[Cos[c + d*x]]) + (10*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (10*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*(9*A + 7*C)*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(45*b^4*d) + (2*B*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d) + (2*C*(b*Cos[c + d*x])^(7/2)*Sin[c + d*x])/(9*b^6*d)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^5(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{\int (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^5}$$

$$= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^6 d} + \frac{2 \int (b \cos(c + dx))^{5/2} \left(\frac{1}{2}b(A + B \cos(c + dx) + C \cos^2(c + dx))\right) dx}{b^6}$$

$$= \frac{2C(b \cos(c + dx))^{7/2} \sin(c + dx)}{9b^6 d} + \frac{B \int (b \cos(c + dx))^{7/2} dx}{b^6} + \frac{2 \int (b \cos(c + dx))^{5/2} C \cos^2(c + dx) dx}{b^6}$$

$$= \frac{2(9A + 7C)(b \cos(c + dx))^{3/2} \sin(c + dx)}{45b^4 d} + \frac{2B(b \cos(c + dx))^{5/2}}{7b^5}$$

$$= \frac{10B\sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^3 d} + \frac{2(9A + 7C)(b \cos(c + dx))^{3/2}}{45b^4 d}$$

$$= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^3 d \sqrt{\cos(c + dx)}} + \frac{10B\sqrt{b \cos(c + dx)}}{21b^3 d}$$

$$= \frac{2(9A + 7C)\sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{15b^3 d \sqrt{\cos(c + dx)}} + \frac{10B\sqrt{\cos(c + dx)}}{21b^2 d \sqrt{b}}$$

Mathematica [A] time = 0.687966, size = 130, normalized size = 0.6

$$\frac{\sin(2(c + dx))(7(36A + 43C) \cos(c + dx) + 5(18B \cos(2(c + dx)) + 78B + 7C \cos(3(c + dx)))) + 168(9A + 7C)\sqrt{\cos(c + dx)}}{1260b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^5*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (168*(9*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 600*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(36*A + 43*C)*Cos[c + d*x] + 5*(78*B + 18*B*Cos[2*(c + d*x)] + 7*C*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]/(1260*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 3.701, size = 384, normalized size = 1.8

$$-\frac{2}{315 b^2 d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right) \right)^2} \left(-1120 C \cos(1/2 dx + c/2) (\sin(1/2 dx + c/2))^{10} + (720 B + \dots \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] -2/315*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(-1120*C*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(720*B+2240*C)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-504*A-1080*B-2072*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(504*A+840*B+952*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-126*A-240*B-168*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))+75*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-147*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^5}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2) \sqrt{b \cos(dx + c)}}{b^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*sqrt(b*cos(d*x + c))/b^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^5}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^5/(b*cos(d*x + c))^(5/2), x)
```


$$3.280 \quad \int \frac{\cos^4(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=188

$$\frac{2(7A+5C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{21b^3d} + \frac{2(7A+5C) \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21b^2d \sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx) (b \cos(c+dx))^{3/2}}{5b^4d}$$

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)

Rubi [A] time = 0.195567, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2635, 2642, 2641, 2640, 2639}

$$\frac{2(7A+5C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{21b^3d} + \frac{2(7A+5C) \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{21b^2d \sqrt{b \cos(c+dx)}} + \frac{2B \sin(c+dx) (b \cos(c+dx))^{3/2}}{5b^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*(7*A + 5*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(21*b^3*d) + (2*B*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d) + (2*C*(b*Cos[c + d*x])^(5/2)*Sin[c + d*x])/(7*b^5*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n-1))/(d*n), x] + Dist[(b^2*(n-1))/n, Int[(b*Sin[c

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Ssin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

Int[1/Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rule 2640

Int[Sqrt[(b_)*sin[(c_.) + (d_)*(x_)]], x_Symbol] := Dist[Sqrt[b*Ssin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

Int[Sqrt[sin[(c_.) + (d_)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{\cos^4(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx &= \frac{\int (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^4} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^5 d} + \frac{2 \int (b \cos(c + dx))^{3/2} \left(\frac{1}{2}b(7A + 5C) + B \cos(c + dx)\right) dx}{b^5} \\ &= \frac{2C(b \cos(c + dx))^{5/2} \sin(c + dx)}{7b^5 d} + \frac{B \int (b \cos(c + dx))^{5/2} dx}{b^5} + \frac{2(7A + 5C) \int (b \cos(c + dx))^{3/2} dx}{5b^4 d} \\ &= \frac{2(7A + 5C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^3 d} + \frac{2B(b \cos(c + dx))^{3/2}}{5b^4 d} \\ &= \frac{2(7A + 5C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{21b^3 d} + \frac{2B(b \cos(c + dx))^{3/2}}{5b^4 d} \\ &= \frac{6B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3 d \sqrt{\cos(c + dx)}} + \frac{2(7A + 5C) \sqrt{\cos(c + dx)}}{21b^2 d \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.591262, size = 111, normalized size = 0.59

$$\frac{\sqrt{\cos(c + dx)} \left(\sin(c + dx) \sqrt{\cos(c + dx)} (70A + 42B \cos(c + dx) + 15C \cos(2(c + dx)) + 65C) + 10(7A + 5C) F\left(\frac{1}{2}(c + dx)\right) \right)}{105b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^4*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]

[Out] (Sqrt[Cos[c + d*x]]*(126*B*EllipticE[(c + d*x)/2, 2] + 10*(7*A + 5*C)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(70*A + 65*C + 42*B*Cos[c + d*x]

$$+ 15*C*\text{Cos}[2*(c + d*x)]*\text{Sin}[c + d*x])/((105*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]))$$

Maple [A] time = 3.27, size = 353, normalized size = 1.9

$$-\frac{2}{105 b^2 d} \sqrt{b \left(2 \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(240 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^8 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-168 B - 360 C) \sin \left(\frac{1}{2} dx + \frac{c}{2} \right)^6 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (140 A + 168 B + 280 C) \sin \left(\frac{1}{2} dx + \frac{c}{2} \right)^4 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + (-70 A - 42 B - 80 C) \sin \left(\frac{1}{2} dx + \frac{c}{2} \right)^2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) + 35 A \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \text{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) - 63 B \text{EllipticE} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} + 25 C \left(\sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 \right)^{1/2} \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^{1/2} \text{EllipticF} \left(\cos \left(\frac{1}{2} dx + \frac{c}{2} \right), 2^{1/2} \right) \right) / \left(-b \left(2 \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^4 - \sin \left(\frac{1}{2} dx + \frac{c}{2} \right)^2 \right)^{1/2} / \sin \left(\frac{1}{2} dx + \frac{c}{2} \right) / \left(b \left(2 \cos \left(\frac{1}{2} dx + \frac{c}{2} \right) \right)^2 - 1 \right)^{1/2} / d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)

[Out] -2/105*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(240*C*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-168*B-360*C)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(140*A+168*B+280*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-70*A-42*B-80*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+35*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+25*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) \sqrt{b \cos(dx + c)}}{b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt(b*cos(d*x + c))/b^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**4*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^4}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^4/(b*cos(d*x + c))^(5/2), x)

$$3.281 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=153

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^3d} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2C}{5b^4d}$$

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)

Rubi [A] time = 0.158059, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3023, 2748, 2640, 2639, 2635, 2642, 2641}

$$\frac{2(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{3b^3d} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{2C}{5b^4d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(5*A + 3*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d) + (2*C*(b*Cos[c + d*x])^(3/2)*Sin[c + d*x])/(5*b^4*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2640

Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x
]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[SIN[c + d*
x]]/Sqrt[b*SIN[c + d*x]], Int[1/Sqrt[SIN[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx &= \frac{\int \sqrt{b\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx)) dx}{b^3} \\ &= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{2\int \sqrt{b\cos(c+dx)}\left(\frac{1}{2}b(5A+3C)\right) dx}{b^4} \\ &= \frac{2C(b\cos(c+dx))^{3/2}\sin(c+dx)}{5b^4d} + \frac{B\int (b\cos(c+dx))^{3/2} dx}{b^4} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} \\ &= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} \\ &= \frac{2(5A+3C)\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d\sqrt{\cos(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}} \end{aligned}$$

Mathematica [A] time = 0.313397, size = 97, normalized size = 0.63

$$\frac{2\sqrt{\cos(c+dx)}\left(3(5A+3C)E\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(c+dx)\sqrt{\cos(c+dx)}(5B+3C\cos(c+dx))+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{15b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2), x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(3*(5*A + 3*C)*EllipticE[(c + d*x)/2, 2] + 5*B*Ellipt
icF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*B + 3*C*Cos[c + d*x])*Sin[c + d
*x]))/(15*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 3.638, size = 319, normalized size = 2.1

$$\frac{2}{15b^2d} \sqrt{b \left(2 (\cos(1/2 dx + c/2))^2 - 1 \right) \left(\sin \left(\frac{dx}{2} + \frac{c}{2} \right) \right)^2} \left(24C (\sin(1/2 dx + c/2))^6 \cos(1/2 dx + c/2) + (-20B - 24C) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(24*C*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(-20*B-24*C)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(10*B+6*C)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))+9*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/b^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(5/2), x)
```


$$3.282 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=120

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}$$

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)

Rubi [A] time = 0.142894, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {16, 3023, 2748, 2642, 2641, 2640, 2639}

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}} + \frac{2C\sin(c+dx)\sqrt{b\cos(c+dx)}}{3b^3d}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*(3*A + C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*b^3*d)

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{\int \frac{A+B\cos(c+dx)+C\cos^2(c+dx)}{\sqrt{b\cos(c+dx)}} dx}{b^2}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{2\int \frac{\frac{1}{2}b(3A+C)+\frac{3}{2}bB\cos(c+dx)}{\sqrt{b\cos(c+dx)}} dx}{3b^3}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{B\int \sqrt{b\cos(c+dx)} dx}{b^3} + \frac{(3A+C)\int \sqrt{b\cos(c+dx)} dx}{3b^3}$$

$$= \frac{2C\sqrt{b\cos(c+dx)}\sin(c+dx)}{3b^3d} + \frac{((3A+C)\sqrt{\cos(c+dx)})\int \sqrt{b\cos(c+dx)} dx}{3b^2\sqrt{b\cos(c+dx)}}$$

$$= \frac{2B\sqrt{b\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d\sqrt{b\cos(c+dx)}} + \frac{2(3A+C)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.1648, size = 85, normalized size = 0.71

$$\frac{2(3A+C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right) + 6B\sqrt{\cos(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right) + C\sin(2(c+dx))}{3b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(5/2), x]
```

```
[Out] (6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 2*(3*A + C)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] + C*Sin[2*(c + d*x)])/(3*b^2*d*Sqrt[b*Cos
[c + d*x]])
```

Maple [A] time = 3.432, size = 285, normalized size = 2.4

$$-\frac{2}{3b^2d}\sqrt{b\left(2\left(\cos\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2 - 1\right)\left(\sin\left(\frac{dx}{2} + \frac{c}{2}\right)\right)^2}\left(4C\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^4\cos\left(\frac{1}{2}dx + \frac{c}{2}\right) + 3A\sqrt{\left(\sin\left(\frac{1}{2}dx + \frac{c}{2}\right)\right)^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)`

[Out]
$$-2/3*(b*(2*\cos(1/2*d*x+1/2*c)^2-1)*\sin(1/2*d*x+1/2*c)^2)^{(1/2)}/b^2*(4*C*\sin(1/2*d*x+1/2*c)^4*\cos(1/2*d*x+1/2*c)+3*A*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-3*B*\text{EllipticE}(\cos(1/2*d*x+1/2*c),2^{(1/2)})*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}+C*(\sin(1/2*d*x+1/2*c)^2)^{(1/2)}*(2*\sin(1/2*d*x+1/2*c)^2-1)^{(1/2)}*\text{EllipticF}(\cos(1/2*d*x+1/2*c),2^{(1/2)})-2*C*\sin(1/2*d*x+1/2*c)^2*\cos(1/2*d*x+1/2*c))/(-b*(2*\sin(1/2*d*x+1/2*c)^4-\sin(1/2*d*x+1/2*c)^2))^{(1/2)}/\sin(1/2*d*x+1/2*c)/(b*(2*\cos(1/2*d*x+1/2*c)^2-1))^{(1/2)}/d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] `integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)), x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

$$3.283 \quad \int \frac{\cos(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=116

$$\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{2B \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.153824, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.18$, Rules used = {16, 3021, 2748, 2642, 2641, 2640, 2639}

$$\frac{2(A-C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b \cos(c+dx)}}{b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{2B \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (-2*(A - C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :=> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2642

Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :=> Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{\int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{3/2}} dx}{b}$$

$$= \frac{2A \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} + \frac{2 \int \frac{\frac{b^2 B}{2} - \frac{1}{2} b^2 (A-C) \cos(c+dx)}{\sqrt{b \cos(c+dx)}} dx}{b^4}$$

$$= \frac{2A \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} + \frac{B \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{b^2} - \frac{(A-C) \int \sqrt{b \cos(c+dx)} dx}{b^3}$$

$$= \frac{2A \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \frac{1}{\sqrt{\cos(c+dx)}} dx}{b^2 \sqrt{b \cos(c + dx)}} - \frac{((A-C) \int \sqrt{\cos(c + dx)} dx)}{b^3 \sqrt{b \cos(c + dx)}}$$

$$= -\frac{2(A-C) \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3 d \sqrt{\cos(c + dx)}} + \frac{2B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^2 d \sqrt{b \cos(c + dx)}}$$

Mathematica [C] time = 6.20734, size = 807, normalized size = 6.96

$$\frac{(B+C \cos(c+dx)+A \sec(c+dx)) \left(\frac{4A \sec(c) \sec(c+dx) \sin(dx)}{d} - \frac{2(-2A+C+C \cos(2c)) \csc(c) \sec(c)}{d} \right) \cos^2(c+dx)}{\sqrt{b \cos(c+dx)} (2A+C+2B \cos(c+dx)+C \cos(2c+2dx))} + \frac{2A \csc(c) (B+C \cos(c+dx)+A \sec(c+dx))}{\sqrt{1-\cos(dx+tc)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]
```

```
[Out] ((Cos[c + d*x]^2*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*((-2*(-2*A + C + C*Cos[2*c]))*Csc[c]*Sec[c])/d + (4*A*Sec[c]*Sec[c + d*x]*Sin[d*x])/d))/(Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (4*B*Cos[c + d*x]^(3/2)*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[b*Cos[c + d*x]]*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*Sqrt[1 + Cot[c]
```

$$\begin{aligned} &]^2) + (2A \cos[c + dx]^{3/2} \operatorname{Csc}[c] (B + C \cos[c + dx] + A \operatorname{Sec}[c + dx]) \\ &) * (\operatorname{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2] * \sin \\ & [dx + \operatorname{ArcTan}[\tan[c]]] * \tan[c]) / (\operatorname{Sqrt}[1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]] * \operatorname{Sqrt}[1 \\ & + \cos[dx + \operatorname{ArcTan}[\tan[c]]]] * \operatorname{Sqrt}[\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]] * \operatorname{Sqrt}[1 + \\ & \tan[c]^2]) * \operatorname{Sqrt}[1 + \tan[c]^2]) - ((\sin[dx + \operatorname{ArcTan}[\tan[c]]] * \tan[c]) / \operatorname{Sqrt}[\\ & 1 + \tan[c]^2] + (2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] * \operatorname{Sqrt}[1 + \tan[c]^2]) / (\\ & \cos[c]^2 + \sin[c]^2)) / \operatorname{Sqrt}[\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] * \operatorname{Sqrt}[1 + \tan[c] \\ & ^2]]) / (d \operatorname{Sqrt}[b \cos[c + dx]] * (2A + C + 2B \cos[c + dx] + C \cos[2c + 2 \\ & dx])) - (2C \cos[c + dx]^{3/2} \operatorname{Csc}[c] (B + C \cos[c + dx] + A \operatorname{Sec}[c + dx]) \\ &) * (\operatorname{HypergeometricPFQ}[\{-1/2, -1/4\}, \{3/4\}, \cos[dx + \operatorname{ArcTan}[\tan[c]]]^2] * \sin \\ & [dx + \operatorname{ArcTan}[\tan[c]]] * \tan[c]) / (\operatorname{Sqrt}[1 - \cos[dx + \operatorname{ArcTan}[\tan[c]]]] * \operatorname{Sqrt}[1 \\ & + \cos[dx + \operatorname{ArcTan}[\tan[c]]]] * \operatorname{Sqrt}[\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]]] * \operatorname{Sqrt}[1 \\ & + \tan[c]^2]) * \operatorname{Sqrt}[1 + \tan[c]^2]) - ((\sin[dx + \operatorname{ArcTan}[\tan[c]]] * \tan[c]) / \operatorname{Sqrt}[\\ & 1 + \tan[c]^2] + (2 \cos[c]^2 \cos[dx + \operatorname{ArcTan}[\tan[c]]] * \operatorname{Sqrt}[1 + \tan[c]^2]) / (\\ & \cos[c]^2 + \sin[c]^2)) / \operatorname{Sqrt}[\cos[c] \cos[dx + \operatorname{ArcTan}[\tan[c]]] * \operatorname{Sqrt}[1 + \tan[c] \\ & ^2]]) / (d \operatorname{Sqrt}[b \cos[c + dx]] * (2A + C + 2B \cos[c + dx] + C \cos[2c + 2 \\ & dx])) / b^2 \end{aligned}$$

Maple [A] time = 3.516, size = 261, normalized size = 2.3

$$-2 \frac{\sqrt{-2b(\sin(1/2 dx + c/2))^4 + (\sin(1/2 dx + c/2))^2 b} \left(A \sqrt{(\sin(1/2 dx + c/2))^2} \sqrt{2(\sin(1/2 dx + c/2))^2 - 1} \operatorname{EllipticE} \right)}{-2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(dx+c)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(5/2),x)`

[Out] `-2/b^2*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(dx+c)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(5/2),x, algorithm="maxima")`

[Out] `integrate((C*cos(dx + c)^2 + B*cos(dx + c) + A)*cos(dx + c)/(b*cos(dx + c))^(5/2), x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*
cos(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
c))^(5/2), x)
```


$$3.284 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=147

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

[Out] $(-2*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*(A + 3*C)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(3*b*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*B*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.162703, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2(A+3C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{2A \sin(c+dx)}{3bd(b \cos(c+dx))^{3/2}} + \frac{2B \sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} - \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{b^3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)/(b*\text{Cos}[c + d*x])^(5/2), x]$

[Out] $(-2*B*\text{Sqrt}[b*\text{Cos}[c + d*x]]*\text{EllipticE}[(c + d*x)/2, 2])/(b^3*d*\text{Sqrt}[\text{Cos}[c + d*x]]) + (2*(A + 3*C)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{EllipticF}[(c + d*x)/2, 2])/(3*b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (2*A*\text{Sin}[c + d*x])/(3*b*d*(b*\text{Cos}[c + d*x])^(3/2)) + (2*B*\text{Sin}[c + d*x])/(b^2*d*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 3021

$\text{Int}[(a_. + (b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_.)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_.)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^(m + 1)]/(b*f*(m + 1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m + 1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^(m + 1)*\text{Simp}[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*\text{Sin}[e + f*x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C\}, x] \&\& \text{LtQ}[m, -1] \&\& \text{NeQ}[a^2 - b^2, 0]$

Rule 2748

$\text{Int}[(b_.)*\text{sin}[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^(m + 1), x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2636

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + \text{Dist}[(n + 2)/(b^2*(n + 1)), \text{Int}[(b*\text{Sin}[c + d*x])^(n + 2), x], x] /; \text{FreeQ}\{b, c, d\}, x] \&\& \text{LtQ}[n, -1] \&\& \text{IntegerQ}[2*n]$

Rule 2640

$\text{Int}[\text{Sqrt}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_.)]], x_Symbol] \rightarrow \text{Dist}[\text{Sqrt}[b*\text{Sin}[c + d*x]]/\text{Sqrt}[\text{Sin}[c + d*x]], \text{Int}[\text{Sqrt}[\text{Sin}[c + d*x]], x], x] /; \text{FreeQ}\{b, c, d\},$

x]

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx &= \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2 \int \frac{\frac{3b^2B}{2} + \frac{1}{2}b^2(A+3C) \cos(c+dx)}{(b \cos(c+dx))^{3/2}} dx}{3b^3} \\
&= \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{B \int \frac{1}{(b \cos(c+dx))^{3/2}} dx}{b} + \frac{(A + 3C) \int \frac{1}{\sqrt{b \cos(c+dx)}} dx}{3b^2} \\
&= \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} - \frac{B \int \sqrt{b \cos(c + dx)} dx}{b^3} + \frac{((A + 3C) \int \frac{1}{\sqrt{b \cos(c + dx)}} dx)}{3b^2} \\
&= \frac{2(A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}} + \frac{2A \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2B \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} \\
&= -\frac{2B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{b^3 d \sqrt{\cos(c + dx)}} + \frac{2(A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^2 d \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.377389, size = 92, normalized size = 0.63

$$\frac{2 \left(\tan(c + dx)(A + 3B \cos(c + dx)) + (A + 3C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) - 3B \sqrt{\cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right) \right)}{3b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (2*(-3*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (A + 3*C)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] + (A + 3*B*Cos[c + d*x])*Tan[c + d*x]))
/(3*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 8.615, size = 508, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x)

[Out] $\frac{2}{3} * (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1) * \sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} / b ^ 3 / \sin(1/2 * d * x + 1/2 * c) ^ 3 / (4 * \sin(1/2 * d * x + 1/2 * c) ^ 4 - 4 * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 1) * (2 * A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 + 6 * B * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 12 * B * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 4 + 6 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \sin(1/2 * d * x + 1/2 * c) ^ 2 - A * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) + 2 * A * \cos(1/2 * d * x + 1/2 * c) * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 3 * B * \text{EllipticE}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} + 6 * B * \sin(1/2 * d * x + 1/2 * c) ^ 2 * \cos(1/2 * d * x + 1/2 * c) - 3 * C * (\sin(1/2 * d * x + 1/2 * c) ^ 2) ^ {1/2} * (2 * \sin(1/2 * d * x + 1/2 * c) ^ 2 - 1) ^ {1/2} * \text{EllipticF}(\cos(1/2 * d * x + 1/2 * c), 2 ^ {1/2})) * (-2 * b * \sin(1/2 * d * x + 1/2 * c) ^ 4 + \sin(1/2 * d * x + 1/2 * c) ^ 2 * b) ^ {1/2} / (b * (2 * \cos(1/2 * d * x + 1/2 * c) ^ 2 - 1)) ^ {1/2} / d$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^3 \cos(dx + c)^3}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^3*cos(d*x + c)^3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(5/2), x)

$$3.285 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=185

$$\frac{2(3A+5C) \sin(c+dx)}{5b^2 d \sqrt{b \cos(c+dx)}} - \frac{2(3A+5C) E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2B \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3b^2 d \sqrt{b \cos(c+dx)}}$$

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.219004, antiderivative size = 185, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.205$, Rules used = {16, 3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2(3A+5C) \sin(c+dx)}{5b^2 d \sqrt{b \cos(c+dx)}} - \frac{2(3A+5C) E\left(\frac{1}{2}(c+dx) \middle| 2\right) \sqrt{b \cos(c+dx)}}{5b^3 d \sqrt{\cos(c+dx)}} + \frac{2A \sin(c+dx)}{5d(b \cos(c+dx))^{5/2}} + \frac{2B \sqrt{\cos(c+dx)} F\left(\frac{1}{2}(c+dx) \middle| 2\right)}{3b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(5/2), x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*B*Sin[c + d*x])/(3*b*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{5/2}} dx &= b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2 \int \frac{\frac{5b^2B}{2} + \frac{1}{2}b^2(3A+5C) \cos(c+dx)}{(b \cos(c+dx))^{5/2}} dx}{5b^2} \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + B \int \frac{1}{(b \cos(c + dx))^{5/2}} dx + \frac{(3A + 5C)}{5b^2} \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^2d\sqrt{b \cos(c + dx)}} \\ &= \frac{2A \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3bd(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^2d\sqrt{b \cos(c + dx)}} \\ &= -\frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3d\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)}}{3b^2d\sqrt{b}} \end{aligned}$$

Mathematica [A] time = 0.389122, size = 119, normalized size = 0.64

$$\frac{2\left(-3(3A + 5C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 9A \sin(c + dx) + 3A \tan(c + dx) \sec(c + dx) + 5B \tan(c + dx) + 5B\sqrt{\cos(c + dx)}\right)}{15b^2d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
d*x])^(5/2), x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d
*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^2*d*Sqrt[b*C
os[c + d*x]])
```

Maple [B] time = 10.203, size = 807, normalized size = 4.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 2/15*(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^3/sin(1/2*
d*x+1/2*c)^3/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-72*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+20*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x
+1/2*c)^4+60*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-120*C*sin(1/2*
d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-36*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1
/2*c)^2+72*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-20*B*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*sin(1/2*d*x+1/2*c)^2+20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-6
0*C*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+120*C*sin(1/2*d*x+1/2*c)^
4*cos(1/2*d*x+1/2*c)+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*A*cos(1/2*d*x+1/2*c)*s
in(1/2*d*x+1/2*c)^2+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-10*B*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)+15*C*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-30*C*sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2
)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)} \sec(dx + c)}{b^3 \cos(dx + c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d
*x + c)/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(5/2)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(5/2),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(5/2), x)
```


$$3.286 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=212

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b \cos(c+dx)}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b \cos(c+dx))^{3/2}} + \frac{2Ab \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{6B \sin(c+dx)}{5b^2d\sqrt{b \cos(c+dx)}} - \frac{6C \sin(c+dx)}{5b^2d\sqrt{b \cos(c+dx)}}$$

[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.270625, antiderivative size = 212, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.195$, Rules used = {16, 3021, 2748, 2636, 2640, 2639, 2642, 2641}

$$\frac{2(5A+7C)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21b^2d\sqrt{b \cos(c+dx)}} + \frac{2(5A+7C)\sin(c+dx)}{21bd(b \cos(c+dx))^{3/2}} + \frac{2Ab \sin(c+dx)}{7d(b \cos(c+dx))^{7/2}} + \frac{6B \sin(c+dx)}{5b^2d\sqrt{b \cos(c+dx)}} - \frac{6C \sin(c+dx)}{5b^2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (-6*B*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^3*d*Sqrt[Cos[c + d*x]]) + (2*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(21*b^2*d*Sqrt[b*Cos[c + d*x]]) + (2*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/2)) + (2*B*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/2)) + (2*(5*A + 7*C)*Sin[c + d*x])/(21*b*d*(b*Cos[c + d*x])^(3/2)) + (6*B*Sin[c + d*x])/(5*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2642

```
Int[1/Sqrt[(b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c -
Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{5/2}} dx = b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{9/2}} dx$$

$$= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2 \int \frac{\frac{7b^2B}{2} + \frac{1}{2}b^2(5A+7C) \cos(c+dx)}{(b \cos(c+dx))^{7/2}} dx}{7b}$$

$$= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + (bB) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx + \frac{1}{7}(5A + 7C) \int \frac{1}{(b \cos(c + dx))^{7/2}} dx$$

$$= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21bd(b \cos(c + dx))^{3/2}}$$

$$= \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}} + \frac{2B \sin(c + dx)}{5d(b \cos(c + dx))^{5/2}} + \frac{2(5A + 7C) \sin(c + dx)}{21bd(b \cos(c + dx))^{3/2}}$$

$$= \frac{2(5A + 7C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{21b^2 d \sqrt{b \cos(c + dx)}} + \frac{2Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/2}}$$

$$= -\frac{6B \sqrt{b \cos(c + dx)} E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^3 d \sqrt{\cos(c + dx)}} + \frac{2(5A + 7C) \sqrt{\cos(c + dx)}}{21b^2 d \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.450528, size = 136, normalized size = 0.64

$$\frac{2 \left(5(5A + 7C) \sqrt{\cos(c + dx)} F\left(\frac{1}{2}(c + dx) \middle| 2\right) + 25A \tan(c + dx) + 15A \tan(c + dx) \sec^2(c + dx) + 63B \sin(c + dx) - 63B \sqrt{\cos(c + dx)} \right)}{105b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(5/2), x]

[Out] (2*(-63*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(5*A + 7*C)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 63*B*Sin[c + d*x] + 25*A*Tan[c + d*x] + 35*C*Tan[c + d*x] + 21*B*Sec[c + d*x]*Tan[c + d*x] + 15*A*Sec[c + d*x]^2*Tan[c + d*x]))/(105*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [B] time = 10.934, size = 729, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2), x)

[Out] -(b*(2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/b^2*(2*A*(-1/56*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2))-2/5*B/b/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)*(12*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2)))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c), 2^(1/2))-8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c))*(-2*b*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2*b)^(1/2)+2*C*(-1/6*cos(1/2*d*x+1/2*c)/b*(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-b*(2*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2))^(1/2)*EllipticF(cos(1/2*d*x+1/2*c), 2^(1/2)))/sin(1/2*d*x+1/2*c)/(b*(2*cos(1/2*d*x+1/2*c)^2-1))^(1/2)/d

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A)\sqrt{b \cos(dx+c)} \sec(dx+c)^2}{b^3 \cos(dx+c)^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))*sec(d*x + c)^2/(b^3*cos(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \sec(dx+c)^2}{(b \cos(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(5/2), x)
```

$$3.287 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{7/2}} dx$$

Optimal. Leaf size=188

$$\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^3}$$

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
^3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/
2)) + (2*B*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*
Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.188985, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3021, 2748, 2636, 2642, 2641, 2640, 2639}

$$\frac{2(3A+5C)\sin(c+dx)}{5b^3d\sqrt{b\cos(c+dx)}} - \frac{2(3A+5C)E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\cos(c+dx)}}{5b^4d\sqrt{\cos(c+dx)}} + \frac{2A\sin(c+dx)}{5bd(b\cos(c+dx))^{5/2}} + \frac{2B\sin(c+dx)}{3b^2d(b\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2), x]
```

```
[Out] (-2*(3*A + 5*C)*Sqrt[b*Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])/(5*b^4*d*Sq
rt[Cos[c + d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])/(3*b
^3*d*Sqrt[b*Cos[c + d*x]]) + (2*A*Sin[c + d*x])/(5*b*d*(b*Cos[c + d*x])^(5/
2)) + (2*B*Sin[c + d*x])/(3*b^2*d*(b*Cos[c + d*x])^(3/2)) + (2*(3*A + 5*C)*
Sin[c + d*x])/(5*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^
(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[Sin[c + d*x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticF[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Dist[Sqrt[b*Sin[c + d*x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d}, x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] := Simp[(2*EllipticE[(1*(c - Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/2}} dx &= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2 \int \frac{\frac{5b^2B}{2} + \frac{1}{2}b^2(3A+5C) \cos(c+dx)}{(b \cos(c+dx))^{5/2}} dx}{5b^3} \\ &= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{B \int \frac{1}{(b \cos(c+dx))^{5/2}} dx}{b} + \frac{(3A + 5C) \int \frac{1}{(b \cos(c+dx))^{3/2}} dx}{5b^2} \\ &= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3b^2d(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3d\sqrt{b \cos(c + dx)}} + \\ &= \frac{2A \sin(c + dx)}{5bd(b \cos(c + dx))^{5/2}} + \frac{2B \sin(c + dx)}{3b^2d(b \cos(c + dx))^{3/2}} + \frac{2(3A + 5C) \sin(c + dx)}{5b^3d\sqrt{b \cos(c + dx)}} + \\ &= \frac{2(3A + 5C)\sqrt{b \cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right)}{5b^4d\sqrt{\cos(c + dx)}} + \frac{2B\sqrt{\cos(c + dx)}F\left(\frac{1}{2}(c + dx) \middle| 2\right)}{3b^3d\sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.118158, size = 119, normalized size = 0.63

$$\frac{2\left(-3(3A + 5C)\sqrt{\cos(c + dx)}E\left(\frac{1}{2}(c + dx) \middle| 2\right) + 9A \sin(c + dx) + 3A \tan(c + dx) \sec(c + dx) + 5B \tan(c + dx) + 5B\sqrt{\cos(c + dx)}\right)}{15b^3d\sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(7/2), x]
```

```
[Out] (2*(-3*(3*A + 5*C)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 9*A*Sin[c + d*x] + 15*C*Sin[c + d*x] + 5*B*Tan[c + d*x] + 3*A*Sec[c + d*x]*Tan[c + d*x]))/(15*b^3*d*Sqrt[b*Cos[c + d*x]])
```

Maple [B] time = 10.546, size = 807, normalized size = 4.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x)

[Out]
$$\frac{2}{15} \cdot (b \cdot (2 \cos(\frac{1}{2} d x + \frac{1}{2} c))^{-2} - 1) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} / b^4 / \sin(\frac{1}{2} d x + \frac{1}{2} c)^3 / (8 \sin(\frac{1}{2} d x + \frac{1}{2} c)^6 - 12 \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 + 6 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1) \cdot (36 A \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 - 72 A \cos(\frac{1}{2} d x + \frac{1}{2} c) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^6 + 20 B \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot \operatorname{EllipticF}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 + 60 C \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 - 120 C \sin(\frac{1}{2} d x + \frac{1}{2} c)^6 \cos(\frac{1}{2} d x + \frac{1}{2} c) - 36 A \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 + 72 A \cos(\frac{1}{2} d x + \frac{1}{2} c) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 - 20 B \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot \operatorname{EllipticF}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 + 20 B \cos(\frac{1}{2} d x + \frac{1}{2} c) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 - 60 C \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 + 120 C \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 \cos(\frac{1}{2} d x + \frac{1}{2} c) + 9 A \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) - 24 A \cos(\frac{1}{2} d x + \frac{1}{2} c) \cdot \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 + 5 B \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot \operatorname{EllipticF}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) - 10 B \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \cos(\frac{1}{2} d x + \frac{1}{2} c) + 15 C \cdot (\sin(\frac{1}{2} d x + \frac{1}{2} c)^2)^{\frac{1}{2}} \cdot (2 \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 - 1)^{\frac{1}{2}} \cdot \operatorname{EllipticE}(\cos(\frac{1}{2} d x + \frac{1}{2} c), 2^{\frac{1}{2}})) - 30 C \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 \cos(\frac{1}{2} d x + \frac{1}{2} c) \cdot (-2 b \sin(\frac{1}{2} d x + \frac{1}{2} c)^4 + \sin(\frac{1}{2} d x + \frac{1}{2} c)^2 b)^{\frac{1}{2}} / (b \cdot (2 \cos(\frac{1}{2} d x + \frac{1}{2} c))^{-2} - 1)^{\frac{1}{2}} / d$$

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{b^4 \cos(dx + c)^4}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/(b^4*cos(d*x + c)^4), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(7/2), x)

3.288 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)} \left(A + B \cos(c+dx) + C \cos^2 \right)$

Optimal. Leaf size=223

$$\frac{(5A+4C)\sin^3(c+dx)\sqrt{b \cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{(5A+4C)\sin(c+dx)\sqrt{b \cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3Bx\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx)}{d}$$

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - ((5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.12537, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2633, 2635, 8}

$$\frac{(5A+4C)\sin^3(c+dx)\sqrt{b \cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{(5A+4C)\sin(c+dx)\sqrt{b \cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3Bx\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (3*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + ((5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (C*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - ((5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*cos[c + d*x]
)*(b*sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{\sqrt{b \cos(c + dx)} \int \cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ = \frac{C \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d} + \frac{B \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{(5A + 4C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} + \frac{3B \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{(5A + 4C) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.285483, size = 109, normalized size = 0.49

$$\frac{\sqrt{b \cos(c + dx)}(60(6A + 5C) \sin(c + dx) + 40A \sin(3(c + dx)) + 120B \sin(2(c + dx)) + 15B \sin(4(c + dx)) + 180Bc + 180Bd)}{480d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[b*cos[c + d*x]]*(A + B*cos[c + d*x] + C*cos[c + d*x]^2),x]
```

```
[Out] (Sqrt[b*cos[c + d*x]]*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x] + 120*B*Ssin[2*(c + d*x)] + 40*A*Ssin[3*(c + d*x)] + 50*C*Ssin[3*(c + d*x)] + 15*B*Ssin[4*(c + d*x)] + 6*C*Ssin[5*(c + d*x)]))/(480*d*Sqrt[Cos[c + d*x]])
```

Maple [A] time = 0.354, size = 134, normalized size = 0.6

$$\frac{24 C (\cos(dx + c))^4 \sin(dx + c) + 30 B (\cos(dx + c))^3 \sin(dx + c) + 40 A \sin(dx + c) (\cos(dx + c))^2 + 32 C \sin(dx + c)}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x)

[Out] 1/120/d*(b*cos(d*x+c))^(1/2)*(24*C*cos(d*x+c)^4*sin(d*x+c)+30*B*cos(d*x+c)^3*sin(d*x+c)+40*A*sin(d*x+c)*cos(d*x+c)^2+32*C*sin(d*x+c)*cos(d*x+c)^2+45*B*sin(d*x+c)*cos(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*sin(d*x+c)*C)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.33069, size = 215, normalized size = 0.96

$$15 \left(12 dx + 12 c + \sin(4 dx + 4 c) + 8 \sin \left(\frac{1}{2} \arctan(\sin(4 dx + 4 c), \cos(4 dx + 4 c)) \right) \right) B \sqrt{b} + 2 C \sqrt{b} \left(3 \sin(5 dx + 5 c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/480*(15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*C*sqrt(b)*(3*sin(5*d*x + 5*c) + 25*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 150*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 40*A*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))))/d

Fricas [A] time = 2.03138, size = 815, normalized size = 3.65

$$\frac{45 B \sqrt{-b} \cos(dx + c) \log(2 b \cos(dx + c)^2 - 2 \sqrt{b \cos(dx + c)} \sqrt{-b \sqrt{\cos(dx + c)}} \sin(dx + c) - b) + 2 (24 C \cos(dx + c) \sin(dx + c) + 30 B \cos(dx + c)^3 + 8 (5 A + 4 C) \cos(dx + c)^2 + 45 B \cos(dx + c) + 80 A + 64 C) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c)}{24 \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/240*(45*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*cos(d*x + c)^4 + 30*B*cos(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 80*A + 64*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/120*(45*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*cos(d*x + c)^4 + 30*B*cos(d*x + c)^3 + 8*(5*A + 4*C)*cos(d*x + c)^2 + 45*B*cos(d*x + c) + 80*A + 64*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))  
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(  
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.289 \quad \int \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)} \left(A + B \cos(c+dx) + C \cos^2 \right)$$

Optimal. Leaf size=184

$$\frac{x(4A+3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A+3C) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{8d} - \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.107217, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2635, 8, 2633}

$$\frac{x(4A+3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{(4A+3C) \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{8d} - \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}} + \frac{B \sin^3(c+dx) \sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + ((4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2633

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rubi steps

$$\int \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{\sqrt{b \cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}}$$

$$= \frac{C \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{(4A + 3C) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d}$$

$$= \frac{(4A + 3C)x \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{B \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.261147, size = 92, normalized size = 0.5

$$\frac{\sqrt{b \cos(c + dx)}(24(A + C) \sin(2(c + dx)) + 48Ac + 48Adx + 72B \sin(c + dx) + 8B \sin(3(c + dx)) + 3C \sin(4(c + dx)) + 3C \sin^3(c + dx))}{96d \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/(96*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.542, size = 114, normalized size = 0.6

$$\frac{6 C (\cos(dx + c))^3 \sin(dx + c) + 8 B \sin(dx + c) (\cos(dx + c))^2 + 12 A \cos(dx + c) \sin(dx + c) + 9 C \cos(dx + c) \sin(dx + c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2), x)

```
[Out] 1/24/d*(b*cos(d*x+c))^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos
(d*x+c)^2+12*A*cos(d*x+c)*sin(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(1/2)
```

Maxima [A] time = 2.34114, size = 157, normalized size = 0.85

$$\frac{24(2dx + 2c + \sin(2dx + 2c))A\sqrt{b} + 3\left(12dx + 12c + \sin(4dx + 4c) + 8\sin\left(\frac{1}{2}\arctan\left(\frac{\sin(4dx + 4c)}{\cos(4dx + 4c)}\right)\right)\right)C\sqrt{b} + 8B\sqrt{b}\left(\sin(3dx + 3c) + 9\sin\left(\frac{1}{3}\arctan\left(\frac{\sin(3dx + 3c)}{\cos(3dx + 3c)}\right)\right)\right)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*sqrt(b) + 3*(12*d*x + 12*c + si
n(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*
sqrt(b) + 8*B*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c)))))/d
```

Fricas [A] time = 2.07715, size = 753, normalized size = 4.09

$$\left[\frac{3(4A + 3C)\sqrt{-b}\cos(dx + c)\log\left(2b\cos(dx + c)^2 - 2\sqrt{b}\cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)}\sin(dx + c) - b\right) + 2(6C\cos(dx + c)^3 + 8B\cos(dx + c)^2 + 3(4A + 3C)\cos(dx + c) + 16B)\sqrt{b}\cos(dx + c)}{48d\cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(
b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*cos(
d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/24*(3*(
4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*
x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*
(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```


3.290 $\int \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=143

$$\frac{(3A + 2C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d \sqrt{\cos(c + dx)}} + \frac{Bx \sqrt{b \cos(c + dx)}}{2 \sqrt{\cos(c + dx)}} + \frac{B \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{2d} + \frac{C \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{2d}$$

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0603876, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {17, 3023, 2734}

$$\frac{(3A + 2C) \sin(c + dx) \sqrt{b \cos(c + dx)}}{3d \sqrt{\cos(c + dx)}} + \frac{Bx \sqrt{b \cos(c + dx)}}{2 \sqrt{\cos(c + dx)}} + \frac{B \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{2d} + \frac{C \sin(c + dx) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx)) dx = \frac{\sqrt{b \cos(c+dx)} \int \cos(c+dx) (A + B \cos(c+dx) + C \cos^2(c+dx)) dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{C \cos^3(c+dx) \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d} + \frac{B \sqrt{b \cos(c+dx)} \sin(c+dx)}{2\sqrt{\cos(c+dx)}} + \frac{(3A + 2C) \sqrt{b \cos(c+dx)} \sin(c+dx)}{3d \sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.184823, size = 75, normalized size = 0.52

$$\frac{\sqrt{b \cos(c+dx)} (3(4A + 3C) \sin(c+dx) + 3B \sin(2(c+dx)) + 6Bc + 6Bdx + C \sin(3(c+dx)))}{12d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.468, size = 83, normalized size = 0.6

$$\frac{2C \sin(dx+c) (\cos(dx+c))^2 + 3B \sin(dx+c) \cos(dx+c) + 6A \sin(dx+c) + 3B(dx+c) + 4 \sin(dx+c) C}{6d} \sqrt{b \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2), x)

[Out] 1/6/d*(b*cos(d*x+c))^(1/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.26473, size = 108, normalized size = 0.76

$$\frac{3(2dx + 2c + \sin(2dx + 2c))B\sqrt{b} + C\sqrt{b} \left(\sin(3dx + 3c) + 9 \sin\left(\frac{1}{3} \arctan\left(\frac{\sin(3dx + 3c)}{\cos(3dx + 3c)}\right)\right) \right) + 12A\sqrt{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*sqrt(b) + C*sqrt(b)*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))) + 12*A*sqrt(b)*sin(d*x + c))/d

Fricas [A] time = 2.00874, size = 655, normalized size = 4.58

$$\left[\frac{3B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right) + 2\left(2C\cos(dx+c)\right)}{12d\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)*(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)*(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: NotImplementedError

$$3.291 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=123

$$\frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0382755, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2637, 2635, 8}

$$\frac{Ax\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (A*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{\sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx = \frac{\sqrt{b \cos(c+dx)} \int (A + B \cos(c+dx) + C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} \\ = \frac{Ax \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{(B \sqrt{b \cos(c+dx)}) \int \cos(c+dx) dx}{\sqrt{\cos(c+dx)}} \\ = \frac{Ax \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} \\ = \frac{Ax \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{Cx \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{B \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.0994938, size = 61, normalized size = 0.5

$$\frac{\sqrt{b \cos(c+dx)}(2(2A+C)(c+dx) + 4B \sin(c+dx) + C \sin(2(c+dx)))}{4d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (Sqrt[b*Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.458, size = 63, normalized size = 0.5

$$\frac{C \cos(dx+c) \sin(dx+c) + 2A(dx+c) + 2B \sin(dx+c) + C(dx+c)}{2d} \sqrt{b \cos(dx+c)} \frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x)

[Out] 1/2/d*(b*cos(d*x+c))^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))/cos(d*x+c)^(1/2)

Maxima [A] time = 2.08128, size = 86, normalized size = 0.7

$$\frac{(2dx + 2c + \sin(2dx + 2c))C\sqrt{b} + 8A\sqrt{b} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 4B\sqrt{b} \sin(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C*sqrt(b) + 8*A*sqrt(b)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*sqrt(b)*sin(d*x + c))/d

Fricas [A] time = 2.04888, size = 591, normalized size = 4.8

$$\left[\frac{(2A + C)\sqrt{-b} \cos(dx + c) \log(2b \cos(dx + c)^2 - 2\sqrt{b} \cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b) + 2(C \cos(dx + c) + 2B)\sqrt{b} \cos(dx + c) \sqrt{\cos(dx + c)} \sin(dx + c)}{4d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{b \cos(dx + c)}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/sqrt(cos(d*x + c)), x)

$$3.292 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=93

$$\frac{A\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{Bx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0568781, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3023, 2735, 3770}

$$\frac{A\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{Bx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Ssin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx = \frac{\sqrt{b \cos(c+dx)} \int (A + B \cos(c+dx) + C \cos^2(c+dx)) \sec(c+dx) dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{C \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{\sqrt{b \cos(c+dx)} \int (A + B \cos(c+dx)) \sec(c+dx) dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Bx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{C \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \sqrt{\cos(c+dx)}} + \frac{(A \sqrt{b}) \int \sec(c+dx) dx}{\sqrt{\cos(c+dx)}}$$

$$= \frac{Bx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A \tanh^{-1}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

Mathematica [A] time = 0.10071, size = 93, normalized size = 1.

$$\frac{\sqrt{b \cos(c+dx)} \left(-A \log \left(\cos \left(\frac{1}{2}(c+dx) \right) - \sin \left(\frac{1}{2}(c+dx) \right) \right) + A \log \left(\sin \left(\frac{1}{2}(c+dx) \right) + \cos \left(\frac{1}{2}(c+dx) \right) \right) + Bc + Bdx + C \right)}{d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.385, size = 63, normalized size = 0.7

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right) - B(dx+c) - \sin(dx+c) C \right) \sqrt{b \cos(dx+c)} \frac{1}{\sqrt{\cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)

Maxima [A] time = 2.07597, size = 140, normalized size = 1.51

$$\frac{A \sqrt{b} \left(\log \left(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1 \right) - \log \left(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1 \right) \right) + Bx \sqrt{b \cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/2*(A*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*B*sqrt(b)*a

$\text{rctan}(\sin(dx + c)/(\cos(dx + c) + 1)) + 2C\sqrt{b}\sin(dx + c)/d$

Fricas [A] time = 2.3972, size = 859, normalized size = 9.24

$$\frac{2A\sqrt{-b}\arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right)\cos(dx+c) - B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\right)}{2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)*(b*cos(dx+c))^(1/2)/cos(dx+c)^(3/2),x, algorithm="fricas")

[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c) - B*sqrt(-b)*cos(dx + c)*log(2*b*cos(dx + c)^2 - 2*sqrt(b*cos(dx + c))*sqrt(-b)*sqrt(cos(dx + c))*sin(dx + c) - b) - 2*sqrt(b*cos(dx + c))*C*sqrt(cos(dx + c))*sin(dx + c)/(d*cos(dx + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(dx + c))*sin(dx + c)/(sqrt(b)*cos(dx + c)^(3/2)))*cos(dx + c) + A*sqrt(b)*cos(dx + c)*log(-(b*cos(dx + c))^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3) + 2*sqrt(b*cos(dx + c))*C*sqrt(cos(dx + c))*sin(dx + c)/(d*cos(dx + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)**2)*(b*cos(dx+c))**(1/2)/cos(dx+c)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + B\cos(dx+c) + A)\sqrt{b\cos(dx+c)}}{\cos(dx+c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)*(b*cos(dx+c))^(1/2)/cos(dx+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + B*cos(dx + c) + A)*sqrt(b*cos(dx + c))/cos(dx + c)^(3/2), x)

$$3.293 \quad \int \frac{\sqrt{b \cos(c+dx)} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=93

$$\frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{B \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0658156, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3021, 2735, 3770}

$$\frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{B \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sine[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx &= \frac{\sqrt{b \cos(c+dx)} \int (A + B \cos(c+dx) + C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} \\
&= \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{\sqrt{b \cos(c+dx)} \int (B + C \cos(c+dx))}{\sqrt{\cos(c+dx)}} \\
&= \frac{Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{A \sqrt{b \cos(c+dx)} \sin(c+dx)}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{(B + C \cos(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}} \\
&= \frac{Cx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{B \tanh^{-1}(\sin(c+dx)) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0783913, size = 60, normalized size = 0.65

$$\frac{\sqrt{b \cos(c+dx)} (A \sin(c+dx) + B \cos(c+dx) \tanh^{-1}(\sin(c+dx)) + C dx \cos(c+dx))}{d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.401, size = 72, normalized size = 0.8

$$\frac{1}{d} \sqrt{b \cos(dx+c)} \left(-2B \cos(dx+c) \operatorname{Arctanh} \left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right) + C \cos(dx+c)(dx+c) + A \sin(dx+c) \right) (\cos(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2), x)

[Out] 1/d*(b*cos(d*x+c))^(1/2)*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c)))+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)

Maxima [A] time = 2.08108, size = 194, normalized size = 2.09

$$\frac{B\sqrt{b}(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2\sin(dx+c) + 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

```
[Out] 1/2*(B*sqrt(b)*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -
log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1)) + 4*C*sqrt(b)*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*sqrt(b)*sin(2*d*x + 2*c)/(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 2.38871, size = 875, normalized size = 9.41

$$\left[\frac{2B\sqrt{-b} \arctan\left(\frac{\sqrt{b}\cos(dx+c)\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - C\sqrt{-b} \cos(dx+c)^2 \log\left(2b \cos(dx+c)^2 - 2\sqrt{b}\cos(dx+c)\sqrt{\cos(dx+c)}\right)}{2d \cos(dx+c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sq
rt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*cos(d*x + c)^2*log(2*b*cos(d
*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c)
- b) - 2*sqrt(b*cos(d*x + c))*A*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(
b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*sqrt(b)*cos(d*x + c)^2*log(-b*c
os(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x +
c) - 2*b*cos(d*x + c))/cos(d*x + c)^3 + 2*sqrt(b*cos(d*x + c))*A*sqrt(cos
(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)
**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A)\sqrt{b \cos(dx+c)}}{\cos(dx+c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(
d*x + c)^(5/2), x)
```

$$3.294 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=111

$$\frac{(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.104333, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3021, 2748, 3767, 8, 3770}

$$\frac{(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx = \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)} + \frac{\sqrt{b \cos(c + dx)} \int (2B + (A + 2C) \cos(c + dx)) \sec^2(c + dx) dx}{2 \sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)} + \frac{(B \sqrt{b \cos(c + dx)}) \int \sec^2(c + dx) dx}{\sqrt{\cos(c + dx)}}$$

$$= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)}}{2d \cos^{\frac{5}{2}}(c + dx)}$$

$$= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)}}{2d \cos^{\frac{5}{2}}(c + dx)}$$

Mathematica [A] time = 0.142708, size = 69, normalized size = 0.62

$$\frac{\sqrt{b \cos(c + dx)} (\sin(c + dx)(A + 2B \cos(c + dx)) + (A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{2d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.43, size = 150, normalized size = 1.4

$$\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right) - 4C \cos(dx + c)^2 \operatorname{arctanh} \left(\frac{\sin(dx + c)}{\cos(dx + c)} \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2), x)

[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-

$$1+\cos(d*x+c))/\sin(d*x+c))+2*B*\sin(d*x+c)*\cos(d*x+c)+A*\sin(d*x+c))*(b*\cos(d*x+c))^(1/2)/\cos(d*x+c)^(5/2)$$

Maxima [B] time = 2.30175, size = 1053, normalized size = 9.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out]
$$\frac{1}{4}*(2*C*\sqrt{b}*(\log(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\sin(d*x + c) + 1) - \log(\cos(d*x + c)^2 + \sin(d*x + c)^2 - 2*\sin(d*x + c) + 1)) - (4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))))*A*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1) + 8*B*\sqrt{b}*\sin(2*d*x + 2*c)/(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1))/d$$

Fricas [A] time = 1.97565, size = 648, normalized size = 5.84

$$\frac{(A + 2C)\sqrt{b} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)}\sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2B \cos(dx + c) + A)\sqrt{b}}{4d \cos(dx + c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out]
$$\left[\frac{1}{4}*((A + 2C)*\sqrt{b}*\cos(d*x + c)^3*\log(-(b*\cos(d*x + c))^3 - 2*\sqrt{b*\cos(d*x + c)}*\sqrt{b*\cos(d*x + c)}*\sin(d*x + c) - 2*b*\cos(d*x + c))/\cos(d*x + c)^3) + 2*(2*B*\cos(d*x + c) + A)*\sqrt{b*\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^3), -1/2*((A + 2C)*\sqrt{-b}*\arctan(\sqrt{b*\cos(d*x + c)}*\sqrt{-b}*\sin(d*x + c)/(b*\sqrt{\cos(d*x + c)})))*\cos(d*x + c)^3 - (2*B*\cos(d*x + c) + A)*\sqrt{b*\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^3)]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(7/2), x)

$$3.295 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{9 \cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=152

$$\frac{(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{B \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{B \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + ((2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.11567, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {17, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{B \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{B \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + ((2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{9}{2}}(c + dx)} dx = \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^7(c + dx) dx}{\sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx)} + \frac{\sqrt{b \cos(c + dx)} \int (3B + (2A + 3C) \cos^2(c + dx)) \sec^3(c + dx) dx}{3\sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx)} + \frac{(B \sqrt{b \cos(c + dx)}) \int \sec^3(c + dx) dx}{\sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx)} + \frac{B \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{\frac{5}{2}}(c + dx)}$$

$$= \frac{B \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{\frac{7}{2}}(c + dx)}$$

Mathematica [A] time = 0.413408, size = 87, normalized size = 0.57

$$\frac{\sqrt{b \cos(c + dx)} \left(\tan(c + dx) ((2A + 3C) \cos(2(c + dx)) + 4A + 3B \cos(c + dx) + 3C) + 3B \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)) \right)}{6d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.444, size = 157, normalized size = 1.

$$\frac{1}{6d} \left(-3B \ln \left(\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 + 3B \ln \left(\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))*cos(d*x+c)^(1/2)/cos(d*x+c)^(7/2)
```

Maxima [B] time = 2.3862, size = 1362, normalized size = 8.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] 1/12*(16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1) + 24*C*sqrt(b)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.97205, size = 722, normalized size = 4.75

$$\frac{3B\sqrt{b}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b\cos(dx+c)}\sqrt{b\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2(2A+3C)\cos(dx+c)^2+3\right)}{12d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(9/2), x)
```

$$3.296 \quad \int \frac{\sqrt{b \cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=193

$$\frac{(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c +
d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (
(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (B*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.117391, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3021, 2748, 3767, 3768, 3770}

$$\frac{(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{A \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c +
d*x]^(11/2), x]
```

```
[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c +
d*x]]) + (A*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (
(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (
B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (B*Sqrt[b*Cos
[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[((A*b^2
- a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(
m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b
- a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B,
C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_.)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{11}{2}}(c + dx)} dx = \frac{\sqrt{b \cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx)}{\sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{\frac{9}{2}}(c + dx)} + \frac{\sqrt{b \cos(c + dx)} \int (4B + (3A + 4C) \cos(c + dx)) \sec^4(c + dx)}{4d \cos^{\frac{9}{2}}(c + dx)}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{\frac{9}{2}}(c + dx)} + \frac{(B \sqrt{b \cos(c + dx)}) \int \sec^4(c + dx)}{\sqrt{\cos(c + dx)}}$$

$$= \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{\frac{9}{2}}(c + dx)} + \frac{(3A + 4C) \sqrt{b \cos(c + dx)} \int \sec^2(c + dx)}{8d \cos^{\frac{5}{2}}(c + dx)}$$

$$= \frac{(3A + 4C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{8d \sqrt{\cos(c + dx)}} + \frac{A \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{\frac{9}{2}}(c + dx)}$$

Mathematica [A] time = 0.336353, size = 110, normalized size = 0.57

$$\frac{\sqrt{b \cos(c + dx)} (\sin(c + dx) (3(3A + 4C) \cos^2(c + dx) + 6A + 24B \cos^3(c + dx) + 8B \sin^2(c + dx) \cos(c + dx)) + 3(3A + 4C) \cos(c + dx))}{24d \cos^{\frac{9}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[b*Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]
```

```
[Out] (Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))
```

Maple [A] time = 0.348, size = 248, normalized size = 1.3

$$\frac{1}{24d} \left(9A \ln \left(\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^4 - 9A (\cos(dx + c))^4 \ln \left(\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2), x)

[Out] 1/24/d*(9*A*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*sin(d*x+c)*cos(d*x+c)^2+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c))*cos(d*x+c)^(1/2)/cos(d*x+c)^(9/2)

Maxima [B] time = 2.66585, size = 3525, normalized size = 18.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2), x, algorithm="maxima")

[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c)

```

+ 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*
d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2
*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*
c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos
(2*d*x + 2*c) + 1) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*c
os(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
- 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4*c) +
3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c
)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x
+ 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*
sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 12*(4*(sin(4*d*x + 4*c) + 2*
sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*
(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*
x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*
d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*c
os(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
C*sqrt(b)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2
+ 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d

```

Fricas [A] time = 1.99341, size = 810, normalized size = 4.2

$$\left[\frac{3(3A + 4C)\sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(16B \cos(dx + c)^3 + 3(3A + 4C)\sqrt{b} \cos(dx + c)^5)}{48d \cos(dx + c)^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(
11/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d*x
+ c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16*B
*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*sq
rt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]
```


Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*(b*cos(d*x+c))**(1/2)/cos(d*x+c)**(11/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{b \cos(dx + c)}}{\cos(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*(b*cos(d*x+c))^(1/2)/cos(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(b*cos(d*x + c))/cos(d*x + c)^(11/2), x)

3.297 $\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=229

$$-\frac{b(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{b(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{bB\sin(c+dx)}{8\sqrt{\cos(c+dx)}}$$

```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*C*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b*(5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.127084, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2633, 2635, 8}

$$-\frac{b(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{b(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3bBx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{bB\sin(c+dx)}{8\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (3*b*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*(5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*B*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b*C*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b*(5*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{(b\sqrt{b \cos(c + dx)}) \int \cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ = \frac{bC \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d} \\ = \frac{bC \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d} \\ = \frac{bB \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\ = \frac{b(5A + 4C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} + \frac{3bBx \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{b(5A + 4C) \sqrt{b \cos(c + dx)}}{5d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.317279, size = 109, normalized size = 0.48

$$\frac{(b \cos(c + dx))^{3/2} (60(6A + 5C) \sin(c + dx) + 40A \sin(3(c + dx)) + 120B \sin(2(c + dx)) + 15B \sin(4(c + dx)) + 180Bc \cos^3(c + dx))}{480d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2), x]
```

```
[Out] ((b*Cos[c + d*x])^(3/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
+ 120*B*Sin[2*(c + d*x)] + 40*A*Sin[3*(c + d*x)] + 50*C*Sin[3*(c + d*x)] +
15*B*Sin[4*(c + d*x)] + 6*C*Sin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(3/2))
```

Maple [A] time = 0.293, size = 134, normalized size = 0.6

$$\frac{24 C (\cos(dx + c))^4 \sin(dx + c) + 30 B (\cos(dx + c))^3 \sin(dx + c) + 40 A \sin(dx + c) (\cos(dx + c))^2 + 32 C \sin(dx + c)}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)`

[Out] $\frac{1}{120d}(b\cos(dx+c))^{3/2}(24C\cos(dx+c)^4\sin(dx+c)+30B\cos(dx+c)^3\sin(dx+c)+40A\sin(dx+c)\cos(dx+c)^2+32C\sin(dx+c)\cos(dx+c)^2+45B\sin(dx+c)\cos(dx+c)+80A\sin(dx+c)+45B(dx+c)+64\sin(dx+c)C)/\cos(dx+c)^{3/2}$

Maxima [A] time = 2.3301, size = 228, normalized size = 1.

$40\left(b\sin(3dx+3c)+9b\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)A\sqrt{b}+15\left(12(dx+c)b+b\sin(4dx+4c)+\dots\right)$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")`

[Out] $\frac{1}{480}(40(b\sin(3dx+3c)+9b\sin(\frac{1}{3}\arctan2(\sin(3dx+3c),\cos(3dx+3c))))A\sqrt{b}+15(12(dx+c)b+b\sin(4dx+4c)+8b\sin(\frac{1}{2}\arctan2(\sin(4dx+4c),\cos(4dx+4c))))B\sqrt{b}+2(3b\sin(5dx+5c)+25b\sin(\frac{3}{5}\arctan2(\sin(5dx+5c),\cos(5dx+5c))))+150b\sin(\frac{1}{5}\arctan2(\sin(5dx+5c),\cos(5dx+5c))))C\sqrt{b})/d$

Fricas [A] time = 2.07001, size = 853, normalized size = 3.72

$$\frac{45B\sqrt{-bb}\cos(dx+c)\log(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b)+2(24Cb\cos(dx+c)+\dots)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")`

[Out] $\left[\frac{1}{240}(45B\sqrt{-b}b\cos(dx+c)\log(2b\cos(dx+c)^2-2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b)+2(24Cb\cos(dx+c)+30Bb\cos(dx+c)^3+8(5A+4C)b\cos(dx+c)^2+45Bb\cos(dx+c)+16(5A+4C)b)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c))/(d\cos(dx+c)),\frac{1}{120}(45Bb^{3/2}\arctan(\sqrt{b\cos(dx+c)}\sin(dx+c)/(\sqrt{b}\cos(dx+c)^{3/2}))\cos(dx+c)+(24Cb\cos(dx+c)^4+30Bb\cos(dx+c)^3+8(5A+4C)b\cos(dx+c)^2+45Bb\cos(dx+c)+16(5A+4C)b)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c))/(d\cos(dx+c))\right]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.298 $\int \sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos$

Optimal. Leaf size=189

$$\frac{bx(4A + 3C)\sqrt{b \cos(c + dx)}}{8\sqrt{\cos(c + dx)}} + \frac{b(4A + 3C) \sin(c + dx)\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)}}{8d} - \frac{bB \sin^3(c + dx)\sqrt{b \cos(c + dx)}}{3d\sqrt{\cos(c + dx)}} + \frac{bC \sin^5(c + dx)\sqrt{b \cos(c + dx)}}{3d\sqrt{\cos(c + dx)}}$$

```
[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.118522, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2635, 8, 2633}

$$\frac{bx(4A + 3C)\sqrt{b \cos(c + dx)}}{8\sqrt{\cos(c + dx)}} + \frac{b(4A + 3C) \sin(c + dx)\sqrt{\cos(c + dx)}\sqrt{b \cos(c + dx)}}{8d} - \frac{bB \sin^3(c + dx)\sqrt{b \cos(c + dx)}}{3d\sqrt{\cos(c + dx)}} + \frac{bC \sin^5(c + dx)\sqrt{b \cos(c + dx)}}{3d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (b*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

$+ d*x])^{(n - 2), x], x] /; \text{FreeQ}[\{b, c, d\}, x] \ \&\& \ \text{GtQ}[n, 1] \ \&\& \ \text{IntegerQ}[2*n]$

Rule 8

$\text{Int}[a_, x_Symbol] \text{ :> } \text{Simp}[a*x, x] /; \text{FreeQ}[a, x]$

Rule 2633

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)]^{(n_.)}, x_Symbol] \text{ :> } -\text{Dist}[d^{(-1)}, \text{Subst}[\text{Int}[\text{Expand}[(1 - x^2)^{(n - 1)/2}], x], x], x, \text{Cos}[c + d*x]], x] /; \text{FreeQ}[\{c, d\}, x] \ \&\& \ \text{IGtQ}[(n - 1)/2, 0]$

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c + dx)} (b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{bC \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx) + (4Ab + 3b^2C) \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx) + (4A^2 + 4Ab + 3b^2C) \cos^{\frac{1}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\ &= \frac{bC \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx) + (4Ab + 3b^2C) \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx) + (4A^2 + 4Ab + 3b^2C) \cos^{\frac{1}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\ &= \frac{b(4A + 3C) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx) + (4Ab + 3b^2C) \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx) + (4A^2 + 4Ab + 3b^2C) \cos^{\frac{1}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} \\ &= \frac{b(4A + 3C)x \sqrt{b \cos(c + dx)}}{8\sqrt{\cos(c + dx)}} + \frac{bB \sqrt{b \cos(c + dx)}}{d} \end{aligned}$$

Mathematica [A] time = 0.214395, size = 92, normalized size = 0.49

$$\frac{(b \cos(c + dx))^{3/2} (24(A + C) \sin(2(c + dx)) + 48Ac + 48Adx + 72B \sin(c + dx) + 8B \sin(3(c + dx)) + 3C \sin(4(c + dx)))}{96d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C *Cos[c + d*x]^2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d*x)]))/(96*d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.485, size = 114, normalized size = 0.6

$$\frac{6C(\cos(dx + c))^3 \sin(dx + c) + 8B \sin(dx + c) (\cos(dx + c))^2 + 12A \cos(dx + c) \sin(dx + c) + 9C \cos(dx + c) \sin(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2), x)

```
[Out] 1/24/d*(b*cos(d*x+c))^(3/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos
(d*x+c)^2+12*A*cos(d*x+c)*sin(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(3/2)
```

Maxima [A] time = 2.29419, size = 170, normalized size = 0.9

$$\frac{24(2(dx+c)b + b\sin(2dx+2c))A\sqrt{b} + 8\left(b\sin(3dx+3c) + 9b\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c), \cos(3dx+3c))\right)\right)B\sqrt{b}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b*sin(3*d*x +
3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*sqrt(b)
+ 3*(12*(d*x + c)*b + b*sin(4*d*x + 4*c) + 8*b*sin(1/2*arctan2(sin(4*d*x +
4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Fricas [A] time = 2.03715, size = 778, normalized size = 4.12

$$\frac{3(4A + 3C)\sqrt{-bb}\cos(dx+c)\log\left(2b\cos(dx+c)^2 - 2\sqrt{b}\cos(dx+c)\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b\right) + 2(6Cb\cos(dx+c) - b^2)}{48d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt
t(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*b*
cos(d*x + c)^3 + 8*B*b*cos(d*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B
*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)),
1/24*(3*(4*A + 3*C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt
(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b*cos(d*x + c)^3 + 8*B*b*cos(d
*x + c)^2 + 3*(4*A + 3*C)*b*cos(d*x + c) + 16*B*b)*sqrt(b*cos(d*x + c))*sqrt
t(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.299 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=147

$$\frac{b(3A+2C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{bBx \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} + \frac{bC \sin(c+dx)}{2d}$$

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*(3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b*C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rubi [A] time = 0.0590462, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {17, 3023, 2734}

$$\frac{b(3A+2C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{bBx \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} + \frac{bC \sin(c+dx)}{2d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*(3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b*C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2734

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx = \frac{(b\sqrt{b \cos(c + dx)}) \int \cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} + \frac{(b\sqrt{b \cos(c + dx)})^3 \sin(c + dx)}{3d} + \frac{bBx\sqrt{b \cos(c + dx)}}{2\sqrt{\cos(c + dx)}} + \frac{b(3A + 2C)\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d\sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.0720573, size = 76, normalized size = 0.52

$$\frac{b\sqrt{b \cos(c + dx)}(3(4A + 3C) \sin(c + dx) + 3B \sin(2(c + dx)) + 6Bc + 6Bdx + C \sin(3(c + dx)))}{12d\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]

[Out] (b*Sqrt[b*Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sqrt[2*(c + d*x)] + C*Sqrt[3*(c + d*x)]))/(12*d*Sqrt[Cos[c + d*x]])

Maple [A] time = 0.418, size = 83, normalized size = 0.6

$$\frac{2C \sin(dx + c) (\cos(dx + c))^2 + 3B \sin(dx + c) \cos(dx + c) + 6A \sin(dx + c) + 3B(dx + c) + 4 \sin(dx + c) C}{6d} (b \cos(dx + c))^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)

[Out] 1/6/d*(b*cos(d*x+c))^(3/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/cos(d*x+c)^(3/2)

Maxima [A] time = 2.32264, size = 116, normalized size = 0.79

$$\frac{12Ab^{\frac{3}{2}} \sin(dx + c) + 3(2(dx + c)b + b \sin(2dx + 2c))B\sqrt{b} + \left(b \sin(3dx + 3c) + 9b \sin\left(\frac{1}{3} \arctan(\sin(3dx + 3c))\right)\right) C}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/12*(12*A*b^(3/2)*sin(d*x + c) + 3*(2*(d*x + c)*b + b*sin(2*d*x + 2*c))*B*sqrt(b) + (b*sin(3*d*x + 3*c) + 9*b*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d

Fricas [A] time = 1.96836, size = 684, normalized size = 4.65

$$\frac{3B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2-2\sqrt{b}\cos(dx+c)\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)+2\left(2Cb\cos(dx+c)\right)}{12d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out] [1/12*(3*B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*(3*A + 2*C)*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A)(b \cos(dx+c))^{\frac{3}{2}}}{\sqrt{\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/sqrt(cos(d*x + c)), x)

$$3.300 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=127

$$\frac{Abx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0381749, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2637, 2635, 8}

$$\frac{Abx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bB \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{bCx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (A*b*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ = \frac{Abx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{(bB\sqrt{b \cos(c + dx)}) \int \cos(c + dx)}{\sqrt{\cos(c + dx)}} \\ = \frac{Abx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{bB\sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{b}{d\sqrt{\cos(c + dx)}} \\ = \frac{Abx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{bCx\sqrt{b \cos(c + dx)}}{2\sqrt{\cos(c + dx)}} + \frac{bB\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.123911, size = 61, normalized size = 0.48

$$\frac{(b \cos(c + dx))^{3/2} (2(2A + C)(c + dx) + 4B \sin(c + dx) + C \sin(2(c + dx)))}{4d \cos^{\frac{3}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(3/2)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2))/cos[c + d*x]^(3/2), x]

[Out] ((b*cos[c + d*x])^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*sin[c + d*x] + C*sin[2*(c + d*x)]))/(4*d*cos[c + d*x]^(3/2))

Maple [A] time = 0.283, size = 63, normalized size = 0.5

$$\frac{C \cos(dx + c) \sin(dx + c) + 2A(dx + c) + 2B \sin(dx + c) + C(dx + c)}{2d} (b \cos(dx + c))^{\frac{3}{2}} (\cos(dx + c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x)

[Out] 1/2/d*(b*cos(d*x+c))^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))/cos(d*x+c)^(3/2)

Maxima [A] time = 2.09056, size = 90, normalized size = 0.71

$$\frac{8Ab^{\frac{3}{2}} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 4Bb^{\frac{3}{2}} \sin(dx+c) + (2(dx+c)b + b \sin(2dx+2c))C\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2), x, algorithm="maxima")

[Out] 1/4*(8*A*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(3/2)*sin(d*x + c) + (2*(d*x + c)*b + b*sin(2*d*x + 2*c))*C*sqrt(b))/d

Fricas [A] time = 1.99187, size = 605, normalized size = 4.76

$$\left[\frac{(2A + C)\sqrt{-b}b \cos(dx + c) \log\left(2b \cos(dx + c)^2 - 2\sqrt{b} \cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b\right) + 2(Cb \cos(dx + c) + 2Bb)\sqrt{b} \cos(dx + c) \sqrt{\cos(dx + c)} \sin(dx + c)}{4d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/4*((2*A + C)*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b*cos(d*x + c) + 2*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/2*((2*A + C)*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b*cos(d*x + c) + 2*B*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(3/2), x)

$$3.301 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=96

$$\frac{Ab\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{bBx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0549559, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3023, 2735, 3770}

$$\frac{Ab\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{\cos(c+dx)}} + \frac{bBx\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{bC \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (b*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{5/2}(c + dx)} dx = \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ = \frac{bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} + \frac{(b\sqrt{b \cos(c + dx)}) \int}{\sqrt{\cos(c + dx)}} \\ = \frac{bBx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{bC\sqrt{b \cos(c + dx)} \sin(c + dx)}{d\sqrt{\cos(c + dx)}} \\ = \frac{bBx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{Ab \tanh^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.121863, size = 93, normalized size = 0.97

$$\frac{(b \cos(c + dx))^{3/2} \left(-A \log \left(\cos \left(\frac{1}{2}(c + dx) \right) - \sin \left(\frac{1}{2}(c + dx) \right) \right) + A \log \left(\sin \left(\frac{1}{2}(c + dx) \right) + \cos \left(\frac{1}{2}(c + dx) \right) \right) + Bc + Bdx \right)}{d \cos^{5/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Cos[c + d*x]^(3/2))

Maple [A] time = 0.278, size = 63, normalized size = 0.7

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - B(dx + c) - \sin(dx + c) C \right) (b \cos(dx + c))^{3/2} (\cos(dx + c))^{-3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(3/2)

Maxima [A] time = 2.12814, size = 144, normalized size = 1.5

$$\frac{4 B b^{3/2} \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) + 2 C b^{3/2} \sin(dx+c) + (b \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - b \log(\cos(dx+c)))}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x, algorithm="maxima")

[Out] 1/2*(4*B*b^(3/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(3/2)*sin(dx + c) + (b*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b

*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*A*sqrt(b))/d

Fricas [A] time = 2.4037, size = 869, normalized size = 9.05

$$\left[\frac{2A\sqrt{-bb} \arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c) - B\sqrt{-bb} \cos(dx+c) \log\left(2b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)\right)}{2d\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [-1/2*(2*A*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) - B*sqrt(-b)*b*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)), 1/2*(2*B*b^(3/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*b^(3/2)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3 + 2*sqrt(b*cos(d*x + c))*C*b*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(5/2), x)

$$3.302 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=96

$$\frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^2(c+dx)} + \frac{bB \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{bCx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0607124, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3021, 2735, 3770}

$$\frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^2(c+dx)} + \frac{bB \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{bCx \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (b*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{7/2}(c + dx)} dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{3/2}(c + dx)} + \frac{(b\sqrt{b \cos(c + dx)}) \int (B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\
&= \frac{bCx\sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{3/2}(c + dx)} + \frac{bB \tan^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{d\sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.0753996, size = 60, normalized size = 0.62

$$\frac{(b \cos(c + dx))^{3/2} (A \sin(c + dx) + B \cos(c + dx) \tanh^{-1}(\sin(c + dx)) + C dx \cos(c + dx))}{d \cos^{5/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2),x]

[Out] ((b*Cos[c + d*x])^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.247, size = 72, normalized size = 0.8

$$\frac{1}{d} (b \cos(dx + c))^{3/2} \left(-2B \cos(dx + c) \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) + C \cos(dx + c)(dx + c) + A \sin(dx + c) \right) (\cos(dx + c))^{1/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

[Out] 1/d*(b*cos(d*x+c))^(3/2)*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(5/2)

Maxima [A] time = 2.10769, size = 198, normalized size = 2.06

$$\frac{4Cb^{\frac{3}{2}} \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) + (b \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - b \log(\cos(dx+c)^2 + \sin(dx+c)^2))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] $\frac{1}{2}*(4*C*b^{(3/2)}*\arctan(\sin(d*x + c)/(\cos(d*x + c) + 1)) + (b*\log(\cos(d*x + c)^2 + \sin(d*x + c)^2 + 2*\sin(d*x + c) + 1) - b*\log(\cos(d*x + c)^2 + \sin(d*x + c)^2 - 2*\sin(d*x + c) + 1))*B*\sqrt{b} + 4*A*b^{(3/2)}*\sin(2*d*x + 2*c)/(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1))/d$

Fricas [A] time = 2.41611, size = 886, normalized size = 9.23

$$\frac{2 B \sqrt{-b} b \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b \sin(dx+c)}}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - C \sqrt{-b} b \cos(dx+c)^2 \log\left(2 b \cos(dx+c)^2 - 2 \sqrt{b \cos(dx+c)}\right)}{2 d \cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="fricas")

[Out] $[-1/2*(2*B*\sqrt{-b}*b*\arctan(\sqrt{b*\cos(d*x + c)}*\sqrt{-b}*\sin(d*x + c)/(b*\sqrt{\cos(d*x + c)})))*\cos(d*x + c)^2 - C*\sqrt{-b}*b*\cos(d*x + c)^2*\log(2*b*\cos(d*x + c)^2 - 2*\sqrt{b*\cos(d*x + c)}*\sqrt{-b}*\sqrt{\cos(d*x + c)}*\sin(d*x + c) - b) - 2*\sqrt{b*\cos(d*x + c)}*A*b*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^2), 1/2*(2*C*b^{(3/2)}*\arctan(\sqrt{b*\cos(d*x + c)}*\sin(d*x + c)/(\sqrt{b}*\cos(d*x + c)^{(3/2)}))*\cos(d*x + c)^2 + B*b^{(3/2)}*\cos(d*x + c)^2*\log(-b*\cos(d*x + c)^3 - 2*\sqrt{b*\cos(d*x + c)}*\sqrt{b}*\sqrt{\cos(d*x + c)}*\sin(d*x + c) - 2*b*\cos(d*x + c))/\cos(d*x + c)^3 + 2*\sqrt{b*\cos(d*x + c)}*A*b*\sqrt{\cos(d*x + c)}*\sin(d*x + c)/(d*\cos(d*x + c)^2)]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) (b \cos(dx+c))^{\frac{3}{2}}}{\cos(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(7/2), x)

$$3.303 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=114

$$\frac{b(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{bB \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0871232, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3021, 2748, 3767, 8, 3770}

$$\frac{b(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{bB \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (b*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{9/2}(c + dx)} dx = \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} + \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} + \frac{(bB\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{b(A + 2C) \tanh^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{2d\sqrt{\cos(c + dx)}} + \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)}$$

$$= \frac{b(A + 2C) \tanh^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{2d\sqrt{\cos(c + dx)}} + \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)}$$

Mathematica [A] time = 0.135879, size = 69, normalized size = 0.61

$$\frac{(b \cos(c + dx))^{3/2} (\sin(c + dx)(A + 2B \cos(c + dx)) + (A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{2d \cos^{7/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] ((b*Cos[c + d*x])^(3/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(7/2))

Maple [A] time = 0.263, size = 150, normalized size = 1.3

$$\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2), x)

[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-

$1+\cos(dx+c)/\sin(dx+c)+2*B*\sin(dx+c)*\cos(dx+c)+A*\sin(dx+c))*(b*\cos(dx+c))^{3/2}/\cos(dx+c)^{7/2}$

Maxima [B] time = 2.34683, size = 1098, normalized size = 9.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(3/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(9/2),x, algorithm="maxima")

[Out] $\frac{1}{4}*(2*(b*\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2*\sin(dx+c) + 1) - b*\log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2*\sin(dx+c) + 1))*C*\sqrt{b} + 8*B*b^{3/2}*\sin(2*d*x + 2*c)/(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1) - (4*(b*\sin(4*d*x + 4*c) + 2*b*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(b*\sin(4*d*x + 4*c) + 2*b*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - (b*\cos(4*d*x + 4*c)^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*d*x + 4*c)^2 + 4*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + 2*(2*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (b*\cos(4*d*x + 4*c)^2 + 4*b*\cos(2*d*x + 2*c)^2 + b*\sin(4*d*x + 4*c)^2 + 4*b*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b*\sin(2*d*x + 2*c)^2 + 2*(2*b*\cos(2*d*x + 2*c) + b)*\cos(4*d*x + 4*c) + 4*b*\cos(2*d*x + 2*c) + b)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(b*\cos(4*d*x + 4*c) + 2*b*\cos(2*d*x + 2*c) + b)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(b*\cos(4*d*x + 4*c) + 2*b*\cos(2*d*x + 2*c) + b)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*A*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1))/d$

Fricas [A] time = 1.99228, size = 662, normalized size = 5.81

$$\left[\frac{(A + 2C)b^{\frac{3}{2}} \cos(dx+c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2Bb \cos(dx+c) + Ab)\sqrt{b \cos(dx+c)}}{4d \cos(dx+c)^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(3/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(9/2),x, algorithm="fricas")

[Out] $[1/4*((A + 2*C)*b^{3/2}*\cos(dx+c)^3*\log(-(b*\cos(dx+c))^3 - 2*\sqrt{b*\cos(dx+c)}*\sqrt{\cos(dx+c)}*\sin(dx+c) - 2*b*\cos(dx+c))/\cos(dx+c)^3) + 2*(2*B*b*\cos(dx+c) + A*b)*\sqrt{b*\cos(dx+c)}*\sqrt{\cos(dx+c)}*\sin(dx+c)/(d*\cos(dx+c)^3), -1/2*((A + 2*C)*\sqrt{-b}*b*\arctan(\sqrt{b*\cos(dx+c)}*\sqrt{-b}*\sin(dx+c)/(b*\sqrt{\cos(dx+c)})))*\cos(dx+c)^3 - (2*B*b*\cos(dx+c) + A*b)*\sqrt{b*\cos(dx+c)}*\sqrt{\cos(dx+c)}*\sin(dx+c)/(d*\cos(dx+c)^3)]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(9/2), x)

$$3.304 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=156

$$\frac{b(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{bB \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{bB \sqrt{b \cos(c+dx)}}{2d}$$

[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.106262, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {17, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{bB \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{bB \sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]

[Out] (b*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{11/2}(c + dx)} dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{(b\sqrt{b \cos(c + dx)}) \int}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{(bB\sqrt{b \cos(c + dx)})}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{bB\sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} \\ &= \frac{bB \tanh^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{2d\sqrt{\cos(c + dx)}} + \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} \end{aligned}$$

Mathematica [A] time = 0.0547504, size = 88, normalized size = 0.56

$$\frac{b\sqrt{b \cos(c + dx)} (\tan(c + dx)((2A + 3C) \cos(2(c + dx)) + 4A + 3B \cos(c + dx) + 3C) + 3B \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{6d \cos^{5/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]

[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.275, size = 157, normalized size = 1.

$$\frac{1}{6d} \left(-3B \ln \left(\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 + 3B \ln \left(\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((b \cdot \cos(dx+c))^{3/2} \cdot (A+B \cdot \cos(dx+c)+C \cdot \cos(dx+c)^2) / \cos(dx+c)^{(11/2)}, x)$

[Out] $\frac{1}{6} \cdot d \cdot (-3 \cdot B \cdot \ln(-(-1+\cos(dx+c)+\sin(dx+c))/\sin(dx+c)) \cdot \cos(dx+c)^3 + 3 \cdot B \cdot \ln(-(-1+\cos(dx+c)-\sin(dx+c))/\sin(dx+c)) \cdot \cos(dx+c)^3 + 4 \cdot A \cdot \sin(dx+c) \cdot \cos(dx+c)^2 + 6 \cdot C \cdot \sin(dx+c) \cdot \cos(dx+c)^2 + 3 \cdot B \cdot \sin(dx+c) \cdot \cos(dx+c) + 2 \cdot A \cdot \sin(dx+c)) \cdot (b \cdot \cos(dx+c))^{3/2} / \cos(dx+c)^{(9/2)}$

Maxima [B] time = 2.36121, size = 1409, normalized size = 9.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((b \cdot \cos(dx+c))^{3/2} \cdot (A+B \cdot \cos(dx+c)+C \cdot \cos(dx+c)^2) / \cos(dx+c)^{(11/2)}, x, \text{algorithm}="maxima")$

[Out] $\frac{1}{12} \cdot (24 \cdot C \cdot b^{3/2} \cdot \sin(2 \cdot dx + 2 \cdot c) / (\cos(2 \cdot dx + 2 \cdot c)^2 + \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) - 16 \cdot (3 \cdot b \cdot \cos(6 \cdot dx + 6 \cdot c) \cdot \sin(2 \cdot dx + 2 \cdot c) + 9 \cdot b \cdot \cos(4 \cdot dx + 4 \cdot c) \cdot \sin(2 \cdot dx + 2 \cdot c) - (3 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \sin(6 \cdot dx + 6 \cdot c) - 3 \cdot (3 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \sin(4 \cdot dx + 4 \cdot c)) \cdot A \cdot \sqrt{b} / (2 \cdot (3 \cdot \cos(4 \cdot dx + 4 \cdot c) + 3 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \cos(6 \cdot dx + 6 \cdot c) + \cos(6 \cdot dx + 6 \cdot c)^2 + 6 \cdot (3 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \cos(4 \cdot dx + 4 \cdot c) + 9 \cdot \cos(4 \cdot dx + 4 \cdot c)^2 + 9 \cdot \cos(2 \cdot dx + 2 \cdot c)^2 + 6 \cdot (\sin(4 \cdot dx + 4 \cdot c) + \sin(2 \cdot dx + 2 \cdot c)) \cdot \sin(6 \cdot dx + 6 \cdot c) + \sin(6 \cdot dx + 6 \cdot c)^2 + 9 \cdot \sin(4 \cdot dx + 4 \cdot c)^2 + 18 \cdot \sin(4 \cdot dx + 4 \cdot c) \cdot \sin(2 \cdot dx + 2 \cdot c) + 9 \cdot \sin(2 \cdot dx + 2 \cdot c)^2 + 6 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) - 3 \cdot (4 \cdot (b \cdot \sin(4 \cdot dx + 4 \cdot c) + 2 \cdot b \cdot \sin(2 \cdot dx + 2 \cdot c)) \cdot \cos(3/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) - 4 \cdot (b \cdot \sin(4 \cdot dx + 4 \cdot c) + 2 \cdot b \cdot \sin(2 \cdot dx + 2 \cdot c)) \cdot \cos(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) - (b \cdot \cos(4 \cdot dx + 4 \cdot c)^2 + 4 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c)^2 + b \cdot \sin(4 \cdot dx + 4 \cdot c)^2 + 4 \cdot b \cdot \sin(4 \cdot dx + 4 \cdot c) \cdot \sin(2 \cdot dx + 2 \cdot c) + 4 \cdot b \cdot \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot (2 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \cos(4 \cdot dx + 4 \cdot c) + 4 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \log(\cos(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + \sin(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + 2 \cdot \sin(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 1) + (b \cdot \cos(4 \cdot dx + 4 \cdot c)^2 + 4 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c)^2 + b \cdot \sin(4 \cdot dx + 4 \cdot c)^2 + 4 \cdot b \cdot \sin(4 \cdot dx + 4 \cdot c) \cdot \sin(2 \cdot dx + 2 \cdot c) + 4 \cdot b \cdot \sin(2 \cdot dx + 2 \cdot c)^2 + 2 \cdot (2 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \cos(4 \cdot dx + 4 \cdot c) + 4 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \log(\cos(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 + \sin(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))^2 - 2 \cdot \sin(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 1) - 4 \cdot (b \cdot \cos(4 \cdot dx + 4 \cdot c) + 2 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \sin(3/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c))) + 4 \cdot (b \cdot \cos(4 \cdot dx + 4 \cdot c) + 2 \cdot b \cdot \cos(2 \cdot dx + 2 \cdot c) + b) \cdot \sin(1/2 \cdot \arctan2(\sin(2 \cdot dx + 2 \cdot c), \cos(2 \cdot dx + 2 \cdot c)))) \cdot B \cdot \sqrt{b} / (2 \cdot (2 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1) \cdot \cos(4 \cdot dx + 4 \cdot c) + \cos(4 \cdot dx + 4 \cdot c)^2 + 4 \cdot \cos(2 \cdot dx + 2 \cdot c)^2 + \sin(4 \cdot dx + 4 \cdot c)^2 + 4 \cdot \sin(4 \cdot dx + 4 \cdot c) \cdot \sin(2 \cdot dx + 2 \cdot c) + 4 \cdot \sin(2 \cdot dx + 2 \cdot c)^2 + 4 \cdot \cos(2 \cdot dx + 2 \cdot c) + 1)) / d$

Fricas [A] time = 2.04521, size = 741, normalized size = 4.75

$$\frac{3 B b^2 \cos(dx+c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2 \left(2(2A+3C)b \cos(dx+c)^2 + 3 B b\right)}{12 d \cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*b^(3/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3 + 2*(2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*b*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b*cos(d*x + c)^2 + 3*B*b*cos(d*x + c) + 2*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(11/2), x)
```

$$3.305 \quad \int \frac{(b \cos(c+dx))^{3/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx$$

Optimal. Leaf size=198

$$\frac{b(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))

Rubi [A] time = 0.124212, antiderivative size = 198, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3021, 2748, 3767, 3768, 3770}

$$\frac{b(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2), x]

[Out] (b*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x] * (b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^{3/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{13/2}(c + dx)} dx &= \frac{(b\sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{(b\sqrt{b \cos(c + dx)}) \int}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{(bB\sqrt{b \cos(c + dx)})}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab\sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{b(3A + 4C)\sqrt{b \cos(c + dx)}}{8d \cos^{5/2}(c + dx)} \\ &= \frac{b(3A + 4C) \tanh^{-1}(\sin(c + dx))\sqrt{b \cos(c + dx)}}{8d\sqrt{\cos(c + dx)}} + \frac{Ab}{\sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.24995, size = 111, normalized size = 0.56

$$\frac{b\sqrt{b \cos(c + dx)} (\sin(c + dx) (3(3A + 4C) \cos^2(c + dx) + 6A + 24B \cos^3(c + dx) + 8B \sin^2(c + dx) \cos(c + dx)) + 3(3A + 4C))}{24d \cos^{9/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2), x]

[Out] (b*Sqrt[b*Cos[c + d*x]]*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(9/2))

Maple [A] time = 0.331, size = 248, normalized size = 1.3

$$\frac{1}{24d} \left(9A \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^4 - 9A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
x)
```

```
[Out] 1/24/d*(9*A*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-9*A*cos
(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)
)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*sin(d*x+c)*cos(d*
x+c)^2+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c
))*(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 2.77528, size = 3688, normalized size = 18.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4
*c) + 4*b*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c
) + 4*b*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 44*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c)
+ 4*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 12*(b*sin(8*d*x + 8*c) + 4*b*sin(6*d*x + 6*c) + 6*b*sin(4*d*x + 4*c) +
4*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)
^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)
^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*
b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b
*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d
*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x
+ 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c)
+ 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) +
3*(b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*x + 4*c)^
2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*x + 6*c)^
2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b
*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) + 4*b*cos(
2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)*cos(4*d*
x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*sin(4*d*x +
4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x + 4*c) +
2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*log(cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) -
12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*b*
cos(2*d*x + 2*c) + b)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) + 4*
b*cos(2*d*x + 2*c) + b)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 44*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 12*(b*cos(8*d*x + 8*c) + 4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*c)
+ 4*b*cos(2*d*x + 2*c) + b)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
```



```

*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) +
4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*c
os(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*
x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)
^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2
*d*x + 2*c) + 1) + 64*(3*b*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) + 9*b*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c) - (3*b*cos(2*d*x + 2*c) + b)*sin(6*d*x + 6*c) - 3
*(3*b*cos(2*d*x + 2*c) + b)*sin(4*d*x + 4*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4
*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*
cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x
+ 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(
6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) + 12*(4*(b*sin(4*d*x + 4
*c) + 2*b*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) - 4*(b*sin(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - (b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2
*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*
sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos
(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (b*cos(4*d*x + 4*c)^2 + 4
*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x
+ 4*c) + 4*b*cos(2*d*x + 2*c) + b)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b*cos(
4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 4*(b*cos(4*d*x + 4*c) + 2*b*cos(2*d*x + 2*c) + b)*sin(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(b)/(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2
*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d

```

Fricas [A] time = 2.09039, size = 834, normalized size = 4.21

$$\left[\frac{3(3A + 4C)b^{\frac{3}{2}} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(16Bb \cos(dx + c)^3 + 48d \cos(dx + c)^5}{48d \cos(dx + c)^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="fricas")

```

```

[Out] [1/48*(3*(3*A + 4*C)*b^(3/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)
^2 + 8*B*b*cos(d*x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b*arctan(sqrt
(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)
^5 - (16*B*b*cos(d*x + c)^3 + 3*(3*A + 4*C)*b*cos(d*x + c)^2 + 8*B*b*cos(d*
x + c) + 6*A*b)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos
(d*x + c)^5)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{3}{2}}}{\cos(dx + c)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(3/2)/cos(d*x + c)^(13/2), x)

3.306 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2} (A + B \cos(c+dx) + C \cos(c+dx)^2) dx$

Optimal. Leaf size=241

$$\frac{b^2(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{b^2(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2B\sin^2(c+dx)\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}$$

```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(5*A + 4*C)*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt
[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*
x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*C*Cos[c + d*x]^(7/
2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b^2*(5*A + 4*C)*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.127851, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2633, 2635, 8}

$$\frac{b^2(5A+4C)\sin^3(c+dx)\sqrt{b\cos(c+dx)}}{15d\sqrt{\cos(c+dx)}} + \frac{b^2(5A+4C)\sin(c+dx)\sqrt{b\cos(c+dx)}}{5d\sqrt{\cos(c+dx)}} + \frac{3b^2Bx\sqrt{b\cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2B\sin^2(c+dx)\sqrt{b\cos(c+dx)}}{8d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2), x]
```

```
[Out] (3*b^2*B*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*(5*A + 4*C)*
Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (3*b^2*B*Sqrt
[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*B*Cos[c + d*
x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) + (b^2*C*Cos[c + d*x]^(7/
2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(5*d) - (b^2*(5*A + 4*C)*Sqrt[b*Cos[c
+ d*x]]*Sin[c + d*x]^3)/(15*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_)
+ (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c + dx)} (b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int \cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 C \cos^2(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d} \\ &= \frac{b^2 C \cos^2(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d} \\ &= \frac{b^2 B \cos^2(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} \\ &= \frac{b^2 (5A + 4C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} + \frac{3b^2 C \cos^2(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} \\ &= \frac{3b^2 B x \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{b^2 (5A + 4C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{5d \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.31914, size = 109, normalized size = 0.45

$$\frac{(b \cos(c + dx))^{5/2} (60(6A + 5C) \sin(c + dx) + 40A \sin(3(c + dx)) + 120B \sin(2(c + dx)) + 15B \sin(4(c + dx)) + 180Bc + 180Bd)}{480d \cos^2(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C
*Cos[c + d*x]^2), x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(180*B*c + 180*B*d*x + 60*(6*A + 5*C)*Sin[c + d*x]
+ 120*B*Ssin[2*(c + d*x)] + 40*A*Ssin[3*(c + d*x)] + 50*C*Ssin[3*(c + d*x)] +
15*B*Ssin[4*(c + d*x)] + 6*C*Ssin[5*(c + d*x)]))/(480*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.303, size = 134, normalized size = 0.6

$$\frac{24C (\cos(dx + c))^4 \sin(dx + c) + 30B (\cos(dx + c))^3 \sin(dx + c) + 40A \sin(dx + c) (\cos(dx + c))^2 + 32C \sin(dx + c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)
```

```
[Out] 1/120/d*(b*cos(d*x+c))^(5/2)*(24*C*cos(d*x+c)^4*sin(d*x+c)+30*B*cos(d*x+c)^3*sin(d*x+c)+40*A*sin(d*x+c)*cos(d*x+c)^2+32*C*sin(d*x+c)*cos(d*x+c)^2+45*B*sin(d*x+c)*cos(d*x+c)+80*A*sin(d*x+c)+45*B*(d*x+c)+64*sin(d*x+c)*C)/cos(d*x+c)^(5/2)
```

Maxima [A] time = 2.31939, size = 250, normalized size = 1.04

$$40 \left(b^2 \sin(3dx + 3c) + 9b^2 \sin\left(\frac{1}{3} \arctan(\sin(3dx + 3c), \cos(3dx + 3c))\right) \right) A\sqrt{b} + 15 \left(12(dx + c)b^2 + b^2 \sin(4dx + 4c) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/480*(40*(b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*A*sqrt(b) + 15*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*B*sqrt(b) + 2*(3*b^2*sin(5*d*x + 5*c) + 25*b^2*sin(3/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c)))) + 150*b^2*sin(1/5*arctan2(sin(5*d*x + 5*c), cos(5*d*x + 5*c))))*C*sqrt(b))/d
```

Fricas [A] time = 2.12293, size = 883, normalized size = 3.66

$$\left[\frac{45B\sqrt{-bb^2} \cos(dx + c) \log\left(2b \cos(dx + c)^2 - 2\sqrt{b} \cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b\right) + 2\left(24Cb^2 \cos(dx + c) \log\left(\frac{\sqrt{b}\cos(dx + c)\sin(dx + c)}{\sqrt{b}\cos(dx + c)}\right) + 24Cb^2 \cos(dx + c)\right)}{\dots} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/240*(45*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(24*C*b^2*cos(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x + c)^2 + 45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)), 1/120*(45*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (24*C*b^2*cos(d*x + c)^4 + 30*B*b^2*cos(d*x + c)^3 + 8*(5*A + 4*C)*b^2*cos(d*x + c)^2 + 45*B*b^2*cos(d*x + c) + 16*(5*A + 4*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

$$3.307 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=199

$$\frac{b^2 x(4A+3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C) \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d} - \frac{b^2 B \sin^3(c+dx)\sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rubi [A] time = 0.112684, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2635, 8, 2633}

$$\frac{b^2 x(4A+3C)\sqrt{b \cos(c+dx)}}{8\sqrt{\cos(c+dx)}} + \frac{b^2(4A+3C) \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{8d} - \frac{b^2 B \sin^3(c+dx)\sqrt{b \cos(c+dx)}}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (b^2*(4*A + 3*C)*x*Sqrt[b*Cos[c + d*x]])/(8*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*(4*A + 3*C)*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d) + (b^2*C*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d) - (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 C \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{(b^2 \sqrt{b \cos(c + dx)}) \int \cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 C \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{(b^2 B \sqrt{b \cos(c + dx)}) \int \cos(c + dx)}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 (4A + 3C) \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)} \sin(c + dx)}{8d} + \frac{(b^2 B \sqrt{b \cos(c + dx)}) \int \cos(c + dx)}{\sqrt{\cos(c + dx)}} \\ &= \frac{b^2 (4A + 3C) x \sqrt{b \cos(c + dx)}}{8 \sqrt{\cos(c + dx)}} + \frac{b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.284591, size = 92, normalized size = 0.46

$$\frac{(b \cos(c + dx))^{5/2} (24(A + C) \sin(2(c + dx)) + 48Ac + 48Adx + 72B \sin(c + dx) + 8B \sin(3(c + dx)) + 3C \sin(4(c + dx)))}{96d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/
Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c
+ d*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c
+ d*x)]))/(96*d*Cos[c + d*x]^(5/2))
```

Maple [A] time = 0.509, size = 114, normalized size = 0.6

$$\frac{6C(\cos(dx + c))^3 \sin(dx + c) + 8B \sin(dx + c) (\cos(dx + c))^2 + 12A \cos(dx + c) \sin(dx + c) + 9C \cos(dx + c) \sin(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x
)
```



```
[Out] 1/24/d*(b*cos(d*x+c))^(5/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos
(d*x+c)^2+12*A*cos(d*x+c)*sin(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)
+16*B*sin(d*x+c)+9*C*(d*x+c))/cos(d*x+c)^(5/2)
```

Maxima [A] time = 2.29901, size = 189, normalized size = 0.95

$$\frac{24(2(dx+c)b^2 + b^2 \sin(2dx+2c))A\sqrt{b} + 8(b^2 \sin(3dx+3c) + 9b^2 \sin(\frac{1}{3} \arctan(\sin(3dx+3c), \cos(3dx+3c))))B\sqrt{b} + 3(12(dx+c)b^2 + b^2 \sin(4dx+4c) + 8b^2 \sin(\frac{1}{2} \arctan(2(\sin(4dx+4c), \cos(4dx+4c))))C\sqrt{b}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="maxima")
```

```
[Out] 1/96*(24*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*A*sqrt(b) + 8*(b^2*sin(3*
d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*B*
sqrt(b) + 3*(12*(d*x + c)*b^2 + b^2*sin(4*d*x + 4*c) + 8*b^2*sin(1/2*arctan
2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C*sqrt(b))/d
```

Fricas [A] time = 2.04767, size = 802, normalized size = 4.03

$$\left[\frac{3(4A + 3C)\sqrt{-bb^2} \cos(dx+c) \log(2b \cos(dx+c)^2 - 2\sqrt{b \cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)} \sin(dx+c) - b) + 2(6C \cos(dx+c) \sqrt{-bb^2} \sin(dx+c) - b)}{48d \cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
1/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(4*A + 3*C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt
(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(6*C*
b^2*cos(d*x + c)^3 + 8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c)
) + 16*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)), 1/24*(3*(4*A + 3*C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*b^2*cos(d*x + c)^3 +
8*B*b^2*cos(d*x + c)^2 + 3*(4*A + 3*C)*b^2*cos(d*x + c) + 16*B*b^2)*sqrt(b
*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{5}{2}}}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/sqrt(cos(d*x + c)), x)
```

$$3.308 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^3(c+dx)} dx$$

Optimal. Leaf size=155

$$\frac{b^2(3A+2C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{b^2 B x \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{b^2 B \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} + \frac{b^2 C \sin^2(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}$$

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*(3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b^2*C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rubi [A] time = 0.0607352, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {17, 3023, 2734}

$$\frac{b^2(3A+2C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}} + \frac{b^2 B x \sqrt{b \cos(c+dx)}}{2 \sqrt{\cos(c+dx)}} + \frac{b^2 B \sin(c+dx) \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}{2d} + \frac{b^2 C \sin^2(c+dx) \sqrt{b \cos(c+dx)}}{3d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]
```

```
[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*(3*A + 2*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d) + (b^2*C*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 17

```
Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2734

```
Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^3(c + dx)} dx = \frac{(b^2 \sqrt{b \cos(c + dx)}) \int \cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ = \frac{b^2 C \cos^3(c + dx) \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d} + \frac{(b^2 \sqrt{b \cos(c + dx)}) \int \cos(c + dx) (A + B \cos(c + dx))}{\sqrt{\cos(c + dx)}} \\ = \frac{b^2 B x \sqrt{b \cos(c + dx)}}{2 \sqrt{\cos(c + dx)}} + \frac{b^2 (3A + 2C) \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.247861, size = 75, normalized size = 0.48

$$\frac{(b \cos(c + dx))^{5/2} (3(4A + 3C) \sin(c + dx) + 3B \sin(2(c + dx)) + 6Bc + 6Bdx + C \sin(3(c + dx)))}{12d \cos^5(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2))/cos[c + d*x]^(3/2),x]

[Out] ((b*cos[c + d*x])^(5/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Sin[2*(c + d*x)] + C*Sin[3*(c + d*x)]))/(12*d*cos[c + d*x]^(5/2))

Maple [A] time = 0.325, size = 83, normalized size = 0.5

$$\frac{2C \sin(dx + c) (\cos(dx + c))^2 + 3B \sin(dx + c) \cos(dx + c) + 6A \sin(dx + c) + 3B(dx + c) + 4 \sin(dx + c) C}{6d} (b \cos(dx + c))^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)

[Out] 1/6/d*(b*cos(d*x+c))^(5/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/cos(d*x+c)^(5/2)

Maxima [A] time = 2.29283, size = 127, normalized size = 0.82

$$\frac{12 A b^5 \sin(dx + c) + 3 \left((dx + c) b^2 + b^2 \sin(2dx + 2c) \right) B \sqrt{b} + \left(b^2 \sin(3dx + 3c) + 9 b^2 \sin\left(\frac{1}{3} \arctan(\sin(3dx + 3c))\right) \right) C \sqrt{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="maxima")

[Out] 1/12*(12*A*b^(5/2)*sin(d*x + c) + 3*(2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*B*sqrt(b) + (b^2*sin(3*d*x + 3*c) + 9*b^2*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*C*sqrt(b))/d

Fricas [A] time = 1.99412, size = 703, normalized size = 4.54

$$\frac{3 B \sqrt{-b b^2} \cos (d x+c) \log \left(2 b \cos (d x+c)^2-2 \sqrt{b} \cos (d x+c) \sqrt{-b} \sqrt{\cos (d x+c)} \sin (d x+c)-b\right)+2\left(2 C b^2 \cos (d x+c)\right)}{12 d \cos (d x+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")

[Out] [1/12*(3*B*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(2*C*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/6*(3*B*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*(3*A + 2*C)*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos (d x+c)^2+B \cos (d x+c)+A)(b \cos (d x+c))^{\frac{5}{2}}}{\cos (d x+c)^{\frac{3}{2}}} d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(3/2), x)

$$3.309 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=135

$$\frac{Ab^2x\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rubi [A] time = 0.0374911, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2637, 2635, 8}

$$\frac{Ab^2x\sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d\sqrt{\cos(c+dx)}} + \frac{b^2Cx\sqrt{b \cos(c+dx)}}{2\sqrt{\cos(c+dx)}} + \frac{b^2C \sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}{2d}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (A*b^2*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*C*x*Sqrt[b*Cos[c + d*x]])/(2*Sqrt[Cos[c + d*x]]) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d)

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_.)])^(n_.), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Sin[c + d*x])^(n - 1)/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ = \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{(b^2 B \sqrt{b \cos(c + dx)}) \int \cos(c + dx)}{\sqrt{\cos(c + dx)}} \\ = \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} \\ = \frac{Ab^2 x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 C x \sqrt{b \cos(c + dx)}}{2 \sqrt{\cos(c + dx)}} + \frac{b^2 B \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.147127, size = 61, normalized size = 0.45

$$\frac{(b \cos(c + dx))^{5/2} (2(2A + C)(c + dx) + 4B \sin(c + dx) + C \sin(2(c + dx)))}{4d \cos^{\frac{5}{2}}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(5/2),x]

[Out] ((b*Cos[c + d*x])^(5/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*d*Cos[c + d*x]^(5/2))

Maple [A] time = 0.277, size = 63, normalized size = 0.5

$$\frac{C \cos(dx + c) \sin(dx + c) + 2A(dx + c) + 2B \sin(dx + c) + C(dx + c)}{2d} (b \cos(dx + c))^{\frac{5}{2}} (\cos(dx + c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x)

[Out] 1/2/d*(b*cos(d*x+c))^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))/cos(d*x+c)^(5/2)

Maxima [A] time = 2.14864, size = 96, normalized size = 0.71

$$\frac{8Ab^{\frac{5}{2}} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + 4Bb^{\frac{5}{2}} \sin(dx+c) + (2(dx+c)b^2 + b^2 \sin(2dx+2c))C\sqrt{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] 1/4*(8*A*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*B*b^(5/2)*sin(d*x + c) + (2*(d*x + c)*b^2 + b^2*sin(2*d*x + 2*c))*C*sqrt(b))/d

Fricas [A] time = 1.97039, size = 618, normalized size = 4.58

$$\left[\frac{(2A + C)\sqrt{-bb^2} \cos(dx + c) \log\left(2b \cos(dx + c)^2 - 2\sqrt{b \cos(dx + c)}\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b\right) + 2(Cb^2 \cos(dx + c) + 2Bb^2)\sqrt{b \cos(dx + c)}\sqrt{\cos(dx + c)}\sin(dx + c)}{4d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] [1/4*((2*A + C)*sqrt(-b)*b^2*cos(d*x + c)*log(2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) + 2*(C*b^2*cos(d*x + c) + 2*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)), 1/2*((2*A + C)*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*b^2*cos(d*x + c) + 2*B*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(5/2), x)

$$3.310 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^2(c+dx)} dx$$

Optimal. Leaf size=102

$$\frac{Ab^2 \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{b^2 B x \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2 C \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rubi [A] time = 0.0608191, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3023, 2735, 3770}

$$\frac{Ab^2 \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{b^2 B x \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}} + \frac{b^2 C \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(7/2), x]

[Out] (b^2*B*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (A*b^2*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (b^2*C*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{7/2}(c + dx)} dx = \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} + \frac{b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} + \frac{b^2 B x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 C \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \sqrt{\cos(c + dx)}} + \frac{b^2 B x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{A b^2 \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.148828, size = 93, normalized size = 0.91

$$\frac{(b \cos(c + dx))^{5/2} \left(-A \log \left(\cos \left(\frac{1}{2}(c + dx) \right) - \sin \left(\frac{1}{2}(c + dx) \right) \right) + A \log \left(\sin \left(\frac{1}{2}(c + dx) \right) + \cos \left(\frac{1}{2}(c + dx) \right) \right) + Bc + Bdx + C \cos^2(c + dx) \right)}{d \cos^{7/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^(5/2)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2))/cos[c + d*x]^(7/2),x]

[Out] ((b*cos[c + d*x])^(5/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*cos[c + d*x]^(5/2))

Maple [A] time = 0.253, size = 63, normalized size = 0.6

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - B(dx + c) - \sin(dx + c) C \right) (b \cos(dx + c))^{5/2} (\cos(dx + c))^{-5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(5/2)

Maxima [A] time = 2.0743, size = 150, normalized size = 1.47

$$\frac{4 B b^{5/2} \arctan \left(\frac{\sin(dx+c)}{\cos(dx+c)+1} \right) + 2 C b^{5/2} \sin(dx+c) + (b^2 \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - b^2 \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2),x, algorithm="maxima")

[Out] 1/2*(4*B*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 2*C*b^(5/2)*sin(d*x + c) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) -

$b^2 \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2\sin(dx + c) + 1) \cdot A \sqrt{b} / d$

Fricas [A] time = 2.45099, size = 880, normalized size = 8.63

$$\frac{2 A \sqrt{-b} b^2 \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c) - B \sqrt{-b} b^2 \cos(dx+c) \log\left(2 b \cos(dx+c)^2 - 2 \sqrt{b \cos(dx+c)}\right)}{2 d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(7/2),x, algorithm="fricas")

[Out] [-1/2*(2*A*sqrt(-b)*b^2*arctan(sqrt(b*cos(dx+c))*sqrt(-b)*sin(dx+c)/(b*sqrt(cos(dx+c))))*cos(dx+c) - B*sqrt(-b)*b^2*cos(dx+c)*log(2*b*cos(dx+c)^2 - 2*sqrt(b*cos(dx+c))*sqrt(-b)*sqrt(cos(dx+c))*sin(dx+c) - b) - 2*sqrt(b*cos(dx+c))*C*b^2*sqrt(cos(dx+c))*sin(dx+c))/(d*cos(dx+c)), 1/2*(2*B*b^(5/2)*arctan(sqrt(b*cos(dx+c))*sin(dx+c)/(sqrt(b)*cos(dx+c)^(3/2)))*cos(dx+c) + A*b^(5/2)*cos(dx+c)*log(-b*cos(dx+c)^3 - 2*sqrt(b*cos(dx+c))*sqrt(b)*sqrt(cos(dx+c))*sin(dx+c) - 2*b*cos(dx+c))/cos(dx+c)^3 + 2*sqrt(b*cos(dx+c))*C*b^2*sqrt(cos(dx+c))*sin(dx+c)/(d*cos(dx+c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))**(5/2)*(A+B*cos(dx+c)+C*cos(dx+c)**2)/cos(dx+c)**(7/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) (b \cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(7/2),x, algorithm="giac")

[Out] integrate((C*cos(dx+c)^2 + B*cos(dx+c) + A)*(b*cos(dx+c))^(5/2)/cos(dx+c)^(7/2), x)

$$3.311 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{9}{2}}(c+dx)} dx$$

Optimal. Leaf size=102

$$\frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 B \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{b^2 C x \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0639631, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3021, 2735, 3770}

$$\frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)} + \frac{b^2 B \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d \sqrt{\cos(c+dx)}} + \frac{b^2 C x \sqrt{b \cos(c+dx)}}{\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2), x]

[Out] (b^2*C*x*Sqrt[b*Cos[c + d*x]])/Sqrt[Cos[c + d*x]] + (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{9/2}(c + dx)} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\
&= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{3/2}(c + dx)} + \frac{(b^2 \sqrt{b \cos(c + dx)})}{d \cos^{3/2}(c + dx)} \\
&= \frac{b^2 C x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{d \cos^{3/2}(c + dx)} \\
&= \frac{b^2 C x \sqrt{b \cos(c + dx)}}{\sqrt{\cos(c + dx)}} + \frac{b^2 B \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{d \sqrt{\cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.1219, size = 60, normalized size = 0.59

$$\frac{(b \cos(c + dx))^{5/2} (A \sin(c + dx) + B \cos(c + dx) \tanh^{-1}(\sin(c + dx)) + C dx \cos(c + dx))}{d \cos^{7/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(9/2),x]

[Out] ((b*Cos[c + d*x])^(5/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*Cos[c + d*x]^(7/2))

Maple [A] time = 0.25, size = 72, normalized size = 0.7

$$\frac{1}{d} (b \cos(dx + c))^{5/2} \left(-2B \cos(dx + c) \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) + C \cos(dx + c) (dx + c) + A \sin(dx + c) \right) (\cos(dx + c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x)

[Out] 1/d*(b*cos(d*x+c))^(5/2)*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c)))+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(7/2)

Maxima [A] time = 2.12526, size = 204, normalized size = 2.

$$\frac{4Cb^{\frac{5}{2}} \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right) + \frac{4Ab^{\frac{5}{2}} \sin(2dx+2c)}{\cos(2dx+2c)^2 + \sin(2dx+2c)^2 + 2\cos(2dx+2c)+1} + (b^2 \log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c)))}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2),x, algorithm="maxima")

```
[Out] 1/2*(4*C*b^(5/2)*arctan(sin(d*x + c)/(cos(d*x + c) + 1)) + 4*A*b^(5/2)*sin(
2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1) + (b^2*log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - b^2
*log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))*B*sqrt(b))/d
```

Fricas [A] time = 2.43867, size = 896, normalized size = 8.78

$$\frac{2B\sqrt{-bb^2} \arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 - C\sqrt{-bb^2} \cos(dx+c)^2 \log\left(2b\cos(dx+c)^2 - 2\sqrt{b\cos(dx+c)}\right)}{2d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(
b*sqrt(cos(d*x + c))))*cos(d*x + c)^2 - C*sqrt(-b)*b^2*cos(d*x + c)^2*log(2
*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(
d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2), 1/2*(2*C*b^(5/2)*arctan(sqrt(b*cos(d*x + c))*sin(d*x
+ c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c)^2 + B*b^(5/2)*cos(d*x + c)
^2*log(-(b*cos(d*x + c)^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c
))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*sqrt(b*cos(d*x + c)
)*A*b^2*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)
**(9/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A)(b \cos(dx+c))^{\frac{5}{2}}}{\cos(dx+c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
9/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/co
s(d*x + c)^(9/2), x)
```

$$3.312 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{11}{2}}(c+dx)} dx$$

Optimal. Leaf size=120

$$\frac{b^2(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.0969051, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3021, 2748, 3767, 8, 3770}

$$\frac{b^2(A+2C)\sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx)\sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B \sin(c+dx)\sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2), x]

[Out] (b^2*(A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]]/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_) + (d_)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,

d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{11/2}(c + dx)} dx = \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} + \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} + \frac{(b^2 B \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{b^2(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}}$$

$$= \frac{b^2(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.138371, size = 69, normalized size = 0.57

$$\frac{(b \cos(c + dx))^{5/2} (\sin(c + dx)(A + 2B \cos(c + dx)) + (A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{2d \cos^{9/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(11/2),x]

[Out] ((b*Cos[c + d*x])^(5/2)*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*Cos[c + d*x]^(9/2))

Maple [A] time = 0.262, size = 151, normalized size = 1.3

$$-\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2), x)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((

$-1+\cos(dx+c)/\sin(dx+c)-2B\sin(dx+c)\cos(dx+c)-A\sin(dx+c))(b\cos(dx+c))^{5/2}/\cos(dx+c)^{9/2}$

Maxima [B] time = 2.37679, size = 1179, normalized size = 9.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(11/2),x, algorithm="maxima")

[Out] $\frac{1}{4}*(8*B*b^{5/2}*\sin(2*d*x + 2*c)/(\cos(2*d*x + 2*c)^2 + \sin(2*d*x + 2*c)^2 + 2*\cos(2*d*x + 2*c) + 1) + 2*(b^2*\log(\cos(dx + c)^2 + \sin(dx + c)^2 + 2*\sin(dx + c) + 1) - b^2*\log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2*\sin(dx + c) + 1))*C*\sqrt{b} - (4*(b^2*\sin(4*d*x + 4*c) + 2*b^2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(b^2*\sin(4*d*x + 4*c) + 2*b^2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))) - (b^2*\cos(4*d*x + 4*c)^2 + 4*b^2*\cos(2*d*x + 2*c)^2 + b^2*\sin(4*d*x + 4*c)^2 + 4*b^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b^2*\sin(2*d*x + 2*c)^2 + 4*b^2*\cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*\cos(2*d*x + 2*c) + b^2)*\cos(4*d*x + 4*c))*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (b^2*\cos(4*d*x + 4*c)^2 + 4*b^2*\cos(2*d*x + 2*c)^2 + b^2*\sin(4*d*x + 4*c)^2 + 4*b^2*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*b^2*\sin(2*d*x + 2*c)^2 + 4*b^2*\cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*\cos(2*d*x + 2*c) + b^2)*\cos(4*d*x + 4*c))*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(b^2*\cos(4*d*x + 4*c) + 2*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(b^2*\cos(4*d*x + 4*c) + 2*b^2*\cos(2*d*x + 2*c) + b^2)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*A*\sqrt{b}/(2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1))/d$

Fricas [A] time = 2.02197, size = 675, normalized size = 5.62

$$\left[\frac{(A + 2C)b^{\frac{5}{2}} \cos(dx + c)^3 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2Bb^2 \cos(dx + c) + Ab^2)}{4d \cos(dx + c)^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(dx+c))^(5/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(11/2),x, algorithm="fricas")

[Out] $[1/4*((A + 2C)*b^{5/2}*\cos(dx + c)^3*\log(-(b*\cos(dx + c))^3 - 2*\sqrt{b*\cos(dx + c)}*\sqrt{b*\cos(dx + c)}*\sin(dx + c) - 2*b*\cos(dx + c))/\cos(dx + c)^3) + 2*(2*B*b^2*\cos(dx + c) + A*b^2)*\sqrt{b*\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c)/(d*\cos(dx + c)^3), -1/2*((A + 2C)*\sqrt{-b}*b^2*\arctan(\sqrt{b*\cos(dx + c)}*\sqrt{-b}*\sin(dx + c)/(b*\sqrt{\cos(dx + c)})))*\cos(dx + c)^3 - (2*B*b^2*\cos(dx + c) + A*b^2)*\sqrt{b*\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c)/(d*\cos(dx + c)^3)]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(11/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{11}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(11/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(11/2), x)

$$3.313 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{13}{2}}(c+dx)} dx$$

Optimal. Leaf size=164

$$\frac{b^2(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{b^2B \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^2*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rubi [A] time = 0.110016, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {17, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{b^2(2A+3C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{3}{2}}(c+dx)} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{3d \cos^{\frac{7}{2}}(c+dx)} + \frac{b^2B \sin(c+dx) \sqrt{b \cos(c+dx)}}{2d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2B \sqrt{b \cos(c+dx)}}{d \cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2), x]

[Out] (b^2*B*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(2*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(7/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)) + (b^2*(2*A + 3*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_.)]*(b_.))^(n_.), x_Symbol] :> -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{13/2}(c + dx)} dx &= \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{(b^2 B \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} \\ &= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} + \frac{b^2 B \sqrt{b \cos(c + dx)} \sin(c + dx)}{2d \cos^{5/2}(c + dx)} \\ &= \frac{b^2 B \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{2d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{3d \cos^{7/2}(c + dx)} \end{aligned}$$

Mathematica [A] time = 0.421793, size = 87, normalized size = 0.53

$$\frac{(b \cos(c + dx))^{5/2} (\tan(c + dx)((2A + 3C) \cos(2(c + dx)) + 4A + 3B \cos(c + dx) + 3C) + 3B \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{6d \cos^{9/2}(c + dx)}$$

Antiderivative was successfully verified.

[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(13/2), x]

[Out] ((b*Cos[c + d*x])^(5/2)*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*Cos[c + d*x]^(9/2))

Maple [A] time = 0.293, size = 157, normalized size = 1.

$$\frac{1}{6d} \left(-3B \ln \left(\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 + 3B \ln \left(\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),
x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(
-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x
+c)^2+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))
*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(11/2)
```

Maxima [B] time = 2.43436, size = 1501, normalized size = 9.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
13/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*b^(5/2)*sin(2*d*x + 2*c)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1) - 16*(3*b^2*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) +
9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*c) + b^2)*s
in(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4*c))*A*sqrt
(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos
(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x
+ 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1) -
3*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
+ 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (b^2*cos(4*
d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*
x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*s
in(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c)
+ b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(b^2*cos(4*d*x +
4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(b)/(2*(2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^
2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x
+ 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 1.99659, size = 760, normalized size = 4.63

$$\frac{3 B b^{\frac{5}{2}} \cos(dx+c)^4 \log\left(-\frac{b \cos(dx+c)^3 - 2 \sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c) - 2 b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2 \left(2 (2 A + 3 C) b^2 \cos(dx+c)^2 + \dots\right)}{12 d \cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*b^(5/2)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*(2*A + 3*C)*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*b^2*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c))/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*b^2*cos(d*x + c)^2 + 3*B*b^2*cos(d*x + c) + 2*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(13/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{13}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(13/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(13/2), x)
```

$$3.314 \quad \int \frac{(b \cos(c+dx))^{5/2} (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{15}{2}}(c+dx)} dx$$

Optimal. Leaf size=208

$$\frac{b^2(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

```
[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rubi [A] time = 0.125884, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3021, 2748, 3767, 3768, 3770}

$$\frac{b^2(3A+4C) \sin(c+dx) \sqrt{b \cos(c+dx)}}{8d \cos^{\frac{5}{2}}(c+dx)} + \frac{b^2(3A+4C) \sqrt{b \cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{\cos(c+dx)}} + \frac{Ab^2 \sin(c+dx) \sqrt{b \cos(c+dx)}}{4d \cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2), x]
```

```
[Out] (b^2*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[b*Cos[c + d*x]])/(8*d*Sqrt[Cos[c + d*x]]) + (A*b^2*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)) + (b^2*(3*A + 4*C)*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(8*d*Cos[c + d*x]^(5/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x])/(d*Cos[c + d*x]^(3/2)) + (b^2*B*Sqrt[b*Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(7/2))
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3021

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(b \cos(c + dx))^{5/2} (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{15/2}(c + dx)} dx = \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{(b^2 \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{(b^2 B \sqrt{b \cos(c + dx)}) \int (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx$$

$$= \frac{Ab^2 \sqrt{b \cos(c + dx)} \sin(c + dx)}{4d \cos^{9/2}(c + dx)} + \frac{b^2(3A + 4C) \sqrt{b \cos(c + dx)}}{8d \cos^{5/2}(c + dx)}$$

$$= \frac{b^2(3A + 4C) \tanh^{-1}(\sin(c + dx)) \sqrt{b \cos(c + dx)}}{8d \sqrt{\cos(c + dx)}} + \frac{Ab^2 \sqrt{b \cos(c + dx)}}{4d \cos^{9/2}(c + dx)}$$

Mathematica [A] time = 0.3762, size = 110, normalized size = 0.53

$$\frac{(b \cos(c + dx))^{5/2} (\sin(c + dx) (3(3A + 4C) \cos^2(c + dx) + 6A + 24B \cos^3(c + dx) + 8B \sin^2(c + dx) \cos(c + dx)) + 3(3A + 4C) \cos(c + dx))}{24d \cos^{13/2}(c + dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Cos[c + d*x])^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(15/2),x]
```

```
[Out] ((b*Cos[c + d*x])^(5/2)*(3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2)))/(24*d*Cos[c + d*x]^(13/2))
```

Maple [A] time = 0.269, size = 248, normalized size = 1.2

$$-\frac{1}{24d} \left(9A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) - 9A \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) \right) (\cos(dx + c))^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.


```
[In] int((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),
x)
```

```
[Out] -1/24/d*(9*A*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-9*A*ln
(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4+12*C*cos(d*x+c)^4*ln(
-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-12*C*ln(-(-1+cos(d*x+c)-sin(d*x+c))
/sin(d*x+c))*cos(d*x+c)^4-16*B*cos(d*x+c)^3*sin(d*x+c)-9*A*sin(d*x+c)*cos(d
*x+c)^2-12*C*sin(d*x+c)*cos(d*x+c)^2-8*B*sin(d*x+c)*cos(d*x+c)-6*A*sin(d*x+
c))*(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(13/2)
```

Maxima [B] time = 2.78234, size = 4012, normalized size = 19.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*sin(4*d
*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6*b^2*s
in(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 44*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*c) + 6
*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 12*(b^2*sin(8*d*x + 8*c) + 4*b^2*sin(6*d*x + 6*
c) + 6*b^2*sin(4*d*x + 4*c) + 4*b^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*
d*x + 6*c)^2 + 36*b^2*cos(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*
sin(8*d*x + 8*c)^2 + 16*b^2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2
+ 48*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x +
4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x
+ 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d
*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4
*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*
x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
+ 1) + 3*(b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*cos(
4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*b^
2*sin(6*d*x + 6*c)^2 + 36*b^2*sin(4*d*x + 4*c)^2 + 48*b^2*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 16*b^2*sin(2*d*x + 2*c)^2 + 8*b^2*cos(2*d*x + 2*c) + b^2
+ 2*(4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2
*c) + b^2)*cos(8*d*x + 8*c) + 8*(6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x +
2*c) + b^2)*cos(6*d*x + 6*c) + 12*(4*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x
+ 4*c) + 4*(2*b^2*sin(6*d*x + 6*c) + 3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 12*(b^2*cos(8*d*x
+ 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x
+ 2*c) + b^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(b^
2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x + 4*c) + 4*b^
2*cos(2*d*x + 2*c) + b^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 44*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2*cos(4*d*x +
4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos
```

```
(2*d*x + 2*c))) + 12*(b^2*cos(8*d*x + 8*c) + 4*b^2*cos(6*d*x + 6*c) + 6*b^2
*cos(4*d*x + 4*c) + 4*b^2*cos(2*d*x + 2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))*A*sqrt(b)/(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x
+ 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8
*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*
d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x
+ 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*
c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(
4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2
+ 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*
d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1) + 64*(3*b^2*cos(6*d*x + 6*c)*sin(2*d
*x + 2*c) + 9*b^2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - (3*b^2*cos(2*d*x + 2*
c) + b^2)*sin(6*d*x + 6*c) - 3*(3*b^2*cos(2*d*x + 2*c) + b^2)*sin(4*d*x + 4
*c))*B*sqrt(b)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1) + 12*(4*(b^2*sin(4*d*x + 4*c) + 2*b^2*sin(2*d*x + 2*c))*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(b^2*sin(4*d*x + 4*c) + 2*b
^2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
(b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^
2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*
b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4
*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) + 1) + (b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*
x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos
(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*log(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(b^2
*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x + 2*c) + b^2)*sin(3/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) + 4*(b^2*cos(4*d*x + 4*c) + 2*b^2*cos(2*d*x +
2*c) + b^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C*sqrt(b)/
(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2
*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1))/d
```

Fricas [A] time = 2.10289, size = 859, normalized size = 4.13

$$\frac{3(3A + 4C)b^{\frac{5}{2}} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(16Bb^2 \cos(dx + c)^3 + 3(3A + 4C)b^2 \cos(dx + c)^2 + 8Bb^2 \cos(dx + c) + 6A b^2) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) - 2b \cos(dx + c)}{\cos(dx + c)^3} + 2(16Bb^2 \cos(dx + c)^3 + 3(3A + 4C)b^2 \cos(dx + c)^2 + 8Bb^2 \cos(dx + c) + 6A b^2) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) - 2b \cos(dx + c)}{48d \cos(dx + c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(
15/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*b^(5/2)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x
+ c)^2 + 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c)/(d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*b^2*a
rctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*co
s(d*x + c)^5 - (16*B*b^2*cos(d*x + c)^3 + 3*(3*A + 4*C)*b^2*cos(d*x + c)^2
```

+ 8*B*b^2*cos(d*x + c) + 6*A*b^2)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(15/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{5}{2}}}{\cos(dx + c)^{\frac{15}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(15/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(5/2)/cos(d*x + c)^(15/2), x)

$$3.315 \quad \int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=184

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b \cos(c+dx)}} + \frac{(4A+3C) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b \cos(c+dx)}} - \frac{B \sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}}$$

[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.139417, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2635, 8, 2633}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8\sqrt{b \cos(c+dx)}} + \frac{(4A+3C) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{8d\sqrt{b \cos(c+dx)}} - \frac{B \sin^3(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(m_))*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx = \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4d \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx))}{4 \sqrt{b \cos(c + dx)}} dx$$

$$= \frac{C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4d \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \cos^3(c + dx)}{\sqrt{b \cos(c + dx)}} dx$$

$$= \frac{(4A + 3C) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{8d \sqrt{b \cos(c + dx)}} + \frac{C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4d \sqrt{b \cos(c + dx)}} + \frac{B \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + \frac{(4A + 3C)x \sqrt{\cos(c + dx)}}{8 \sqrt{b \cos(c + dx)}} + \frac{B \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + \dots$$

Mathematica [A] time = 0.221741, size = 92, normalized size = 0.5

$$\frac{\sqrt{\cos(c + dx)}(24(A + C) \sin(2(c + dx)) + 48Ac + 48Adx + 72B \sin(c + dx) + 8B \sin(3(c + dx)) + 3C \sin(4(c + dx)))}{96d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt
[b*Cos[c + d*x]], x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x]]))/(96*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.524, size = 114, normalized size = 0.6

$$\frac{6 C (\cos(dx + c))^3 \sin(dx + c) + 8 B \sin(dx + c) (\cos(dx + c))^2 + 12 A \cos(dx + c) \sin(dx + c) + 9 C \cos(dx + c) \sin(dx + c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x)

[Out] 1/24/d*cos(d*x+c)^(1/2)*(6*C*cos(d*x+c)^3*sin(d*x+c)+8*B*sin(d*x+c)*cos(d*x+c)^2+12*A*cos(d*x+c)*sin(d*x+c)+9*C*cos(d*x+c)*sin(d*x+c)+12*A*(d*x+c)+16*B*sin(d*x+c)+9*C*(d*x+c))/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.33505, size = 157, normalized size = 0.85

$$\frac{24(2dx+2c+\sin(2dx+2c))A}{\sqrt{b}} + \frac{3(12dx+12c+\sin(4dx+4c)+8\sin(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))))C}{\sqrt{b}} + \frac{8B(\sin(3dx+3c)+9\sin(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))))}{\sqrt{b}}$$

96d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/96*(24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A/sqrt(b) + 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c))))*C/sqrt(b) + 8*B*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b))/d

Fricas [A] time = 2.08674, size = 760, normalized size = 4.13

$$\left[\frac{3(4A + 3C)\sqrt{-b}\cos(dx + c)\log(2b\cos(dx + c)^2 + 2\sqrt{b}\cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)}\sin(dx + c) - b) - 2(6C\cos(dx + c) + 8B\sin(dx + c))\sqrt{-b}\sqrt{\cos(dx + c)}}{48bd\cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/48*(3*(4*A + 3*C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)), 1/24*(3*(4*A + 3*C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (6*C*cos(d*x + c)^3 + 8*B*cos(d*x + c)^2 + 3*(4*A + 3*C)*cos(d*x + c) + 16*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{\sqrt{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*cos(d*x + c)), x)

$$3.316 \quad \int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx$$

Optimal. Leaf size=143

$$\frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\cos^3(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^5(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.062233, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {17, 3023, 2734}

$$\frac{(3A+2C)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\cos^3(c+dx)}{2d\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\cos^5(c+dx)}{3d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2734

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} \\ = \frac{C \cos^5(c+dx) \sin(c+dx)}{3d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)}{3\sqrt{b\cos(c+dx)}} \\ = \frac{Bx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)} \sin(c+dx)}{3d\sqrt{b\cos(c+dx)}} +$$

Mathematica [A] time = 0.178772, size = 75, normalized size = 0.52

$$\frac{\sqrt{\cos(c+dx)}(3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+6Bc+6Bdx+C\sin(3(c+dx)))}{12d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Ssin[2*(c + d*x)] + C*Ssin[3*(c + d*x)]))/(12*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.489, size = 83, normalized size = 0.6

$$\frac{2C\sin(dx+c)(\cos(dx+c))^2 + 3B\sin(dx+c)\cos(dx+c) + 6A\sin(dx+c) + 3B(dx+c) + 4\sin(dx+c)C}{6d} \sqrt{\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/6/d*cos(d*x+c)^(1/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.24654, size = 108, normalized size = 0.76

$$\frac{\frac{3(2dx+2c+\sin(2dx+2c))B}{\sqrt{b}} + \frac{C\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan\left(\frac{\sin(3dx+3c)}{\cos(3dx+3c)}\right)\right)\right)}{\sqrt{b}}}{12d} + \frac{12A\sin(dx+c)}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/sqrt(b) + C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/sqrt(b) + 12*A*sin(d*x + c)/sqrt(b))/d

Fricas [A] time = 2.22087, size = 662, normalized size = 4.63

$$\left[\frac{3B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2\left(2C\cos(dx+c)\right)^2}{12bd\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + B\cos(dx+c) + A)\cos(dx+c)^{\frac{3}{2}}}{\sqrt{b\cos(dx+c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*cos(d*x + c)), x)

$$3.317 \quad \int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=123

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0330262, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2637, 2635, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]],x]

[Out] (A*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (C*x*Sqrt[Cos[c + d*x]])/(2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt{b\cos(c+dx)}} dx = \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)+C\cos^2(c+dx)) dx}{\sqrt{b\cos(c+dx)}} \\ = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{(B\sqrt{\cos(c+dx)}) \int \cos(c+dx) dx}{\sqrt{b\cos(c+dx)}} + \frac{C \int \cos^2(c+dx) dx}{\sqrt{b\cos(c+dx)}} \\ = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{b\cos(c+dx)}} + \frac{C \cos^3(c+dx)}{2d\sqrt{b\cos(c+dx)}} \\ = \frac{Ax\sqrt{\cos(c+dx)}}{\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0882564, size = 61, normalized size = 0.5

$$\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[b*Cos[c + d*x]], x]

[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.452, size = 63, normalized size = 0.5

$$\frac{C \cos(dx+c) \sin(dx+c) + 2A(dx+c) + 2B \sin(dx+c) + C(dx+c)}{2d} \sqrt{\cos(dx+c)} \frac{1}{\sqrt{b \cos(dx+c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/2/d*cos(d*x+c)^(1/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.28292, size = 86, normalized size = 0.7

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))C}{\sqrt{b}} + \frac{8A \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{4B \sin(dx+c)}{\sqrt{b}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/sqrt(b) + 8*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/sqrt(b) + 4*B*sin(d*x + c)/sqrt(b))/d

Fricas [A] time = 2.02116, size = 598, normalized size = 4.86

$$\left[\frac{(2A + C)\sqrt{-b} \cos(dx + c) \log\left(2b \cos(dx + c)^2 + 2\sqrt{b} \cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b\right) - 2(C \cos(dx + c) + 2B)\sqrt{b} \cos(dx + c)}{4bd \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)\sqrt{\cos(dx + c)}}{\sqrt{b} \cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*cos(d*x + c)), x)

$$3.318 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=93

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}}$$

[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0576249, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {18, 3023, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]), x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}}$$

$$= \frac{C \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx)) \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}}$$

$$= \frac{Bx \sqrt{\cos(c + dx)}}{\sqrt{b \cos(c + dx)}} + \frac{C \sqrt{\cos(c + dx)} \sin(c + dx)}{d \sqrt{b \cos(c + dx)}} + \frac{(A \sqrt{\cos(c + dx)}) \int \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}}$$

$$= \frac{Bx \sqrt{\cos(c + dx)}}{\sqrt{b \cos(c + dx)}} + \frac{A \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}} + \frac{C \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.0959353, size = 93, normalized size = 1.

$$\frac{\sqrt{\cos(c + dx)} \left(-A \log \left(\cos \left(\frac{1}{2}(c + dx) \right) - \sin \left(\frac{1}{2}(c + dx) \right) \right) + A \log \left(\sin \left(\frac{1}{2}(c + dx) \right) + \cos \left(\frac{1}{2}(c + dx) \right) \right) + Bc + Bdx + C \right)}{d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]),x]

[Out] (Sqrt[Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.441, size = 63, normalized size = 0.7

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) - B(dx + c) - \sin(dx + c) C \right) \sqrt{\cos(dx + c)} \frac{1}{\sqrt{b \cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.12963, size = 140, normalized size = 1.51

$$\frac{A(\log(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1)-\log(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1))}{\sqrt{b}} + \frac{4B \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{2C \sin(dx+c)}{\sqrt{b}}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*B*arctan(si

$n(dx + c)/(\cos(dx + c) + 1)/\sqrt{b} + 2C\sin(dx + c)/\sqrt{b})/d$

Fricas [A] time = 2.41314, size = 864, normalized size = 9.29

$$\left[\frac{2A\sqrt{-b} \arctan\left(\frac{\sqrt{b}\cos(dx+c)\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c) + B\sqrt{-b} \cos(dx+c) \log\left(2b\cos(dx+c)^2 + 2\sqrt{b}\cos(dx+c)\sqrt{-b}\right)}{2bd\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(1/2)/(b*cos(dx+c))^(1/2),x, algorithm="fricas")

[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c) + B*sqrt(-b)*cos(dx + c)*log(2*b*cos(dx + c)^2 + 2*sqrt(b*cos(dx + c))*sqrt(-b)*sqrt(cos(dx + c))*sin(dx + c) - b) - 2*sqrt(b*cos(dx + c))*C*sqrt(cos(dx + c))*sin(dx + c))/(b*d*cos(dx + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(dx + c))*sin(dx + c)/(sqrt(b)*cos(dx + c)^(3/2)))*cos(dx + c) + A*sqrt(b)*cos(dx + c)*log(-(b*cos(dx + c)^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3) + 2*sqrt(b*cos(dx + c))*C*sqrt(cos(dx + c))*sin(dx + c))/(b*d*cos(dx + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)**2)/cos(dx+c)**(1/2)/(b*cos(dx+c))**1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(1/2)/(b*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + B*cos(dx + c) + A)/(sqrt(b*cos(dx + c))*sqrt(cos(dx + c))), x)

$$3.319 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{3 \cos^2(c+dx)\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=93

$$\frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}}$$

[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0732515, antiderivative size = 93, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {18, 3021, 2735, 3770}

$$\frac{A \sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{d\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] (C*x*Sqrt[Cos[c + d*x]])/Sqrt[b*Cos[c + d*x]] + (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned}
\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^3(c + dx) \sqrt{b \cos(c + dx)}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\
&= \frac{A \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (B + C \cos(c + dx)) \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\
&= \frac{Cx \sqrt{\cos(c + dx)}}{\sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\
&= \frac{Cx \sqrt{\cos(c + dx)}}{\sqrt{b \cos(c + dx)}} + \frac{B \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}
\end{aligned}$$

Mathematica [A] time = 0.070765, size = 60, normalized size = 0.65

$$\frac{A \sin(c + dx) + B \cos(c + dx) \tanh^{-1}(\sin(c + dx)) + C dx \cos(c + dx)}{d \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.421, size = 72, normalized size = 0.8

$$\frac{1}{d} \left(-2B \cos(dx + c) \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) + C \cos(dx + c) (dx + c) + A \sin(dx + c) \right) \frac{1}{\sqrt{\cos(dx + c)}} \frac{1}{\sqrt{b \cos(dx + c)}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x)

[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(1/2)

Maxima [A] time = 2.15017, size = 201, normalized size = 2.16

$$\frac{B(\log(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1)-\log(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1))}{\sqrt{b}} + \frac{4C \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{\sqrt{b}} + \frac{4A\sqrt{b}\sin(2c)}{b\cos(2dx+2c)^2+b\sin(2dx+2c)}$$

2d

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*(B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/sqrt(b) + 4*C*arctan(si

$n(dx + c)/(\cos(dx + c) + 1)/\sqrt{b} + 4A\sqrt{b}\sin(2dx + 2c)/(b\cos(2dx + 2c)^2 + b\sin(2dx + 2c)^2 + 2b\cos(2dx + 2c) + b)/d$

Fricas [A] time = 2.41093, size = 880, normalized size = 9.46

$$\frac{2B\sqrt{-b} \arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 + C\sqrt{-b} \cos(dx+c)^2 \log\left(2b\cos(dx+c)^2 + 2\sqrt{b\cos(dx+c)}\right)}{2bd\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(3/2)/(b*cos(dx+c))^(1/2),x, algorithm="fricas")

[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(dx+c))*sqrt(-b)*sin(dx+c)/(b*sqrt(cos(dx+c))))*cos(dx+c)^2 + C*sqrt(-b)*cos(dx+c)^2*log(2*b*cos(dx+c)^2 + 2*sqrt(b*cos(dx+c))*sqrt(-b)*sqrt(cos(dx+c))*sin(dx+c) - b) - 2*sqrt(b*cos(dx+c))*A*sqrt(cos(dx+c))*sin(dx+c)/(b*d*cos(dx+c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(dx+c))*sin(dx+c)/(sqrt(b*cos(dx+c))^(3/2)))*cos(dx+c)^2 + B*sqrt(b)*cos(dx+c)^2*log(-b*cos(dx+c)^3 - 2*sqrt(b*cos(dx+c))*sqrt(b)*sqrt(cos(dx+c))*sin(dx+c) - 2*b*cos(dx+c))/cos(dx+c)^3 + 2*sqrt(b*cos(dx+c))*A*sqrt(cos(dx+c))*sin(dx+c)/(b*d*cos(dx+c)^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)**2)/cos(dx+c)**(3/2)/(b*cos(dx+c))**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(3/2)/(b*cos(dx+c))^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(dx+c)^2 + B*cos(dx+c) + A)/(sqrt(b*cos(dx+c))*cos(dx+c)^(3/2)), x)

$$3.320 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{5 \cos^2(c+dx) \sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=111

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{d\sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*SIN[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*SIN[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0951999, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {18, 3021, 2748, 3767, 8, 3770}

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{d\sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*SIN[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*SIN[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*SIN[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] :> Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (2B + (A + 2C) \cos(c + dx)) \sec^2(c + dx) dx}{2\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{\sqrt{b \cos(c + dx)}} + \frac{C \int \sec^4(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \\ &= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.088331, size = 69, normalized size = 0.62

$$\frac{\sin(c + dx)(A + 2B \cos(c + dx)) + (A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx))}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.448, size = 150, normalized size = 1.4

$$\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2), x)

[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(1/2)

Maxima [B] time = 2.32784, size = 1060, normalized size = 9.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")

[Out]
$$\frac{1}{4} \cdot (2C \cdot (\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2\sin(dx+c) + 1)) / \sqrt{b} + 8B\sqrt{b} \cdot \sin(2dx+2c) / (b\cos(2dx+2c)^2 + b\sin(2dx+2c)^2 + 2b\cos(2dx+2c) + b) - (4(\sin(4dx+4c) + 2\sin(2dx+2c)) \cdot \cos(3/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c)))) - 4(\sin(4dx+4c) + 2\sin(2dx+2c)) \cdot \cos(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c)))) - (2(2\cos(2dx+2c) + 1) \cdot \cos(4dx+4c) + \cos(4dx+4c)^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c) \cdot \sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1) \cdot \log(\cos(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c))))^2 + \sin(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c))))^2 + 2\sin(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c)))) + 1) + (2(2\cos(2dx+2c) + 1) \cdot \cos(4dx+4c) + \cos(4dx+4c)^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c) \cdot \sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1) \cdot \log(\cos(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c))))^2 + \sin(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c))))^2 - 2\sin(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c)))) + 1) - 4(\cos(4dx+4c) + 2\cos(2dx+2c) + 1) \cdot \sin(3/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c)))) + 4(\cos(4dx+4c) + 2\cos(2dx+2c) + 1) \cdot \sin(1/2 \arctan(2(\sin(2dx+2c)/\cos(2dx+2c)))) \cdot A / ((2(2\cos(2dx+2c) + 1) \cdot \cos(4dx+4c) + \cos(4dx+4c)^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c) \cdot \sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1) \cdot \sqrt{b})) / d$$

Fricas [A] time = 2.06991, size = 653, normalized size = 5.88

$$\frac{(A + 2C)\sqrt{b} \cos(dx+c)^3 \log\left(\frac{-b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2B \cos(dx+c) + A)\sqrt{b} \cos(dx+c)}{4bd \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")

[Out]
$$\left[\frac{1}{4} \cdot ((A + 2C) \cdot \sqrt{b} \cdot \cos(dx+c)^3 \cdot \log(-b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c) \sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)) / \cos(dx+c)^3 + 2(2B \cos(dx+c) + A) \cdot \sqrt{b} \cdot \cos(dx+c) \sqrt{\cos(dx+c)} \cdot \sin(dx+c) / (b \cdot d \cdot \cos(dx+c)^3), -1/2 \cdot ((A + 2C) \cdot \sqrt{-b} \cdot \arctan(\sqrt{b \cos(dx+c)} \sqrt{-b} \cdot \sin(dx+c) / (b \cdot \sqrt{\cos(dx+c)}))) \cdot \cos(dx+c)^3 - (2B \cos(dx+c) + A) \cdot \sqrt{b \cos(dx+c)} \cdot \sqrt{\cos(dx+c)} \cdot \sin(dx+c) / (b \cdot d \cdot \cos(dx+c)^3) \right]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(5/2)), x)
```

$$3.321 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^2(c+dx)\sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=152

$$\frac{(2A+3C) \sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \tan(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.118751, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {18, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2A+3C) \sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \tan(c+dx)}{2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_.)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_.)]^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (3B + (2A + 3C) \cos(c + dx)) \sec^3(c + dx) dx}{3\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(B\sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{\sqrt{b \cos(c + dx)}} + \frac{C \int \sec^5(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(B\sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{B \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{3d \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{2d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.175263, size = 87, normalized size = 0.57

$$\frac{\tan(c + dx)((2A + 3C) \cos(2(c + dx)) + 4A + 3B \cos(c + dx) + 3C) + 3B \cos^2(c + dx) \tanh^{-1}(\sin(c + dx))}{6d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]),x]

[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)]*Tan[c + d*x])/(6*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.479, size = 157, normalized size = 1.

$$\frac{1}{6d} \left(-3B \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 + 3B \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{int}((A+B*\cos(d*x+c)+C*\cos(d*x+c)^2)/\cos(d*x+c)^{(7/2)}/(b*\cos(d*x+c))^{(1/2)},x)$

[Out] $1/6/d*(-3*B*\ln(-(-1+\cos(d*x+c)+\sin(d*x+c))/\sin(d*x+c))*\cos(d*x+c)^3+3*B*\ln(-(-1+\cos(d*x+c)-\sin(d*x+c))/\sin(d*x+c))*\cos(d*x+c)^3+4*A*\sin(d*x+c)*\cos(d*x+c)^2+6*C*\sin(d*x+c)*\cos(d*x+c)^2+3*B*\sin(d*x+c)*\cos(d*x+c)+2*A*\sin(d*x+c))/\cos(d*x+c)^{(5/2)}/(b*\cos(d*x+c))^{(1/2)}$

Maxima [B] time = 2.37698, size = 1369, normalized size = 9.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] $\text{integrate}((A+B*\cos(d*x+c)+C*\cos(d*x+c)^2)/\cos(d*x+c)^{(7/2)}/(b*\cos(d*x+c))^{(1/2)},x, \text{algorithm}="maxima")$

[Out] $1/12*(24*C*\sqrt{b}*\sin(2*d*x + 2*c)/(b*\cos(2*d*x + 2*c)^2 + b*\sin(2*d*x + 2*c)^2 + 2*b*\cos(2*d*x + 2*c) + b) + 16*((3*\cos(2*d*x + 2*c) + 1)*\sin(6*d*x + 6*c) + 3*(3*\cos(2*d*x + 2*c) + 1)*\sin(4*d*x + 4*c) - 3*\cos(6*d*x + 6*c)*\sin(2*d*x + 2*c) - 9*\cos(4*d*x + 4*c)*\sin(2*d*x + 2*c))*A/((2*(3*\cos(4*d*x + 4*c) + 3*\cos(2*d*x + 2*c) + 1)*\cos(6*d*x + 6*c) + \cos(6*d*x + 6*c)^2 + 6*(3*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + 9*\cos(4*d*x + 4*c)^2 + 9*\cos(2*d*x + 2*c)^2 + 6*(\sin(4*d*x + 4*c) + \sin(2*d*x + 2*c))*\sin(6*d*x + 6*c) + \sin(6*d*x + 6*c)^2 + 9*\sin(4*d*x + 4*c)^2 + 18*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 9*\sin(2*d*x + 2*c)^2 + 6*\cos(2*d*x + 2*c) + 1)*\sqrt{b}) - 3*(4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - 4*(\sin(4*d*x + 4*c) + 2*\sin(2*d*x + 2*c))*\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) - (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) + (2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\log(\cos(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 + \sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c)))^2 - 2*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 1) - 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(3/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))) + 4*(\cos(4*d*x + 4*c) + 2*\cos(2*d*x + 2*c) + 1)*\sin(1/2*\arctan2(\sin(2*d*x + 2*c), \cos(2*d*x + 2*c))))*B/((2*(2*\cos(2*d*x + 2*c) + 1)*\cos(4*d*x + 4*c) + \cos(4*d*x + 4*c)^2 + 4*\cos(2*d*x + 2*c)^2 + \sin(4*d*x + 4*c)^2 + 4*\sin(4*d*x + 4*c)*\sin(2*d*x + 2*c) + 4*\sin(2*d*x + 2*c)^2 + 4*\cos(2*d*x + 2*c) + 1)*\sqrt{b}))$

Fricas [A] time = 2.23572, size = 728, normalized size = 4.79

$$\frac{3B\sqrt{b}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b}\cos(dx+c)\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2(2A+3C)\cos(dx+c)^2+3B\cos(dx+c)\right)}{12bd\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b*d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos(d*x + c)^(7/2)), x)
```

$$3.322 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{9 \cos^2(c+dx) \sqrt{b \cos(c+dx)}} dx$$

Optimal. Leaf size=193

$$\frac{(3A+4C) \sin(c+dx)}{8d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{(3A+4C) \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4d \cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.117843, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {18, 3021, 2748, 3767, 3768, 3770}

$$\frac{(3A+4C) \sin(c+dx)}{8d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{(3A+4C) \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4d \cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{3d \cos^{\frac{5}{2}}(c+dx) \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]), x]

[Out] ((3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(8*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]]) + ((3*A + 4*C)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x]^3)/(3*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{9}{2}}(c + dx) \sqrt{b \cos(c + dx)}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx}{\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (4B + (3A + 4C) \cos(c + dx)) \sec^4(c + dx) dx}{4\sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(B\sqrt{\cos(c + dx)}) \int \sec^4(c + dx) dx}{\sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \sin(c + dx)}{8d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \\ &= \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \sin(c + dx)}{8d \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{8d \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{4d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.219793, size = 110, normalized size = 0.57

$$\frac{\sin(c + dx) \left(3(3A + 4C) \cos^2(c + dx) + 6A + 24B \cos^3(c + dx) + 8B \sin^2(c + dx) \cos(c + dx) \right) + 3(3A + 4C) \cos^4(c + dx)}{24d \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(9/2)*Sqrt[b*Cos[c + d*x]]), x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2))/(24*d*Cos[c + d*x]^(7/2)*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.523, size = 248, normalized size = 1.3

$$\frac{1}{24d} \left(9A \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^4 - 9A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x)
```

```
[Out] 1/24/d*(9*A*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-9*A*cos
(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)
)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*sin(d*x+c)*cos(d*
x+c)^2+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c
))/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(1/2)
```

Maxima [B] time = 2.51991, size = 3525, normalized size = 18.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
+ 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*A/((2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c)
) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*co
```

```

s(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*
*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 +
48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x
+ 2*c) + 1)*sqrt(b)) - 64*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3
*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*
c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*B/((2*(3*cos(4*d*x + 4*c) + 3*cos
(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c) + 9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 +
6*(sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c
)^2 + 9*sin(4*d*x + 4*c)^2 + 18*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2
*d*x + 2*c)^2 + 6*cos(2*d*x + 2*c) + 1)*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c)
+ 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
- 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(
4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(
cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + co
s(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*lo
g(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*s
in(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) +
2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*C/((2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*sqrt(b)))/d

```

Fricas [A] time = 2.37677, size = 815, normalized size = 4.22

$$\left[\frac{3(3A + 4C)\sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)}\sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(16B \cos(dx + c)^3 + 3A \cos(dx + c)^2 + 8B \cos(dx + c) + 6A) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) - 2b \cos(dx + c)}{\cos(dx + c)^3} + 2(16B \cos(dx + c)^3 + 3A \cos(dx + c)^2 + 8B \cos(dx + c) + 6A) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c) - 2b \cos(dx + c)}{\cos(dx + c)^3} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

```

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="fricas")

```

```

[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c))^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(d
*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (16
*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A)*
sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b*d*cos(d*x + c)^5)]

```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(9/2)/(b*cos(d*x+c))
**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{\sqrt{b \cos(dx + c)} \cos(dx + c)^{\frac{9}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(9/2)/(b*cos(d*x+c))^(
1/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(sqrt(b*cos(d*x + c))*cos
(d*x + c)^(9/2)), x)

$$3.323 \quad \int \frac{\cos^7(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=199

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^3(c+dx)}{8bd\sqrt{b\cos(c+dx)}} - \frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.109347, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2635, 8, 2633}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b\sqrt{b\cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^3(c+dx)}{8bd\sqrt{b\cos(c+dx)}} - \frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3bd\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*b*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{b \sqrt{b \cos(c + dx)}} \\ = \frac{C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4bd \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{4b \sqrt{b \cos(c + dx)}} \\ = \frac{C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4bd \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \cos^3(c + dx)}{b \sqrt{b \cos(c + dx)}} \\ = \frac{(4A + 3C) \cos^{\frac{3}{2}}(c + dx) \sin(c + dx)}{8bd \sqrt{b \cos(c + dx)}} + \frac{C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4bd \sqrt{b \cos(c + dx)}} \\ = \frac{(4A + 3C)x \sqrt{\cos(c + dx)}}{8b \sqrt{b \cos(c + dx)}} + \frac{B \sqrt{\cos(c + dx)} \sin(c + dx)}{bd \sqrt{b \cos(c + dx)}} + \frac{4C \cos^{\frac{7}{2}}(c + dx) \sin(c + dx)}{4bd \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.178611, size = 92, normalized size = 0.46

$$\frac{\cos^{\frac{3}{2}}(c + dx) (24(A + C) \sin(2(c + dx)) + 48Ac + 48Adx + 72B \sin(c + dx) + 8B \sin(3(c + dx)) + 3C \sin(4(c + dx)) + 36C \cos^2(c + dx))}{96d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(3/2), x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.373, size = 114, normalized size = 0.6

$$\frac{6C(\cos(dx + c))^3 \sin(dx + c) + 8B \sin(dx + c) (\cos(dx + c))^2 + 12A \cos(dx + c) \sin(dx + c) + 9C \cos(dx + c) \sin(dx + c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x
)
```

[Out] $\frac{1}{24}d\cos(dx+c)^{(3/2)}*(6*C*\cos(dx+c)^3*\sin(dx+c)+8*B*\sin(dx+c)*\cos(dx+c)^2+12*A*\cos(dx+c)*\sin(dx+c)+9*C*\cos(dx+c)*\sin(dx+c)+12*A*(dx+c)+16*B*\sin(dx+c)+9*C*(dx+c))/(b*\cos(dx+c))^{(3/2)}$

Maxima [A] time = 2.42629, size = 157, normalized size = 0.79

$$\frac{\frac{24(2dx+2c+\sin(2dx+2c))A}{b^{\frac{3}{2}}} + \frac{3\left(12dx+12c+\sin(4dx+4c)+8\sin\left(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))\right)\right)C}{b^{\frac{3}{2}}} + \frac{8B\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))\right)\right)}{b^{\frac{3}{2}}}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(3/2),x, algorithm="maxima")

[Out] $\frac{1}{96}*(24*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A/b^{(3/2)} + 3*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))*C/b^{(3/2)} + 8*B*(\sin(3*d*x + 3*c) + 9*\sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))/b^{(3/2)})/d$

Fricas [A] time = 2.52065, size = 765, normalized size = 3.84

$$\frac{3(4A + 3C)\sqrt{-b}\cos(dx+c)\log(2b\cos(dx+c)^2 + 2\sqrt{b}\cos(dx+c)\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c) - b) - 2(6C\cos(dx+c)^3 + 8B\cos(dx+c)^2 + 3(4A + 3C)\cos(dx+c) + 16B)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{48b^2d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(7/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(3/2),x, algorithm="fricas")

[Out] $[-1/48*(3*(4*A + 3*C)*\sqrt{-b}*\cos(dx+c)*\log(2*b*\cos(dx+c)^2 + 2*\sqrt{b}*\cos(dx+c)*\sqrt{-b}*\sqrt{\cos(dx+c)}*\sin(dx+c) - b) - 2*(6*C*\cos(dx+c)^3 + 8*B*\cos(dx+c)^2 + 3*(4*A + 3*C)*\cos(dx+c) + 16*B)*\sqrt{b*\cos(dx+c)}*\sqrt{\cos(dx+c)}*\sin(dx+c))/(b^2*d*\cos(dx+c)), 1/24*(3*(4*A + 3*C)*\sqrt{b}*\arctan(\sqrt{b*\cos(dx+c)}*\sin(dx+c)/(\sqrt{b}*\cos(dx+c)^{(3/2)}))*\cos(dx+c) + (6*C*\cos(dx+c)^3 + 8*B*\cos(dx+c)^2 + 3*(4*A + 3*C)*\cos(dx+c) + 16*B)*\sqrt{b*\cos(dx+c)}*\sqrt{\cos(dx+c)}*\sin(dx+c))/(b^2*d*\cos(dx+c))]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**(7/2)*(A+B*cos(dx+c)+C*cos(dx+c)**2)/(b*cos(dx+c))** (3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^{\frac{7}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(3/2), x)
```

$$3.324 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=155

$$\frac{(3A+2C) \sin(c+dx) \sqrt{\cos(c+dx)}}{3bd \sqrt{b \cos(c+dx)}} + \frac{Bx \sqrt{\cos(c+dx)}}{2b \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^3(c+dx)}{2bd \sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^5(c+dx)}{3bd \sqrt{b \cos(c+dx)}}$$

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rubi [A] time = 0.0600347, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {17, 3023, 2734}

$$\frac{(3A+2C) \sin(c+dx) \sqrt{\cos(c+dx)}}{3bd \sqrt{b \cos(c+dx)}} + \frac{Bx \sqrt{\cos(c+dx)}}{2b \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^3(c+dx)}{2bd \sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^5(c+dx)}{3bd \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
d*x])^(3/2), x]
```

```
[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c
+ d*x])/(3*b*d*Sqrt[b*Cos[c + d*x]])
```

Rule 17

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/
2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b
, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rule 2734

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_.)]), x_Symbol] :> Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Co
s[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{\frac{3}{2}}} dx = \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{b\sqrt{b\cos(c+dx)}}$$

$$= \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)(3A+B\cos(c+dx))}{3b\sqrt{b\cos(c+dx)}}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{2b\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3bd\sqrt{b\cos(c+dx)}} + \dots$$

Mathematica [A] time = 0.169384, size = 75, normalized size = 0.48

$$\frac{\cos^{\frac{3}{2}}(c+dx)(3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+6Bc+6Bdx+C\sin(3(c+dx)))}{12d(b\cos(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (Cos[c + d*x]^(3/2)*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*SIN[2*(c + d*x)] + C*SIN[3*(c + d*x)]))/(12*d*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.313, size = 83, normalized size = 0.5

$$\frac{2C\sin(dx+c)(\cos(dx+c))^2 + 3B\sin(dx+c)\cos(dx+c) + 6A\sin(dx+c) + 3B(dx+c) + 4\sin(dx+c)C}{6d} (\cos(dx+c))$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/6/d*cos(d*x+c)^(3/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.33386, size = 108, normalized size = 0.7

$$\frac{\frac{3(2dx+2c+\sin(2dx+2c))B}{b^{\frac{3}{2}}} + \frac{C(\sin(3dx+3c)+9\sin(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))))}{b^{\frac{3}{2}}} + \frac{12A\sin(dx+c)}{b^{\frac{3}{2}}}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(3/2) + C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(3/2) + 12*A*sin(d*x + c)/b^(3/2))/d

Fricas [A] time = 2.05172, size = 667, normalized size = 4.3

$$\left[\frac{3 B \sqrt{-b} \cos(dx + c) \log\left(2 b \cos(dx + c)^2 + 2 \sqrt{b \cos(dx + c)} \sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b\right) - 2\left(2 C \cos(dx + c) + 3 B \cos(dx + c) + 6 A + 4 C\right) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c)}{12 b^2 d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(3/2), x)

$$3.325 \quad \int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{\frac{3}{2}}} dx$$

Optimal. Leaf size=135

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b \cos(c+dx)}}$$

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0325222, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2637, 2635, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*SIN[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*SIN[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{\frac{3}{2}}} dx = \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)+C\cos^2(c+dx)) dx}{b\sqrt{b}\cos(c+dx)}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b}\cos(c+dx)} + \frac{(B\sqrt{\cos(c+dx)}) \int \cos(c+dx) dx}{b\sqrt{b}\cos(c+dx)} + \frac{C \int \cos^2(c+dx) dx}{2bd\sqrt{b}\cos(c+dx)}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b}\cos(c+dx)} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b}\cos(c+dx)} + \frac{C \cos^{\frac{3}{2}}(c+dx)}{2bd\sqrt{b}\cos(c+dx)}$$

$$= \frac{Ax\sqrt{\cos(c+dx)}}{b\sqrt{b}\cos(c+dx)} + \frac{Cx\sqrt{\cos(c+dx)}}{2b\sqrt{b}\cos(c+dx)} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{bd\sqrt{b}\cos(c+dx)}$$

Mathematica [A] time = 0.105524, size = 61, normalized size = 0.45

$$\frac{\cos^{\frac{3}{2}}(c+dx)(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4d(b\cos(c+dx))^{\frac{3}{2}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (Cos[c + d*x]^(3/2)*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*d*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.28, size = 63, normalized size = 0.5

$$\frac{C \cos(dx+c) \sin(dx+c) + 2A(dx+c) + 2B \sin(dx+c) + C(dx+c)}{2d} (\cos(dx+c))^{\frac{3}{2}} (b \cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/2/d*cos(d*x+c)^(3/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.18561, size = 86, normalized size = 0.64

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))C}{b^{\frac{3}{2}}} + \frac{8A \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}} + \frac{4B \sin(dx+c)}{b^{\frac{3}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(3/2) + 8*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(3/2) + 4*B*sin(d*x + c)/b^(3/2))/d

Fricas [A] time = 1.70373, size = 603, normalized size = 4.47

$$\left[\frac{(2A + C)\sqrt{-b} \cos(dx + c) \log\left(2b \cos(dx + c)^2 + 2\sqrt{b} \cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)} \sin(dx + c) - b\right) - 2(C \cos(dx + c) + 2B)\sqrt{b} \cos(dx + c) \sqrt{\cos(dx + c)} \sin(dx + c)}{4b^2d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^{\frac{3}{2}}}{(b \cos(dx + c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*cos(d*x + c))^(3/2), x)

$$3.326 \quad \int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=102

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0598896, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3023, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{bd\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{3/2}} dx = \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec(c+dx) dx}{b\sqrt{b\cos(c+dx)}} \\ = \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)) \sec(c+dx) dx}{b\sqrt{b\cos(c+dx)}} \\ = \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{bd\sqrt{b\cos(c+dx)}} + \frac{(A\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{b\sqrt{b\cos(c+dx)}} \\ = \frac{Bx\sqrt{\cos(c+dx)}}{b\sqrt{b\cos(c+dx)}} + \frac{A \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}} + \frac{C \sin(c+dx)\sqrt{\cos(c+dx)}}{bd\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.104879, size = 93, normalized size = 0.91

$$\frac{\cos^{\frac{3}{2}}(c+dx) \left(-A \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + A \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right) + Bc + Bdx + C \sin(c+dx) \right)}{d(b\cos(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(3/2), x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.377, size = 63, normalized size = 0.6

$$-\frac{1}{d} \left(2 A \operatorname{Arctanh}\left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)}\right) - B(dx+c) - \sin(dx+c) C \right) (\cos(dx+c))^{\frac{3}{2}} (b\cos(dx+c))^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2), x)
```

```
[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [A] time = 2.19775, size = 140, normalized size = 1.37

$$\frac{A(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2\sin(dx+c) + 1))}{b^{\frac{3}{2}}} + \frac{4B \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{3}{2}}} + \frac{2C \sin(dx+c)}{b^{\frac{3}{2}}}$$

$2d$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2), x, algorithm="maxima")
```

```
[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(3/2) + 4*B*arctan(sin(dx+c)/(cos(dx+c)+1))
```

$$n(d*x + c)/(\cos(d*x + c) + 1)/b^{(3/2)} + 2*C*\sin(d*x + c)/b^{(3/2)}/d$$

Fricas [A] time = 2.10174, size = 869, normalized size = 8.52

$$\frac{2 A \sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)} \sqrt{-b} \sin(dx+c)}{b \sqrt{\cos(dx+c)}}\right) \cos(dx+c) + B \sqrt{-b} \cos(dx+c) \log\left(2 b \cos(dx+c)^2 + 2 \sqrt{b \cos(dx+c)}\right)}{2 b^2 d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c) + B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + A*sqrt(b)*cos(d*x + c)*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3 + 2*sqrt(b*cos(d*x + c))*C*sqrt(cos(d*x + c))*sin(d*x + c)/(b^2*d*cos(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \sqrt{\cos(dx+c)}}{(b \cos(dx+c))^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sqrt(cos(d*x + c))/(b*cos(d*x + c))^(3/2), x)
```

$$3.327 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=102

$$\frac{A \sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{bd\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}}$$

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0835293, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {18, 3021, 2735, 3770}

$$\frac{A \sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{bd\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{b\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)), x]

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx}{b \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (B + C \cos(c + dx)) \sec^2(c + dx) dx}{b \sqrt{b \cos(c + dx)}}$$

$$= \frac{Cx \sqrt{\cos(c + dx)}}{b \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)} \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{b \sqrt{b \cos(c + dx)}}$$

$$= \frac{Cx \sqrt{\cos(c + dx)}}{b \sqrt{b \cos(c + dx)}} + \frac{B \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{bd \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{bd \sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.0793612, size = 60, normalized size = 0.59

$$\frac{\sqrt{\cos(c + dx)} (A \sin(c + dx) + B \cos(c + dx) \tanh^{-1}(\sin(c + dx)) + C dx \cos(c + dx))}{d(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2)),x]

[Out] (Sqrt[Cos[c + d*x]]*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.378, size = 72, normalized size = 0.7

$$\frac{1}{d} \left(-2 B \cos(dx + c) \operatorname{Arctanh} \left(\frac{-1 + \cos(dx + c)}{\sin(dx + c)} \right) + C \cos(dx + c) (dx + c) + A \sin(dx + c) \right) \sqrt{\cos(dx + c)} (b \cos(dx + c))^{3/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x)

[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)

Maxima [A] time = 2.38537, size = 212, normalized size = 2.08

$$\frac{4 A \sqrt{b} \sin(2 dx + 2 c)}{b^2 \cos(2 dx + 2 c)^2 + b^2 \sin(2 dx + 2 c)^2 + 2 b^2 \cos(2 dx + 2 c) + b^2} + \frac{B(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1))}{2 d b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))

$(x + c) + 1)/b^{(3/2)} + 4*C*\arctan(\sin(dx + c)/(\cos(dx + c) + 1))/b^{(3/2)}/d$

Fricas [A] time = 2.09103, size = 886, normalized size = 8.69

$$\left[\frac{2B\sqrt{-b} \arctan\left(\frac{\sqrt{b \cos(dx+c)}\sqrt{-b} \sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 + C\sqrt{-b} \cos(dx+c)^2 \log\left(2b \cos(dx+c)^2 + 2\sqrt{b \cos(dx+c)}\sqrt{\cos(dx+c)}\right)}{2b^2d \cos(dx+c)^2} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(3/2)/cos(dx+c)^(1/2),x, algorithm="fricas")

[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c)^2 + C*sqrt(-b)*cos(dx + c)^2*log(2*b*cos(dx + c)^2 + 2*sqrt(b*cos(dx + c))*sqrt(-b)*sqrt(cos(dx + c))*sin(dx + c) - b) - 2*sqrt(b*cos(dx + c))*A*sqrt(cos(dx + c))*sin(dx + c))/(b^2*d*cos(dx + c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(dx + c))*sin(dx + c)/(sqrt(b)*cos(dx + c)^(3/2)))*cos(dx + c)^2 + B*sqrt(b)*cos(dx + c)^2*log(-(b*cos(dx + c)^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3) + 2*sqrt(b*cos(dx + c))*A*sqrt(cos(dx + c))*sin(dx + c))/(b^2*d*cos(dx + c)^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)**2)/(b*cos(dx+c))**(3/2)/cos(dx+c)**(1/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(3/2)/cos(dx+c)^(1/2),x, algorithm="giac")

[Out] integrate((C*cos(dx + c)^2 + B*cos(dx + c) + A)/((b*cos(dx + c))^(3/2)*sqrt(cos(dx + c))), x)

$$3.328 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=120

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2bd \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0927031, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {18, 3021, 2748, 3767, 8, 3770}

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2bd\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2bd \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{bd\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

`Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]`

Rule 3770

`Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]`

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx}{b\sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)\sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (2B + (A + 2C) \cos(c + dx)) \sec^2(c + dx) dx}{2b\sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)\sqrt{b \cos(c + dx)}} + \frac{(B\sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{b\sqrt{b \cos(c + dx)}} + \frac{(A + 2C) \int \sec^2(c + dx) dx}{2b\sqrt{b \cos(c + dx)}}$$

$$= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx))\sqrt{\cos(c + dx)}}{2bd\sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)\sqrt{b \cos(c + dx)}}$$

$$= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx))\sqrt{\cos(c + dx)}}{2bd\sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx)\sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.0880675, size = 69, normalized size = 0.57

$$\frac{\sin(c + dx)(A + 2B \cos(c + dx)) + (A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx))}{2d\sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.289, size = 150, normalized size = 1.3

$$\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2), x)

[Out] 1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))+2*B*sin(d*x+c)*cos(d*x+c)+A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(3/2)

Maxima [B] time = 2.48035, size = 1083, normalized size = 9.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")

[Out]
$$\frac{1}{4} \cdot (8B\sqrt{b}\sin(2dx+2c)/(b^2\cos(2dx+2c)^2 + b^2\sin(2dx+2c)^2 + 2b^2\cos(2dx+2c) + b^2) - (4(\sin(4dx+4c) + 2\sin(2dx+2c))\cos(3/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))) - 4(\sin(4dx+4c) + 2\sin(2dx+2c))\cos(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))) - (2(2\cos(2dx+2c) + 1)\cos(4dx+4c) + \cos(4dx+4c))^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c)\sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1)\log(\cos(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 + \sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 + 2\sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))) + 1) + (2(2\cos(2dx+2c) + 1)\cos(4dx+4c) + \cos(4dx+4c))^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c)\sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1)\log(\cos(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 + \sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 - 2\sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))) + 1) - 4(\cos(4dx+4c) + 2\cos(2dx+2c) + 1)\sin(3/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))) + 4(\cos(4dx+4c) + 2\cos(2dx+2c) + 1)\sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))) \cdot A / ((b\cos(4dx+4c)^2 + 4b\cos(2dx+2c)^2 + b\sin(4dx+4c)^2 + 4b\sin(4dx+4c)\sin(2dx+2c) + 4b\sin(2dx+2c)^2 + 2(2b\cos(2dx+2c) + b)\cos(4dx+4c) + 4b\cos(2dx+2c) + b)\sqrt{b}) + 2C(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2\sin(dx+c) + 1))/b^{3/2})/d$$

Fricas [A] time = 1.68912, size = 659, normalized size = 5.49

$$\frac{(A + 2C)\sqrt{b}\cos(dx+c)^3 \log\left(\frac{-b\cos(dx+c)^3 - 2\sqrt{b}\cos(dx+c)\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2B\cos(dx+c) + A)\sqrt{b}}{4b^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")

[Out]
$$\left[\frac{1}{4} \cdot ((A + 2C)\sqrt{b}\cos(dx+c)^3 \log(-b\cos(dx+c)^3 - 2\sqrt{b}\cos(dx+c)\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c) - 2b\cos(dx+c))/\cos(dx+c)^3 + 2(2B\cos(dx+c) + A)\sqrt{b}\cos(dx+c)\sqrt{\cos(dx+c)}\sin(dx+c)/(b^2d\cos(dx+c)^3), -1/2 \cdot ((A + 2C)\sqrt{-b}\arctan(\sqrt{b}\cos(dx+c)\sqrt{-b}\sin(dx+c)/(b\sqrt{\cos(dx+c)}))\cos(dx+c)^3 - (2B\cos(dx+c) + A)\sqrt{b}\cos(dx+c)\sqrt{\cos(dx+c)}\sin(dx+c)/(b^2d\cos(dx+c)^3)) \right]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))
**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(3/2)), x)
```

$$3.329 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=164

$$\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}}{2bd}$$

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.123828, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {18, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3bd\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3bd\cos^{\frac{5}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}}{2bd}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx}{b \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (3B + (2A + 3C) \cos(c + dx)) \sec^3(c + dx) dx}{3b \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{b \sqrt{b \cos(c + dx)}} + \frac{(2A + 3C) \int \sec^3(c + dx) dx}{2b \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{B \sin(c + dx)}{2bd \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(2A + 3C) \tan^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2bd \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{3bd \cos^{\frac{5}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \dots$$

Mathematica [A] time = 0.148537, size = 87, normalized size = 0.53

$$\frac{\tan(c + dx)((2A + 3C) \cos(2(c + dx)) + 4A + 3B \cos(c + dx) + 3C) + 3B \cos^2(c + dx) \tanh^{-1}(\sin(c + dx))}{6d \sqrt{\cos(c + dx)}(b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2)), x]

[Out] (3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(3/2))

Maple [A] time = 0.3, size = 157, normalized size = 1.

$$\frac{1}{6d} \left(-3B \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 + 3B \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c)) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.46096, size = 1415, normalized size = 8.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^2*cos(2*d*x + 2*c)^2 + b^2*sin(2*d*x + 2*c)^2 + 2*b^2*cos(2*d*x + 2*c) + b^2) + 16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x + 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) + 3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b))/d
```

Fricas [A] time = 1.68115, size = 733, normalized size = 4.47

$$\frac{3B\sqrt{b}\cos(dx+c)^4\log\left(-\frac{b\cos(dx+c)^3-2\sqrt{b}\cos(dx+c)\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2(2A+3C)\cos(dx+c)^2+3\right)}{12b^2d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*cos(d*x + c)^(5/2)), x)
```


$$3.330 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{3/2}} dx$$

Optimal. Leaf size=208

$$\frac{(3A+4C) \sin(c+dx)}{8bd \cos^3(c+dx) \sqrt{b \cos(c+dx)}} + \frac{(3A+4C) \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8bd \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4bd \cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} +$$

[Out] $((3A + 4C) \cdot \text{ArcTanh}[\text{Sin}[c + d*x]] \cdot \text{Sqrt}[\text{Cos}[c + d*x]]) / (8*b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (A*\text{Sin}[c + d*x]) / (4*b*d*\text{Cos}[c + d*x]^{(7/2)}*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + ((3A + 4C)*\text{Sin}[c + d*x]) / (8*b*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Sin}[c + d*x]) / (b*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Sin}[c + d*x]^3) / (3*b*d*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rubi [A] time = 0.127982, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {18, 3021, 2748, 3767, 3768, 3770}

$$\frac{(3A+4C) \sin(c+dx)}{8bd \cos^3(c+dx) \sqrt{b \cos(c+dx)}} + \frac{(3A+4C) \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8bd \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4bd \cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} +$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2) / (\text{Cos}[c + d*x]^{(7/2)}*(b*\text{Cos}[c + d*x])^{(3/2)}), x]$

[Out] $((3A + 4C) \cdot \text{ArcTanh}[\text{Sin}[c + d*x]] \cdot \text{Sqrt}[\text{Cos}[c + d*x]]) / (8*b*d*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (A*\text{Sin}[c + d*x]) / (4*b*d*\text{Cos}[c + d*x]^{(7/2)}*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + ((3A + 4C)*\text{Sin}[c + d*x]) / (8*b*d*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Sin}[c + d*x]) / (b*d*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[b*\text{Cos}[c + d*x]]) + (B*\text{Sin}[c + d*x]^3) / (3*b*d*\text{Cos}[c + d*x]^{(5/2)}*\text{Sqrt}[b*\text{Cos}[c + d*x]])$

Rule 18

$\text{Int}[(u_*)*((a_*)*(v_*))^{(m_*)}*((b_*)*(v_*))^{(n_*)}, x_Symbol] \rightarrow \text{Dist}[(a^{(m-1/2)}*b^{(n+1/2)}*\text{Sqrt}[a*v]) / \text{Sqrt}[b*v], \text{Int}[u*v^{(m+n)}, x], x] /;$ FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

$\text{Int}[(a_*) + (b_*)*\text{sin}[(e_*) + (f_*)*(x_*)]^{(m_*)}*((A_*) + (B_*)*\text{sin}[(e_*) + (f_*)*(x_*)] + (C_*)*\text{sin}[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C)*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m+1)} / (b*f*(m+1)*(a^2 - b^2)), x] + \text{Dist}[1/(b*(m+1)*(a^2 - b^2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^{(m+1)}*\text{Simp}[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*\text{Sin}[e + f*x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

$\text{Int}[(b_*)*\text{sin}[(e_*) + (f_*)*(x_*)]^{(m_*)}*((c_*) + (d_*)*\text{sin}[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{7}{2}}(c + dx)(b \cos(c + dx))^{3/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx}{b \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int (4B + (3A + 4C) \cos(c + dx)) \sec^4(c + dx) dx}{4b \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^4(c + dx) dx}{b \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \int \sec^2(c + dx) dx}{b \sqrt{b \cos(c + dx)}}$$

$$= \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \sin(c + dx)}{8bd \cos^{\frac{3}{2}}(c + dx) \sqrt{b \cos(c + dx)}} + \frac{(3A + 4C) \tan^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{8bd \sqrt{b \cos(c + dx)}} + \frac{A \sin(c + dx)}{4bd \cos^{\frac{7}{2}}(c + dx) \sqrt{b \cos(c + dx)}}$$

Mathematica [A] time = 0.213139, size = 110, normalized size = 0.53

$$\frac{\sin(c + dx) (3(3A + 4C) \cos^2(c + dx) + 6A + 24B \cos^3(c + dx) + 8B \sin^2(c + dx) \cos(c + dx)) + 3(3A + 4C) \cos^4(c + dx)}{24d \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^(3/2)), x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2))/(24*d*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(3/2))
```

Maple [A] time = 0.362, size = 248, normalized size = 1.2

$$\frac{1}{24d} \left(9A \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^4 - 9A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(3/2),x)
```

```
[Out] 1/24/d*(9*A*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-9*A*cos
(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)
)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*sin(d*x+c)*cos(d*
x+c)^2+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c
))/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(3/2)
```

Maxima [B] time = 2.58859, size = 3591, normalized size = 17.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
+ 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*A/((b*cos(8*d*x + 8*c)^2 + 16*b*cos(6*d*x + 6*c)^2 + 36*b*cos(4*d*
x + 4*c)^2 + 16*b*cos(2*d*x + 2*c)^2 + b*sin(8*d*x + 8*c)^2 + 16*b*sin(6*d*
```

```
x + 6*c)^2 + 36*b*sin(4*d*x + 4*c)^2 + 48*b*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 16*b*sin(2*d*x + 2*c)^2 + 2*(4*b*cos(6*d*x + 6*c) + 6*b*cos(4*d*x + 4*
c) + 4*b*cos(2*d*x + 2*c) + b)*cos(8*d*x + 8*c) + 8*(6*b*cos(4*d*x + 4*c) +
4*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 12*(4*b*cos(2*d*x + 2*c) + b)
*cos(4*d*x + 4*c) + 8*b*cos(2*d*x + 2*c) + 4*(2*b*sin(6*d*x + 6*c) + 3*b*si
n(4*d*x + 4*c) + 2*b*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*b*sin(4*d*x
+ 4*c) + 2*b*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) - 64*((3*cos
(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x
+ 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x
+ 2*c))*B/((b*cos(6*d*x + 6*c)^2 + 9*b*cos(4*d*x + 4*c)^2 + 9*b*cos(2*d*x
+ 2*c)^2 + b*sin(6*d*x + 6*c)^2 + 9*b*sin(4*d*x + 4*c)^2 + 18*b*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) + 9*b*sin(2*d*x + 2*c)^2 + 2*(3*b*cos(4*d*x + 4*c) +
3*b*cos(2*d*x + 2*c) + b)*cos(6*d*x + 6*c) + 6*(3*b*cos(2*d*x + 2*c) + b)*
cos(4*d*x + 4*c) + 6*b*cos(2*d*x + 2*c) + 6*(b*sin(4*d*x + 4*c) + b*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + b)*sqrt(b)) + 12*(4*(sin(4*d*x + 4*c) + 2*sin
(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(si
n(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x +
4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x
+ 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 4*(cos(4*d*x + 4*c) + 2*cos(
2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*C/(
(b*cos(4*d*x + 4*c)^2 + 4*b*cos(2*d*x + 2*c)^2 + b*sin(4*d*x + 4*c)^2 + 4*b
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b*sin(2*d*x + 2*c)^2 + 2*(2*b*cos(2*
d*x + 2*c) + b)*cos(4*d*x + 4*c) + 4*b*cos(2*d*x + 2*c) + b)*sqrt(b))/d
```

Fricas [A] time = 1.74656, size = 821, normalized size = 3.95

$$\frac{3(3A + 4C)\sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b} \cos(dx+c)\sqrt{b} \sqrt{\cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(16B \cos(dx + c)^3 + 3(3A + 4C)\cos(dx + c)^2 + 8B \cos(dx + c) + 6A)\sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c)}{48b^2d \cos(dx + c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(d*x + c)^5*log(-(b*cos(d*x + c)^3 - 2*sqrt
(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c)
)/cos(d*x + c)^3) + 2*(16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 +
8*B*cos(d*x + c) + 6*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(b^2*d*cos(d*x + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos
(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^5 - (
16*B*cos(d*x + c)^3 + 3*(3*A + 4*C)*cos(d*x + c)^2 + 8*B*cos(d*x + c) + 6*A
)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^2*d*cos(d*x + c)
^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2)/(b*cos(d*x+c))
**(3/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{3}{2}} \cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2)/(b*cos(d*x+c))^(
3/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(3/2)*c
os(d*x + c)^(7/2)), x)

$$3.331 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=199

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b \cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b \cos(c+dx)}} - \frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b \cos(c+dx)}}$$

[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.11657, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {17, 3023, 2748, 2635, 8, 2633}

$$\frac{x(4A+3C)\sqrt{\cos(c+dx)}}{8b^2\sqrt{b \cos(c+dx)}} + \frac{(4A+3C)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{8b^2d\sqrt{b \cos(c+dx)}} - \frac{B\sin^3(c+dx)\sqrt{\cos(c+dx)}}{3b^2d\sqrt{b \cos(c+dx)}} + \frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] ((4*A + 3*C)*x*Sqrt[Cos[c + d*x]])/(8*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + ((4*A + 3*C)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(8*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(4*b^2*d*Sqrt[b*Cos[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x]^3)/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]
)*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cos^9(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{b^2 \sqrt{b \cos(c + dx)}} \\ = \frac{C \cos^7(c + dx) \sin(c + dx)}{4b^2 d \sqrt{b \cos(c + dx)}} + \frac{\sqrt{\cos(c + dx)} \int \cos^2(c + dx)}{4b^2 \sqrt{b}} \\ = \frac{C \cos^7(c + dx) \sin(c + dx)}{4b^2 d \sqrt{b \cos(c + dx)}} + \frac{(B \sqrt{\cos(c + dx)}) \int \cos^3(c + dx)}{b^2 \sqrt{b \cos(c + dx)}} \\ = \frac{(4A + 3C) \cos^3(c + dx) \sin(c + dx)}{8b^2 d \sqrt{b \cos(c + dx)}} + \frac{C \cos^7(c + dx) \sin(c + dx)}{4b^2 d \sqrt{b \cos(c + dx)}} \\ = \frac{(4A + 3C)x \sqrt{\cos(c + dx)}}{8b^2 \sqrt{b \cos(c + dx)}} + \frac{B \sqrt{\cos(c + dx)} \sin(c + dx)}{b^2 d \sqrt{b \cos(c + dx)}} +$$

Mathematica [A] time = 0.143248, size = 95, normalized size = 0.48

$$\frac{\sqrt{\cos(c + dx)}(24(A + C) \sin(2(c + dx)) + 48Ac + 48Adx + 72B \sin(c + dx) + 8B \sin(3(c + dx)) + 3C \sin(4(c + dx)))}{96b^2 d \sqrt{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(9/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*C
os[c + d*x])^(5/2), x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(48*A*c + 36*c*C + 48*A*d*x + 36*C*d*x + 72*B*Sin[c + d
*x] + 24*(A + C)*Sin[2*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*C*Sin[4*(c + d
*x)]))/(96*b^2*d*Sqrt[b*Cos[c + d*x]])
```

Maple [A] time = 0.377, size = 114, normalized size = 0.6

$$\frac{6 C (\cos(dx + c))^3 \sin(dx + c) + 8 B \sin(dx + c) (\cos(dx + c))^2 + 12 A \cos(dx + c) \sin(dx + c) + 9 C \cos(dx + c) \sin(dx + c)}{24 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x
)
```

[Out] $1/24/d*\cos(d*x+c)^{(5/2)}*(6*C*\cos(d*x+c)^3*\sin(d*x+c)+8*B*\sin(d*x+c)*\cos(d*x+c)^2+12*A*\cos(d*x+c)*\sin(d*x+c)+9*C*\cos(d*x+c)*\sin(d*x+c)+12*A*(d*x+c)+16*B*\sin(d*x+c)+9*C*(d*x+c))/(b*\cos(d*x+c))^{(5/2)}$

Maxima [A] time = 2.33658, size = 157, normalized size = 0.79

$$\frac{24(2dx+2c+\sin(2dx+2c))A}{b^{\frac{5}{2}}} + \frac{3(12dx+12c+\sin(4dx+4c)+8\sin(\frac{1}{2}\arctan(\sin(4dx+4c),\cos(4dx+4c))))C}{b^{\frac{5}{2}}} + \frac{8B(\sin(3dx+3c)+9\sin(\frac{1}{3}\arctan(\sin(3dx+3c),\cos(3dx+3c))))}{b^{\frac{5}{2}}}$$

$96d$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")`

[Out] $1/96*(24*(2*d*x + 2*c + \sin(2*d*x + 2*c))*A/b^{(5/2)} + 3*(12*d*x + 12*c + \sin(4*d*x + 4*c) + 8*\sin(1/2*\arctan2(\sin(4*d*x + 4*c), \cos(4*d*x + 4*c))))*C/b^{(5/2)} + 8*B*(\sin(3*d*x + 3*c) + 9*\sin(1/3*\arctan2(\sin(3*d*x + 3*c), \cos(3*d*x + 3*c))))/b^{(5/2)})/d$

Fricas [A] time = 1.71896, size = 765, normalized size = 3.84

$$\left[\frac{3(4A + 3C)\sqrt{-b}\cos(dx + c)\log(2b\cos(dx + c)^2 + 2\sqrt{b}\cos(dx + c)\sqrt{-b}\sqrt{\cos(dx + c)}\sin(dx + c) - b) - 2(6C\cos(dx + c)^3 + 8B\cos(dx + c)^2 + 3(4A + 3C)\cos(dx + c) + 16B)\sqrt{b*\cos(dx + c)}*\sqrt{\cos(dx + c)}*\sin(dx + c)}}{48b^3d\cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")`

[Out] $[-1/48*(3*(4*A + 3*C)*\sqrt{-b}*\cos(d*x + c)*\log(2*b*\cos(d*x + c)^2 + 2*\sqrt{b}*\cos(d*x + c)*\sqrt{-b}*\sqrt{\cos(d*x + c)}*\sin(d*x + c) - b) - 2*(6*C*\cos(d*x + c)^3 + 8*B*\cos(d*x + c)^2 + 3*(4*A + 3*C)*\cos(d*x + c) + 16*B)*\sqrt{b*\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(b^3*d*\cos(d*x + c)), 1/24*(3*(4*A + 3*C)*\sqrt{b}*\arctan(\sqrt{b*\cos(d*x + c)}*\sin(d*x + c)/(\sqrt{b}*\cos(d*x + c)^{(3/2)}))*\cos(d*x + c) + (6*C*\cos(d*x + c)^3 + 8*B*\cos(d*x + c)^2 + 3*(4*A + 3*C)*\cos(d*x + c) + 16*B)*\sqrt{b*\cos(d*x + c)}*\sqrt{\cos(d*x + c)}*\sin(d*x + c))/(b^3*d*\cos(d*x + c))]$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate(cos(d*x+c)**(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^{\frac{9}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(9/2)/(b*cos(d*x + c))^(5/2), x)
```

$$3.332 \quad \int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=155

$$\frac{(3A+2C) \sin(c+dx) \sqrt{\cos(c+dx)}}{3b^2 d \sqrt{b \cos(c+dx)}} + \frac{Bx \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{3b^2 d \sqrt{b \cos(c+dx)}}$$

[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0636998, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.07$, Rules used = {17, 3023, 2734}

$$\frac{(3A+2C) \sin(c+dx) \sqrt{\cos(c+dx)}}{3b^2 d \sqrt{b \cos(c+dx)}} + \frac{Bx \sqrt{\cos(c+dx)}}{2b^2 \sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2 d \sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)}{3b^2 d \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + ((3*A + 2*C)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]]) + (B*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2734

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := Simp[((2*a*c + b*d)*x)/2, x] + (-Simp[((b*c + a*d)*Cos[e + f*x])/f, x] - Simp[(b*d*Cos[e + f*x]*Sin[e + f*x])/(2*f), x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{b^2\sqrt{b\cos(c+dx)}} \\ = \frac{C\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \int \cos(c+dx)}{3b^2\sqrt{b\cos(c+dx)}} \\ = \frac{Bx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{(3A+2C)\sqrt{\cos(c+dx)}\sin(c+dx)}{3b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.124164, size = 78, normalized size = 0.5

$$\frac{\sqrt{\cos(c+dx)}(3(4A+3C)\sin(c+dx)+3B\sin(2(c+dx))+6Bc+6Bdx+C\sin(3(c+dx)))}{12b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(6*B*c + 6*B*d*x + 3*(4*A + 3*C)*Sin[c + d*x] + 3*B*Ssin[2*(c + d*x)] + C*Ssin[3*(c + d*x)]))/(12*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.325, size = 83, normalized size = 0.5

$$\frac{2C\sin(dx+c)(\cos(dx+c))^2+3B\sin(dx+c)\cos(dx+c)+6A\sin(dx+c)+3B(dx+c)+4\sin(dx+c)C}{6d} (\cos$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] 1/6/d*cos(d*x+c)^(5/2)*(2*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c)+3*B*(d*x+c)+4*sin(d*x+c)*C)/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.31996, size = 108, normalized size = 0.7

$$\frac{3(2dx+2c+\sin(2dx+2c))B}{b^{\frac{5}{2}}} + \frac{C\left(\sin(3dx+3c)+9\sin\left(\frac{1}{3}\arctan\left(\frac{\sin(3dx+3c)}{\cos(3dx+3c)}\right)\right)\right)}{b^{\frac{5}{2}}} + \frac{12A\sin(dx+c)}{b^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 1/12*(3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B/b^(5/2) + C*(sin(3*d*x + 3*c) + 9*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))/b^(5/2) + 12*A*sin(d*x + c)/b^(5/2))/d

Fricas [A] time = 1.69352, size = 667, normalized size = 4.3

$$\left[\frac{3B\sqrt{-b}\cos(dx+c)\log\left(2b\cos(dx+c)^2+2\sqrt{b\cos(dx+c)}\sqrt{-b}\sqrt{\cos(dx+c)}\sin(dx+c)-b\right)-2\left(2C\cos(dx+c)\right)^2}{12b^3d\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/12*(3*B*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/6*(3*B*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (2*C*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 6*A + 4*C)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C\cos(dx+c)^2 + B\cos(dx+c) + A)\cos(dx+c)^{\frac{7}{2}}}{(b\cos(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(7/2)/(b*cos(d*x + c))^(5/2), x)

$$3.333 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=135

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b \cos(c+dx)}}$$

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0361816, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 2637, 2635, 8}

$$\frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b \cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b \cos(c+dx)}} + \frac{C \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(5/2)), x]

[Out] (A*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (C*x*Sqrt[Cos[c + d*x]])/(2*b^2*Sqrt[b*Cos[c + d*x]]) + (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 2637

Int[sin[Pi/2 + (c_.) + (d_)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2635

Int[((b_)*sin[(c_.) + (d_)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x])*(b*Ssin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Ssin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 8

Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx) (A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{\frac{5}{2}}} dx = \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)+C\cos^2(c+dx)) dx}{b^2\sqrt{b\cos(c+dx)}} \\ = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{(B\sqrt{\cos(c+dx)}) \int \cos(c+dx) dx}{b^2\sqrt{b\cos(c+dx)}} + \frac{C \int \cos^3(c+dx) dx}{2b^2d\sqrt{b\cos(c+dx)}} \\ = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}} + \frac{C \cos^{\frac{3}{2}}(c+dx)}{2b^2d\sqrt{b\cos(c+dx)}} \\ = \frac{Ax\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{Cx\sqrt{\cos(c+dx)}}{2b^2\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)} \sin(c+dx)}{b^2d\sqrt{b\cos(c+dx)}}$$

Mathematica [A] time = 0.0779192, size = 64, normalized size = 0.47

$$\frac{\sqrt{\cos(c+dx)}(2(2A+C)(c+dx)+4B\sin(c+dx)+C\sin(2(c+dx)))}{4b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(2*(2*A + C)*(c + d*x) + 4*B*Sin[c + d*x] + C*Sin[2*(c + d*x)]))/(4*b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.303, size = 63, normalized size = 0.5

$$\frac{C \cos(dx+c) \sin(dx+c) + 2A(dx+c) + 2B \sin(dx+c) + C(dx+c)}{2d} (\cos(dx+c))^{\frac{5}{2}} (b \cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] 1/2/d*cos(d*x+c)^(5/2)*(C*cos(d*x+c)*sin(d*x+c)+2*A*(d*x+c)+2*B*sin(d*x+c)+C*(d*x+c))/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.11075, size = 86, normalized size = 0.64

$$\frac{\frac{(2dx+2c+\sin(2dx+2c))C}{b^{\frac{5}{2}}} + \frac{8A \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}} + \frac{4B \sin(dx+c)}{b^{\frac{5}{2}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*C/b^(5/2) + 8*A*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/b^(5/2) + 4*B*sin(d*x + c)/b^(5/2))/d

Fricas [A] time = 1.68487, size = 603, normalized size = 4.47

$$\left[\frac{(2A + C)\sqrt{-b} \cos(dx + c) \log\left(2b \cos(dx + c)^2 + 2\sqrt{b \cos(dx + c)}\sqrt{-b} \sqrt{\cos(dx + c)} \sin(dx + c) - b\right) - 2(C \cos(dx + c) + 2B)\sqrt{b \cos(dx + c)}\sqrt{\cos(dx + c)}\sin(dx + c)}{4b^3d \cos(dx + c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")

[Out] [-1/4*((2*A + C)*sqrt(-b)*cos(d*x + c)*log(2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c))*sqrt(-b)*sqrt(cos(d*x + c))*sin(d*x + c) - b) - 2*(C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c)), 1/2*((2*A + C)*sqrt(b)*arctan(sqrt(b*cos(d*x + c))*sin(d*x + c)/(sqrt(b)*cos(d*x + c)^(3/2)))*cos(d*x + c) + (C*cos(d*x + c) + 2*B)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(b^3*d*cos(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^{\frac{5}{2}}}{(b \cos(dx + c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(5/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*cos(d*x + c))^(5/2), x)

$$3.334 \quad \int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=102

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0615652, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3023, 2735, 3770}

$$\frac{A\sqrt{\cos(c+dx)}\tanh^{-1}(\sin(c+dx))}{b^2d\sqrt{b\cos(c+dx)}} + \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b\cos(c+dx)}} + \frac{C\sin(c+dx)\sqrt{\cos(c+dx)}}{b^2d\sqrt{b\cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (B*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (A*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (C*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b^2*d*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2735

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)+C\cos^2(c+dx)) \sec(c+dx) dx}{b^2\sqrt{b}\cos(c+dx)}$$

$$= \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b}\cos(c+dx)} + \frac{\sqrt{\cos(c+dx)} \int (A+B\cos(c+dx)) \sec(c+dx) dx}{b^2\sqrt{b}\cos(c+dx)}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b}\cos(c+dx)} + \frac{C\sqrt{\cos(c+dx)}\sin(c+dx)}{b^2d\sqrt{b}\cos(c+dx)} + \frac{(A\sqrt{\cos(c+dx)}) \int \sec(c+dx) dx}{b^2\sqrt{b}\cos(c+dx)}$$

$$= \frac{Bx\sqrt{\cos(c+dx)}}{b^2\sqrt{b}\cos(c+dx)} + \frac{A \tanh^{-1}(\sin(c+dx))\sqrt{\cos(c+dx)}}{b^2d\sqrt{b}\cos(c+dx)}$$

Mathematica [A] time = 0.0922713, size = 96, normalized size = 0.94

$$\frac{\sqrt{\cos(c+dx)} \left(-A \log\left(\cos\left(\frac{1}{2}(c+dx)\right) - \sin\left(\frac{1}{2}(c+dx)\right)\right) + A \log\left(\sin\left(\frac{1}{2}(c+dx)\right) + \cos\left(\frac{1}{2}(c+dx)\right)\right) + Bc + Bdx + C \right)}{b^2d\sqrt{b}\cos(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (Sqrt[Cos[c + d*x]]*(B*c + B*d*x - A*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + C*Sin[c + d*x]))/(b^2*d*Sqrt[b*Cos[c + d*x]])

Maple [A] time = 0.257, size = 63, normalized size = 0.6

$$-\frac{1}{d} \left(2A \operatorname{Arctanh}\left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)}\right) - B(dx+c) - \sin(dx+c)C \right) (\cos(dx+c))^{\frac{5}{2}} (b\cos(dx+c))^{-\frac{5}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x)

[Out] -1/d*(2*A*arctanh((-1+cos(d*x+c))/sin(d*x+c))-B*(d*x+c)-sin(d*x+c)*C)*cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.20159, size = 140, normalized size = 1.37

$$\frac{A(\log(\cos(dx+c)^2+\sin(dx+c)^2+2\sin(dx+c)+1)-\log(\cos(dx+c)^2+\sin(dx+c)^2-2\sin(dx+c)+1))}{b^{\frac{5}{2}}} + \frac{4B\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{b^{\frac{5}{2}}} + \frac{2C\sin(dx+c)}{b^{\frac{5}{2}}}$$

$$2d$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2), x, algorithm="maxima")

[Out] 1/2*(A*(log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1))/b^(5/2) + 4*B*arctan(si

$n(dx + c)/(\cos(dx + c) + 1)/b^{(5/2)} + 2C\sin(dx + c)/b^{(5/2)}/d$

Fricas [A] time = 2.1084, size = 869, normalized size = 8.52

$$\left[\frac{2A\sqrt{-b} \arctan\left(\frac{\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c) + B\sqrt{-b} \cos(dx+c) \log\left(2b\cos(dx+c)^2 + 2\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)\right)}{2b^3d\cos(dx+c)} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(3/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(5/2),x, algorithm="fricas")

[Out] [-1/2*(2*A*sqrt(-b)*arctan(sqrt(b*cos(dx+c))*sqrt(-b)*sin(dx+c)/(b*sqrt(cos(dx+c))))*cos(dx+c) + B*sqrt(-b)*cos(dx+c)*log(2*b*cos(dx+c)^2 + 2*sqrt(b*cos(dx+c))*sqrt(-b)*sqrt(cos(dx+c))*sin(dx+c) - b) - 2*sqrt(b*cos(dx+c))*C*sqrt(cos(dx+c))*sin(dx+c))/(b^3*d*cos(dx+c)), 1/2*(2*B*sqrt(b)*arctan(sqrt(b*cos(dx+c))*sin(dx+c)/(sqrt(b)*cos(dx+c)^(3/2)))*cos(dx+c) + A*sqrt(b)*cos(dx+c)*log(-b*cos(dx+c)^3 - 2*sqrt(b*cos(dx+c))*sqrt(b)*sqrt(cos(dx+c))*sin(dx+c) - 2*b*cos(dx+c))/cos(dx+c)^3 + 2*sqrt(b*cos(dx+c))*C*sqrt(cos(dx+c))*sin(dx+c))/(b^3*d*cos(dx+c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)**(3/2)*(A+B*cos(dx+c)+C*cos(dx+c)**2)/(b*cos(dx+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx+c)^2 + B \cos(dx+c) + A) \cos(dx+c)^{\frac{3}{2}}}{(b \cos(dx+c))^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(dx+c)^(3/2)*(A+B*cos(dx+c)+C*cos(dx+c)^2)/(b*cos(dx+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(dx+c)^2 + B*cos(dx+c) + A)*cos(dx+c)^(3/2)/(b*cos(dx+c))^(5/2), x)

$$3.335 \quad \int \frac{\sqrt{\cos(c+dx)}(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=102

$$\frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{C x \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}}$$

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0621749, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.093$, Rules used = {17, 3021, 2735, 3770}

$$\frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{B \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{C x \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2), x]

[Out] (C*x*Sqrt[Cos[c + d*x]])/(b^2*Sqrt[b*Cos[c + d*x]]) + (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 17

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[(a^(m + 1/2)*b^(n - 1/2)*Sqrt[b*v])/Sqrt[a*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && IGtQ[n + 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2735

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])/((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]

Rule 3770

Int[csc[(c_) + (d_)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx))}{(b \cos(c+dx))^{5/2}} dx = \frac{\sqrt{\cos(c+dx)} \int (A + B \cos(c+dx) + C \cos^2(c+dx)) \sec^2(c+dx)}{b^2 \sqrt{b \cos(c+dx)}} \\ = \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{\sqrt{\cos(c+dx)} \int (B + C \cos(c+dx)) \sec^2(c+dx)}{b^2 \sqrt{b \cos(c+dx)}} \\ = \frac{Cx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}} + \frac{(B \sqrt{\cos(c+dx)} \tan^{-1}(\sin(c+dx)))}{b^2 \sqrt{b \cos(c+dx)}} \\ = \frac{Cx \sqrt{\cos(c+dx)}}{b^2 \sqrt{b \cos(c+dx)}} + \frac{B \tanh^{-1}(\sin(c+dx)) \sqrt{\cos(c+dx)}}{b^2 d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{b^2 d \sqrt{\cos(c+dx)} \sqrt{b \cos(c+dx)}}$$

Mathematica [A] time = 0.0691534, size = 60, normalized size = 0.59

$$\frac{\cos^{\frac{3}{2}}(c+dx) (A \sin(c+dx) + B \cos(c+dx) \tanh^{-1}(\sin(c+dx)) + C dx \cos(c+dx))}{d (b \cos(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(5/2),x]

[Out] (Cos[c + d*x]^(3/2)*(C*d*x*Cos[c + d*x] + B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x] + A*Sin[c + d*x]))/(d*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.384, size = 72, normalized size = 0.7

$$\frac{1}{d} \left(-2 B \cos(dx+c) \operatorname{Arctanh} \left(\frac{-1 + \cos(dx+c)}{\sin(dx+c)} \right) + C \cos(dx+c) (dx+c) + A \sin(dx+c) \right) (\cos(dx+c))^{\frac{3}{2}} (b \cos(dx+c))^{5/2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x)

[Out] 1/d*(-2*B*cos(d*x+c)*arctanh((-1+cos(d*x+c))/sin(d*x+c))+C*cos(d*x+c)*(d*x+c)+A*sin(d*x+c))*cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)

Maxima [A] time = 2.08213, size = 212, normalized size = 2.08

$$\frac{4 A \sqrt{b} \sin(2 dx+2 c)}{b^3 \cos(2 dx+2 c)^2 + b^3 \sin(2 dx+2 c)^2 + 2 b^3 \cos(2 dx+2 c) + b^3} + \frac{B (\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2 \sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2 \sin(dx+c) + 1))}{b^2}$$

$2 d$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/2*(4*A*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + B*(log(cos(d*x + c)^2 + sin(d*x + c)^2 - 2*sin(d*x + c) + 1) - log(cos(d*x + c)^2 + sin(d*x + c)^2 + 2*sin(d*x + c) + 1))

$$\frac{c^2 + 2\sin(dx + c) + 1 - \log(\cos(dx + c)^2 + \sin(dx + c)^2 - 2\sin(dx + c) + 1)}{b^{5/2}} + \frac{4C \arctan(\sin(dx + c)/(\cos(dx + c) + 1))}{b^{5/2}} / d$$

Fricas [A] time = 2.19824, size = 886, normalized size = 8.69

$$\frac{2B\sqrt{-b} \arctan\left(\frac{\sqrt{b}\cos(dx+c)\sqrt{-b}\sin(dx+c)}{b\sqrt{\cos(dx+c)}}\right) \cos(dx+c)^2 + C\sqrt{-b} \cos(dx+c)^2 \log\left(2b\cos(dx+c)^2 + 2\sqrt{b}\cos(dx+c)\right)}{2b^3d \cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)*cos(dx+c)^(1/2)/(b*cos(dx+c))^(5/2),x, algorithm="fricas")

[Out] [-1/2*(2*B*sqrt(-b)*arctan(sqrt(b*cos(dx+c))*sqrt(-b)*sin(dx+c)/(b*sqrt(cos(dx+c))))*cos(dx+c)^2 + C*sqrt(-b)*cos(dx+c)^2*log(2*b*cos(dx+c)^2 + 2*sqrt(b*cos(dx+c))*sqrt(-b)*sqrt(cos(dx+c))*sin(dx+c) - b) - 2*sqrt(b*cos(dx+c))*A*sqrt(cos(dx+c))*sin(dx+c)/(b^3*d*cos(dx+c)^2), 1/2*(2*C*sqrt(b)*arctan(sqrt(b*cos(dx+c))*sin(dx+c)/(sqrt(b)*cos(dx+c)^(3/2)))*cos(dx+c)^2 + B*sqrt(b)*cos(dx+c)^2*log(-b*cos(dx+c)^3 - 2*sqrt(b*cos(dx+c))*sqrt(b)*sqrt(cos(dx+c))*sin(dx+c) - 2*b*cos(dx+c))/cos(dx+c)^3 + 2*sqrt(b*cos(dx+c))*A*sqrt(cos(dx+c))*sin(dx+c)/(b^3*d*cos(dx+c)^2)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)**2)*cos(dx+c)**(1/2)/(b*cos(dx+c))**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sqrt{\cos(dx + c)}}{(b \cos(dx + c))^{5/2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)*cos(dx+c)^(1/2)/(b*cos(dx+c))^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(dx+c)^2 + B*cos(dx+c) + A)*sqrt(cos(dx+c))/(b*cos(dx+c))^(5/2), x)

$$3.336 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt{\cos(c+dx)}(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=120

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}$$

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*SIN[c + d*x])/(2*b^2*d*cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*SIN[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.0919308, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {18, 3021, 2748, 3767, 8, 3770}

$$\frac{(A+2C)\sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{2b^2d\sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{2b^2d \cos^{\frac{3}{2}}(c+dx)\sqrt{b \cos(c+dx)}} + \frac{B \sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}\sqrt{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)), x]

[Out] ((A + 2*C)*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*SIN[c + d*x])/(2*b^2*d*cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + (B*SIN[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*SIN[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*SIN[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*SIN[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*SIN[e + f*x])^m, x], x] + Dist[d/b, Int[(b*SIN[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt{\cos(c + dx)}(b \cos(c + dx))^{5/2}} dx &= \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx}{b^2 \sqrt{b} \cos(c + dx)} \\ &= \frac{A \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{\sqrt{\cos(c + dx)} \int (2B + (A + 2C) \cos(c + dx)) \sec^2(c + dx) dx}{2b^2 \sqrt{b} \cos(c + dx)} \\ &= \frac{A \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^2(c + dx) dx}{b^2 \sqrt{b} \cos(c + dx)} \\ &= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2b^2 d \sqrt{b} \cos(c + dx)} + \frac{A \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} \\ &= \frac{(A + 2C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2b^2 d \sqrt{b} \cos(c + dx)} + \frac{A \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} \end{aligned}$$

Mathematica [A] time = 0.0942971, size = 69, normalized size = 0.57

$$\frac{\sqrt{\cos(c + dx)} (\sin(c + dx)(A + 2B \cos(c + dx)) + (A + 2C) \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{2d(b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^(5/2)),x]

[Out] (Sqrt[Cos[c + d*x]]*((A + 2*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (A + 2*B*Cos[c + d*x])*Sin[c + d*x]))/(2*d*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.404, size = 151, normalized size = 1.3

$$-\frac{1}{2d} \left(A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) - A (\cos(dx + c))^2 \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x)

[Out] -1/2/d*(A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))-A*cos(d*x+c)^2*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))+4*C*cos(d*x+c)^2*arctanh((-1+cos(d*x+c))/sin(d*x+c))-2*B*sin(d*x+c)*cos(d*x+c)-A*sin(d*x+c))*cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2)

Maxima [B] time = 2.34179, size = 1107, normalized size = 9.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out]
$$\frac{1}{4} \cdot \frac{(8B\sqrt{b}\sin(2dx+2c)/(b^3\cos(2dx+2c)^2 + b^3\sin(2dx+2c)^2 + 2b^3\cos(2dx+2c) + b^3) - (4(\sin(4dx+4c) + 2\sin(2dx+2c))\cos(3/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))) - 4(\sin(4dx+4c) + 2\sin(2dx+2c))\cos(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))) - (2(2\cos(2dx+2c) + 1)\cos(4dx+4c) + \cos(4dx+4c))^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c)\sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1)\log(\cos(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 + \sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 + 2\sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))) + 1) + (2(2\cos(2dx+2c) + 1)\cos(4dx+4c) + \cos(4dx+4c))^2 + 4\cos(2dx+2c)^2 + \sin(4dx+4c)^2 + 4\sin(4dx+4c)\sin(2dx+2c) + 4\sin(2dx+2c)^2 + 4\cos(2dx+2c) + 1)\log(\cos(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 + \sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))^2 - 2\sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c)))) + 1) - 4(\cos(4dx+4c) + 2\cos(2dx+2c) + 1)\sin(3/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))) + 4(\cos(4dx+4c) + 2\cos(2dx+2c) + 1)\sin(1/2\arctan2(\sin(2dx+2c), \cos(2dx+2c))))}{A((b^2\cos(4dx+4c)^2 + 4b^2\cos(2dx+2c)^2 + b^2\sin(4dx+4c)^2 + 4b^2\sin(4dx+4c)\sin(2dx+2c) + 4b^2\sin(2dx+2c)^2 + 4b^2\cos(2dx+2c) + b^2 + 2(2b^2\cos(2dx+2c) + b^2)\cos(4dx+4c))\sqrt{b}) + 2C(\log(\cos(dx+c)^2 + \sin(dx+c)^2 + 2\sin(dx+c) + 1) - \log(\cos(dx+c)^2 + \sin(dx+c)^2 - 2\sin(dx+c) + 1))/b^{5/2}}/d$$

Fricas [A] time = 1.71653, size = 659, normalized size = 5.49

$$\frac{(A + 2C)\sqrt{b}\cos(dx+c)^3 \log\left(\frac{-b\cos(dx+c)^3 - 2\sqrt{b\cos(dx+c)}\sqrt{b\cos(dx+c)}\sin(dx+c) - 2b\cos(dx+c)}{\cos(dx+c)^3}\right) + 2(2B\cos(dx+c) + A)\sqrt{b\cos(dx+c)}}{4b^3d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algorithm="fricas")

[Out]
$$\left[\frac{1}{4} \cdot \frac{(A + 2C)\sqrt{b}\cos(dx+c)^3 \log(-b\cos(dx+c)^3 - 2\sqrt{b\cos(dx+c)}\sqrt{b\cos(dx+c)}\sin(dx+c) - 2b\cos(dx+c))/\cos(dx+c)^3 + 2(2B\cos(dx+c) + A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{(b^3d\cos(dx+c)^3)}, -\frac{1}{2} \cdot \frac{(A + 2C)\sqrt{-b}\arctan(\sqrt{b\cos(dx+c)}\sqrt{-b}\sin(dx+c)/(b\sqrt{\cos(dx+c)}))\cos(dx+c)^3 - (2B\cos(dx+c) + A)\sqrt{b\cos(dx+c)}\sqrt{\cos(dx+c)}\sin(dx+c)}{(b^3d\cos(dx+c)^3)} \right]$$

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(5/2)/cos(d*x+c)
**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}} \sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(5/2)/cos(d*x+c)^(
1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*s
qrt(cos(d*x + c))), x)
```

$$3.337 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=164

$$\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3b^2d\cos^2(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2b^2d\cos^2(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}}{2b^2d}$$

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rubi [A] time = 0.110315, antiderivative size = 164, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {18, 3021, 2748, 3768, 3770, 3767, 8}

$$\frac{(2A+3C)\sin(c+dx)}{3b^2d\sqrt{\cos(c+dx)}\sqrt{b\cos(c+dx)}} + \frac{A\sin(c+dx)}{3b^2d\cos^2(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sin(c+dx)}{2b^2d\cos^2(c+dx)\sqrt{b\cos(c+dx)}} + \frac{B\sqrt{\cos(c+dx)}}{2b^2d}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)), x]

[Out] (B*ArcTanh[Sin[c + d*x]]*Sqrt[Cos[c + d*x]])/(2*b^2*d*Sqrt[b*Cos[c + d*x]]) + (A*Sin[c + d*x])/(3*b^2*d*Cos[c + d*x]^(5/2)*Sqrt[b*Cos[c + d*x]]) + (B*Sin[c + d*x])/(2*b^2*d*Cos[c + d*x]^(3/2)*Sqrt[b*Cos[c + d*x]]) + ((2*A + 3*C)*Sin[c + d*x])/(3*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Cos[c + d*x]])

Rule 18

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(a^(m - 1/2)*b^(n + 1/2)*Sqrt[a*v])/Sqrt[b*v], Int[u*v^(m + n), x], x] /; FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 3768

Int[(csc[(c_.) + (d_.)*(x_)]*(b_))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x])*(b*Csc[c + d*x])^(n - 1)/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I

nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 3770

Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]

Rule 3767

Int[csc[(c_.) + (d_.)*(x_)^(n_.), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx}{b^2 \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{\sqrt{\cos(c + dx)} \int (3B + (2A + 3C) \cos(c + dx)) \sec^3(c + dx) dx}{3b^2 \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^3(c + dx) dx}{b^2 \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{B \sin(c + dx)}{2b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)}$$

$$= \frac{B \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{2b^2 d \sqrt{b} \cos(c + dx)} + \frac{A \sin(c + dx)}{3b^2 d \cos^{\frac{5}{2}}(c + dx) \sqrt{b} \cos(c + dx)}$$

Mathematica [A] time = 0.295958, size = 87, normalized size = 0.53

$$\frac{\sqrt{\cos(c + dx)} (\tan(c + dx)((2A + 3C) \cos(2(c + dx)) + 4A + 3B \cos(c + dx) + 3C) + 3B \cos^2(c + dx) \tanh^{-1}(\sin(c + dx)))}{6d(b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2)), x]

[Out] (Sqrt[Cos[c + d*x]]*(3*B*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^2 + (4*A + 3*C + 3*B*Cos[c + d*x] + (2*A + 3*C)*Cos[2*(c + d*x)])*Tan[c + d*x]))/(6*d*(b*Cos[c + d*x])^(5/2))

Maple [A] time = 0.434, size = 157, normalized size = 1.

$$\frac{1}{6d} \left(-3B \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 + 3B \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^3 \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/6/d*(-3*B*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+3*B*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^3+4*A*sin(d*x+c)*cos(d*x+c)^2+6*C*sin(d*x+c)*cos(d*x+c)^2+3*B*sin(d*x+c)*cos(d*x+c)+2*A*sin(d*x+c))/cos(d*x+c)^(1/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] time = 2.38673, size = 1482, normalized size = 9.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(24*C*sqrt(b)*sin(2*d*x + 2*c)/(b^3*cos(2*d*x + 2*c)^2 + b^3*sin(2*d*x + 2*c)^2 + 2*b^3*cos(2*d*x + 2*c) + b^3) + 16*((3*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(3*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 9*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*A/((b^2*cos(6*d*x + 6*c)^2 + 9*b^2*cos(4*d*x + 4*c)^2 + 9*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(6*d*x + 6*c)^2 + 9*b^2*sin(4*d*x + 4*c)^2 + 18*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*b^2*sin(2*d*x + 2*c)^2 + 6*b^2*cos(2*d*x + 2*c) + b^2 + 2*(3*b^2*cos(4*d*x + 4*c) + 3*b^2*cos(2*d*x + 2*c) + b^2)*cos(6*d*x + 6*c) + 6*(3*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c) + 6*(b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*sqrt(b)) - 3*(4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + (2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))^2 - 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/((b^2*cos(4*d*x + 4*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(4*d*x + 4*c)^2 + 4*b^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*b^2*sin(2*d*x + 2*c)^2 + 4*b^2*cos(2*d*x + 2*c) + b^2 + 2*(2*b^2*cos(2*d*x + 2*c) + b^2)*cos(4*d*x + 4*c))*sqrt(b))/d
```

Fricas [A] time = 1.60051, size = 733, normalized size = 4.47

$$\frac{3B\sqrt{b}\cos(dx+c)^4\log\left(\frac{-b\cos(dx+c)^3-2\sqrt{b}\cos(dx+c)\sqrt{b}\sqrt{\cos(dx+c)}\sin(dx+c)-2b\cos(dx+c)}{\cos(dx+c)^3}\right)+2\left(2(2A+3C)\cos(dx+c)^2+3B\cos(dx+c)\right)}{12b^3d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*B*sqrt(b)*cos(d*x + c)^4*log(-(b*cos(d*x + c))^3 - 2*sqrt(b*cos(d*x + c))*sqrt(b)*sqrt(cos(d*x + c))*sin(d*x + c) - 2*b*cos(d*x + c))/cos(d*x + c)^3) + 2*(2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^4), -1/6*(3*B*sqrt(-b)*arctan(sqrt(b*cos(d*x + c))*sqrt(-b)*sin(d*x + c)/(b*sqrt(cos(d*x + c))))*cos(d*x + c)^4 - (2*(2*A + 3*C)*cos(d*x + c)^2 + 3*B*cos(d*x + c) + 2*A)*sqrt(b*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(b^3*d*cos(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2)/(b*cos(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}} \cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x+ c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*cos(d*x + c)^(3/2)), x)
```

$$3.338 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\cos^2(c+dx)(b \cos(c+dx))^{5/2}} dx$$

Optimal. Leaf size=208

$$\frac{(3A+4C) \sin(c+dx)}{8b^2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{(3A+4C) \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8b^2d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4b^2d \cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \dots$$

[Out] $((3A + 4C) \text{ArcTanh}[\text{Sin}[c + dx]] \text{Sqrt}[\text{Cos}[c + dx]]) / (8b^2d \text{Sqrt}[b \text{Cos}[c + dx]]) + (A \text{Sin}[c + dx]) / (4b^2d \text{Cos}[c + dx]^{7/2} \text{Sqrt}[b \text{Cos}[c + dx]]) + ((3A + 4C) \text{Sin}[c + dx]) / (8b^2d \text{Cos}[c + dx]^{3/2} \text{Sqrt}[b \text{Cos}[c + dx]]) + (B \text{Sin}[c + dx]) / (b^2d \text{Sqrt}[\text{Cos}[c + dx]] \text{Sqrt}[b \text{Cos}[c + dx]]) + (B \text{Sin}[c + dx]^3) / (3b^2d \text{Cos}[c + dx]^{5/2} \text{Sqrt}[b \text{Cos}[c + dx]])$

Rubi [A] time = 0.134648, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 43, $\frac{\text{number of rules}}{\text{integrand size}} = 0.14$, Rules used = {18, 3021, 2748, 3767, 3768, 3770}

$$\frac{(3A+4C) \sin(c+dx)}{8b^2d \cos^{\frac{3}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \frac{(3A+4C) \sqrt{\cos(c+dx)} \tanh^{-1}(\sin(c+dx))}{8b^2d \sqrt{b \cos(c+dx)}} + \frac{A \sin(c+dx)}{4b^2d \cos^{\frac{7}{2}}(c+dx) \sqrt{b \cos(c+dx)}} + \dots$$

Antiderivative was successfully verified.

[In] $\text{Int}[(A + B \text{Cos}[c + dx] + C \text{Cos}[c + dx]^2) / (\text{Cos}[c + dx]^{5/2} (b \text{Cos}[c + dx])^{5/2}), x]$

[Out] $((3A + 4C) \text{ArcTanh}[\text{Sin}[c + dx]] \text{Sqrt}[\text{Cos}[c + dx]]) / (8b^2d \text{Sqrt}[b \text{Cos}[c + dx]]) + (A \text{Sin}[c + dx]) / (4b^2d \text{Cos}[c + dx]^{7/2} \text{Sqrt}[b \text{Cos}[c + dx]]) + ((3A + 4C) \text{Sin}[c + dx]) / (8b^2d \text{Cos}[c + dx]^{3/2} \text{Sqrt}[b \text{Cos}[c + dx]]) + (B \text{Sin}[c + dx]) / (b^2d \text{Sqrt}[\text{Cos}[c + dx]] \text{Sqrt}[b \text{Cos}[c + dx]]) + (B \text{Sin}[c + dx]^3) / (3b^2d \text{Cos}[c + dx]^{5/2} \text{Sqrt}[b \text{Cos}[c + dx]])$

Rule 18

$\text{Int}[(u_.) * ((a_.) * (v_))^{(m_)} * ((b_.) * (v_))^{(n_)}, x_Symbol] \rightarrow \text{Dist}[(a^{(m-1/2)} * b^{(n+1/2)} \text{Sqrt}[a*v]) / \text{Sqrt}[b*v], \text{Int}[u*v^{(m+n)}, x], x] /;$ FreeQ[{a, b, m}, x] && !IntegerQ[m] && ILtQ[n - 1/2, 0] && IntegerQ[m + n]

Rule 3021

$\text{Int}[(a_.) + (b_.) * \sin[(e_.) + (f_.) * (x_)]]^{(m_)} * ((A_.) + (B_.) * \sin[(e_.) + (f_.) * (x_)] + (C_.) * \sin[(e_.) + (f_.) * (x_)]^2), x_Symbol] \rightarrow -\text{Simp}[(A*b^2 - a*b*B + a^2*C) * \text{Cos}[e + f*x] * (a + b * \text{Sin}[e + f*x])^{(m+1)} / (b*f*(m+1) * (a^2 - b^2)), x] + \text{Dist}[1 / (b*(m+1) * (a^2 - b^2)), \text{Int}[(a + b * \text{Sin}[e + f*x])^{(m+1)} * \text{Simp}[b*(a*A - b*B + a*C) * (m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C) * (m+1)) * \text{Sin}[e + f*x], x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

$\text{Int}[(b_.) * \sin[(e_.) + (f_.) * (x_)]]^{(m_)} * ((c_.) + (d_.) * \sin[(e_.) + (f_.) * (x_)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b * \text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b * \text{Sin}[e + f*x])^{(m+1)}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := -Dist[d^(-1), Subst[Int[ExpandIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] := -Simp[(b*Cos[c + d*x]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] := -Simp[ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\cos^{\frac{5}{2}}(c + dx)(b \cos(c + dx))^{5/2}} dx = \frac{\sqrt{\cos(c + dx)} \int (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^5(c + dx) dx}{b^2 \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{\sqrt{\cos(c + dx)} \int (4B + (3A + 4C) \cos(c + dx)) \sec^4(c + dx) dx}{4b^2 \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{(B \sqrt{\cos(c + dx)}) \int \sec^4(c + dx) dx}{b^2 \sqrt{b} \cos(c + dx)}$$

$$= \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{(3A + 4C) \sin(c + dx)}{8b^2 d \cos^{\frac{3}{2}}(c + dx) \sqrt{b} \cos(c + dx)} + \frac{(3A + 4C) \tanh^{-1}(\sin(c + dx)) \sqrt{\cos(c + dx)}}{8b^2 d \sqrt{b} \cos(c + dx)} + \frac{A \sin(c + dx)}{4b^2 d \cos^{\frac{7}{2}}(c + dx) \sqrt{b} \cos(c + dx)}$$

Mathematica [A] time = 0.214122, size = 110, normalized size = 0.53

$$\frac{\sin(c + dx) (3(3A + 4C) \cos^2(c + dx) + 6A + 24B \cos^3(c + dx) + 8B \sin^2(c + dx) \cos(c + dx)) + 3(3A + 4C) \cos^4(c + dx)}{24d \cos^{\frac{3}{2}}(c + dx)(b \cos(c + dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^(5/2)), x]
```

```
[Out] (3*(3*A + 4*C)*ArcTanh[Sin[c + d*x]]*Cos[c + d*x]^4 + Sin[c + d*x]*(6*A + 3*(3*A + 4*C)*Cos[c + d*x]^2 + 24*B*Cos[c + d*x]^3 + 8*B*Cos[c + d*x]*Sin[c + d*x]^2))/(24*d*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^(5/2))
```

Maple [A] time = 0.293, size = 248, normalized size = 1.2

$$\frac{1}{24d} \left(9A \ln \left(-\frac{-1 + \cos(dx + c) - \sin(dx + c)}{\sin(dx + c)} \right) (\cos(dx + c))^4 - 9A (\cos(dx + c))^4 \ln \left(-\frac{-1 + \cos(dx + c) + \sin(dx + c)}{\sin(dx + c)} \right) \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x)
```

```
[Out] 1/24/d*(9*A*ln(-(-1+cos(d*x+c)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-9*A*cos
(d*x+c)^4*ln(-(-1+cos(d*x+c)+sin(d*x+c))/sin(d*x+c))+12*C*ln(-(-1+cos(d*x+c)
)-sin(d*x+c))/sin(d*x+c))*cos(d*x+c)^4-12*C*cos(d*x+c)^4*ln(-(-1+cos(d*x+c)
+sin(d*x+c))/sin(d*x+c))+16*B*cos(d*x+c)^3*sin(d*x+c)+9*A*sin(d*x+c)*cos(d*
x+c)^2+12*C*sin(d*x+c)*cos(d*x+c)^2+8*B*sin(d*x+c)*cos(d*x+c)+6*A*sin(d*x+c
))/cos(d*x+c)^(3/2)/(b*cos(d*x+c))^(5/2)
```

Maxima [B] time = 2.60893, size = 3726, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(12*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) +
4*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
44*(sin(8*d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*
x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 44*(sin(8*
d*x + 8*c) + 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*
cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 12*(sin(8*d*x + 8*c)
+ 4*sin(6*d*x + 6*c) + 6*sin(4*d*x + 4*c) + 4*sin(2*d*x + 2*c))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*(2*(4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + cos(8*d*x + 8*
c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 1
6*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 36*co
s(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 + 4*(2*sin(6*d*x + 6*c) + 3*sin(4*
d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 16
*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 16*sin(6*d*x
+ 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 1
6*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) + 1)*log(cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))^2 + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1)
+ 3*(2*(4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*c
os(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 8*(6*cos(4*d*x + 4*c) + 4*cos(2*d*x
+ 2*c) + 1)*cos(6*d*x + 6*c) + 16*cos(6*d*x + 6*c)^2 + 12*(4*cos(2*d*x + 2*
c) + 1)*cos(4*d*x + 4*c) + 36*cos(4*d*x + 4*c)^2 + 16*cos(2*d*x + 2*c)^2 +
4*(2*sin(6*d*x + 6*c) + 3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*sin(8*d*x
+ 8*c) + sin(8*d*x + 8*c)^2 + 16*(3*sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 16*sin(6*d*x + 6*c)^2 + 36*sin(4*d*x + 4*c)^2 + 48*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 16*sin(2*d*x + 2*c)^2 + 8*cos(2*d*x + 2*c) +
1)*log(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 1) - 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c)
+ 6*cos(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(7/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*c
os(4*d*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(5/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 44*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d
*x + 4*c) + 4*cos(2*d*x + 2*c) + 1)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*(cos(8*d*x + 8*c) + 4*cos(6*d*x + 6*c) + 6*cos(4*d*x + 4
*c) + 4*cos(2*d*x + 2*c) + 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))))*A/((b^2*cos(8*d*x + 8*c)^2 + 16*b^2*cos(6*d*x + 6*c)^2 + 36*b^2*co
s(4*d*x + 4*c)^2 + 16*b^2*cos(2*d*x + 2*c)^2 + b^2*sin(8*d*x + 8*c)^2 + 16*
```


$$\begin{aligned}
& b^2 \sin(6dx + 6c)^2 + 36b^2 \sin(4dx + 4c)^2 + 48b^2 \sin(4dx + 4c) \\
& \sin(2dx + 2c) + 16b^2 \sin(2dx + 2c)^2 + 8b^2 \cos(2dx + 2c) + b^2 \\
& + 2(4b^2 \cos(6dx + 6c) + 6b^2 \cos(4dx + 4c) + 4b^2 \cos(2dx + 2c) \\
& + b^2) \cos(8dx + 8c) + 8(6b^2 \cos(4dx + 4c) + 4b^2 \cos(2dx + 2c) \\
& + b^2) \cos(6dx + 6c) + 12(4b^2 \cos(2dx + 2c) + b^2) \cos(4dx + 4c) \\
& + 4(2b^2 \sin(6dx + 6c) + 3b^2 \sin(4dx + 4c) + 2b^2 \sin(2dx + 2c)) \\
& \sin(8dx + 8c) + 16(3b^2 \sin(4dx + 4c) + 2b^2 \sin(2dx + 2c)) \\
& \sin(6dx + 6c) \sqrt{b} - 64((3 \cos(2dx + 2c) + 1) \sin(6dx + 6c) \\
& + 3(3 \cos(2dx + 2c) + 1) \sin(4dx + 4c) - 3 \cos(6dx + 6c) \sin(2dx + 2c) \\
& - 9 \cos(4dx + 4c) \sin(2dx + 2c)) B / ((b^2 \cos(6dx + 6c)^2 + 9b^2 \cos(4dx + 4c)^2 \\
& + 9b^2 \cos(2dx + 2c)^2 + b^2 \sin(6dx + 6c)^2 + 9b^2 \sin(4dx + 4c)^2 \\
& + 18b^2 \sin(4dx + 4c) \sin(2dx + 2c) + 9b^2 \sin(2dx + 2c)^2 + 6b^2 \cos(2dx + 2c) \\
& + b^2 + 2(3b^2 \cos(4dx + 4c) + 3b^2 \cos(2dx + 2c) + b^2) \cos(6dx + 6c) \\
& + 6(3b^2 \cos(2dx + 2c) + b^2) \cos(4dx + 4c) + 6(b^2 \sin(4dx + 4c) + b^2 \sin(2dx + 2c)) \\
& \sin(6dx + 6c) \sqrt{b} + 12(4(\sin(4dx + 4c) + 2 \sin(2dx + 2c)) \cos(3/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& - 4(\sin(4dx + 4c) + 2 \sin(2dx + 2c)) \cos(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c))) \\
& - (2(2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4 \cos(2dx + 2c)^2 \\
& + \sin(4dx + 4c)^2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c)^2 + 4 \cos(2dx + 2c) + 1) \log(\cos(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + 2 \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) + (2(2 \cos(2dx + 2c) + 1) \cos(4dx + 4c) + \cos(4dx + 4c)^2 + 4 \cos(2dx + 2c)^2 + \sin(4dx + 4c)^2 + 4 \sin(4dx + 4c) \sin(2dx + 2c) + 4 \sin(2dx + 2c)^2 + 4 \cos(2dx + 2c) + 1) \log(\cos(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 + \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c)))^2 - 2 \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c))) + 1) - 4(\cos(4dx + 4c) + 2 \cos(2dx + 2c) + 1) \sin(3/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c))) + 4(\cos(4dx + 4c) + 2 \cos(2dx + 2c) + 1) \sin(1/2 \arctan 2(\sin(2dx + 2c), \cos(2dx + 2c)))) C / ((b^2 \cos(4dx + 4c)^2 + 4b^2 \cos(2dx + 2c)^2 + b^2 \sin(4dx + 4c)^2 + 4b^2 \sin(4dx + 4c) \sin(2dx + 2c) + 4b^2 \sin(2dx + 2c)^2 + 4b^2 \cos(2dx + 2c) + b^2 + 2(2b^2 \cos(2dx + 2c) + b^2) \cos(4dx + 4c)) \sqrt{b}) / d
\end{aligned}$$

Fricas [A] time = 1.73283, size = 821, normalized size = 3.95

$$\left[\frac{3(3A + 4C) \sqrt{b} \cos(dx + c)^5 \log\left(-\frac{b \cos(dx+c)^3 - 2\sqrt{b \cos(dx+c)} \sqrt{b \cos(dx+c)} \sin(dx+c) - 2b \cos(dx+c)}{\cos(dx+c)^3}\right) + 2(16B \cos(dx + c)^3 + 3(3A + 4C) \cos(dx + c)^2 + 8B \cos(dx + c) + 6A) \sqrt{b \cos(dx + c)} \sqrt{\cos(dx + c)} \sin(dx + c)}{48b^3 d \cos(dx + c)^5} \right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(dx+c)+C*cos(dx+c)^2)/cos(dx+c)^(5/2)/(b*cos(dx+c))^(5/2),x, algorithm="fricas")

[Out] [1/48*(3*(3*A + 4*C)*sqrt(b)*cos(dx + c)^5*log(-(b*cos(dx + c))^3 - 2*sqrt(b*cos(dx + c))*sqrt(b)*sqrt(cos(dx + c))*sin(dx + c) - 2*b*cos(dx + c))/cos(dx + c)^3 + 2*(16*B*cos(dx + c)^3 + 3*(3*A + 4*C)*cos(dx + c)^2 + 8*B*cos(dx + c) + 6*A)*sqrt(b*cos(dx + c))*sqrt(cos(dx + c))*sin(dx + c))/(b^3*d*cos(dx + c)^5), -1/24*(3*(3*A + 4*C)*sqrt(-b)*arctan(sqrt(b*cos(dx + c))*sqrt(-b)*sin(dx + c)/(b*sqrt(cos(dx + c))))*cos(dx + c)^5 - (16*B*cos(dx + c)^3 + 3*(3*A + 4*C)*cos(dx + c)^2 + 8*B*cos(dx + c) + 6*A)*sqrt(b*cos(dx + c))*sqrt(cos(dx + c))*sin(dx + c))/(b^3*d*cos(dx + c)^5)]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2)/(b*cos(d*x+c))
**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{5}{2}} \cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2)/(b*cos(d*x+c))^(
5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/((b*cos(d*x + c))^(5/2)*c
os(d*x + c)^(5/2)), x)

3.339 $\int \cos(c+dx)(b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2$

Optimal. Leaf size=154

$$\frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^2d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)}{11b^3d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.153096, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^2d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)}{11b^3d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^2*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{5/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^2d} + \frac{3 \int (b \cos(c + dx))^{5/3} dx}{11b^2d} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^2d} + \frac{B \int (b \cos(c + dx))^{5/3} dx}{11b^2d} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^2d} - \frac{3(11A + 8C)}{11b^2d} \int (b \cos(c + dx))^{5/3} dx \end{aligned}$$

Mathematica [A] time = 0.243082, size = 109, normalized size = 0.71

$$\frac{3 \sin(c + dx)(b \cos(c + dx))^{8/3} \left((11A + 8C) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) + 8B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right) - 8C \right)}{88b^2d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2), x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x]*((11*A + 8*C)*Hypergeometric2F1[1/2
, 4/3, 7/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6,
17/6, Cos[c + d*x]^2] - 8*C*Sqrt[Sin[c + d*x]^2]))/(88*b^2*d*Sqrt[Sin[c +
d*x]^2])
```

Maple [F] time = 0.316, size = 0, normalized size = 0.

$$\int \cos(dx + c)(b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{2/3} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)\right)(b \cos(dx + c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c), x)

3.340 $\int (b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=154

$$\frac{3(8A + 5C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40bd\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{8b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.135087, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3023, 2748, 2643}

$$\frac{3(8A + 5C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40bd\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{8b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{3 \int (b \cos(c + dx))^2}{8bd} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{B \int (b \cos(c + dx))}{b} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} - \frac{3(8A + 5C)(b \cos(c + dx))^{5/3}}{8bd} \end{aligned}$$

Mathematica [A] time = 0.2199, size = 109, normalized size = 0.71

$$\frac{3 \sin(c + dx)(b \cos(c + dx))^{5/3} \left((8A + 5C) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) + 5B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) - 5C \right)}{40bd \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^(2/3)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2), x]

[Out] (-3*(b*cos[c + d*x])^(5/3)*Sin[c + d*x]*((8*A + 5*C)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] + 5*B*cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2] - 5*C*Sqrt[Sin[c + d*x]^2]))/(40*b*d*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.31, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3), x)

$$3.341 \quad \int (b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal. Leaf size=148

$$\frac{3(5A + 2C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{10d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.156137, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(5A + 2C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{10d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5d} + \frac{3}{5} \int \frac{\frac{1}{3}b(5A + 2C)}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5d} + B \int (b \cos(c + dx))^{2/3} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5d} - \frac{3(5A + 2C)}{5d} \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx \end{aligned}$$

Mathematica [A] time = 0.220542, size = 109, normalized size = 0.74

$$\frac{3b \sin(2(c + dx)) \left((5A + 2C) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) + 2B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) - 2C \sqrt{\sin^2(c + dx)} \right)}{20d \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec
c[c + d*x], x]
```

```
[Out] (-3*b*((5*A + 2*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*C
os[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] - 2*C*Sqrt[Si
n[c + d*x]^2])*Sin[2*(c + d*x)])/(20*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Si
n[c + d*x]^2])
```

Maple [F] time = 0.378, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{2/3} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c), x)

3.342 $\int (b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=147

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}} + \frac{3Ab \sin(c + dx)}{d\sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3}}{2d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.185734, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5bd\sqrt{\sin^2(c + dx)}} + \frac{3Ab \sin(c + dx)}{d\sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3}}{2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (3*A*b*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx = b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx$$

$$= \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + \frac{3 \int \frac{b^{2/3} - \frac{1}{3} b^2 (2A - C) \cos(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx}{b}$$

$$= \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + (bB) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx$$

$$= \frac{3Ab \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} - \frac{3B(b \cos(c + dx))^{2/3}}{2d}$$

Mathematica [A] time = 0.347397, size = 116, normalized size = 0.79

$$\frac{3b \sqrt{\sin^2(c + dx)} \left(\cot(c + dx) \left(5B {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) + 2C \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \right) - 10A \cos(c + dx) \right)}{10d \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]
```

```
[Out] (-3*b*(-10*A*Csc[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] + Cot[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(1/3))
```

Maple [F] time = 0.401, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{2/3} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2, x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2, x)

$$3.343 \quad \int (b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal. Leaf size=145

$$\frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}} + \frac{3bB \sin(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}\sqrt[3]{b \cos(c + dx)}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*b*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.192641, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} - \frac{3(A + 4C) \sin(c + dx)(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}} + \frac{3bB \sin(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt{\sin^2(c + dx)}\sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*b*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)^2], x_Symbol] :=> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{3}{4} \int \frac{\frac{4b^2B}{3} + \frac{1}{3}b^2(A + C \cos^2(c + dx))}{(b \cos(c + dx))^{4/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + (b^2B) \int \frac{1}{(b \cos(c + dx))^{4/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{3bB {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{d\sqrt[3]{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.237174, size = 123, normalized size = 0.85

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^2(c + dx) (b \cos(c + dx))^{2/3} \left(2 \cos(c + dx) \left(C \cos(c + dx) {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) - 2B\right)\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(-(A*Hypergeometric2F1[-2/3, 1/2, 1
/3, Cos[c + d*x]^2]) + 2*Cos[c + d*x]*(-2*B*Hypergeometric2F1[-1/6, 1/2, 5/
6, Cos[c + d*x]^2] + C*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c
+ d*x]^2]))*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]^2])/(4*d)
```

Maple [F] time = 0.41, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{2/3} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3, x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3, x)

3.344 $\int (b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=152

$$\frac{3Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3b(4A + 7C) \sin(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{7d\sqrt{\sin^2(c + dx)}\sqrt[3]{b \cos(c + dx)}} + \frac{3b^2B \sin(c + dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}(b \cos(c + dx))^{4/3}}$$

[Out] (3*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b^2*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.193104, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3b(4A + 7C) \sin(c + dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right)}{7d\sqrt{\sin^2(c + dx)}\sqrt[3]{b \cos(c + dx)}} + \frac{3b^2B \sin(c + dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right)}{4d\sqrt{\sin^2(c + dx)}(b \cos(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4, x]

[Out] (3*A*b^3*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b^2*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Ssin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx = b^4 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{10/3}} dx$$

$$= \frac{3Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{1}{7}(3b) \int \frac{\frac{7b^2B}{3} + \frac{1}{3}b^3}{(b \cos(c + dx))^{10/3}} dx$$

$$= \frac{3Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + (b^3B) \int \frac{1}{(b \cos(c + dx))^{10/3}} dx$$

$$= \frac{3Ab^3 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3b^2B {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right)}{4d(b \cos(c + dx))^{7/3}}$$

Mathematica [A] time = 0.205749, size = 123, normalized size = 0.81

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^3(c + dx) (b \cos(c + dx))^{2/3} \left(4A {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right) + 7 \cos(c + dx) \left(B {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}, \frac{1}{3}, \cos^2(c + dx)\right) + 4C \cos(c + dx) \operatorname{Hypergeometric2F1}\left[-\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \cos^2(c + dx)\right]\right)\right) \sec^4(c + dx)}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] (3*(b*Cos[c + d*x])^(2/3)*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(28*d)
```

Maple [F] time = 0.432, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{2/3} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4, x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^4, x)

3.345 $\int \cos(c+dx)(b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2$

Optimal. Leaf size=154

$$\frac{3(13A + 10C) \sin(c + dx)(b \cos(c + dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)}{130b^2d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{13/3} {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c + dx)\right)}{13b^3d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.150349, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(13A + 10C) \sin(c + dx)(b \cos(c + dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)}{130b^2d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{13/3} {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c + dx)\right)}{13b^3d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*C*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x])/(13*b^2*d) - (3*(13*A + 10*C)*(b*Cos[c + d*x])^(10/3)*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2]*Sin[c + d*x])/(130*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(13/3)*Hypergeometric2F1[1/2, 13/6, 19/6, Cos[c + d*x]^2]*Sin[c + d*x])/(13*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \cos(c + dx)(b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{7/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{3C(b \cos(c + dx))^{10/3} \sin(c + dx)}{13b^2d} + \frac{3 \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx)) dx}{13b^2d} \\ &= \frac{3C(b \cos(c + dx))^{10/3} \sin(c + dx)}{13b^2d} + \frac{B \int (b \cos(c + dx))^{4/3} dx}{13b^2d} \\ &= \frac{3C(b \cos(c + dx))^{10/3} \sin(c + dx)}{13b^2d} - \frac{3(13A + 10C)}{13b^2d} \int (b \cos(c + dx))^{4/3} dx \end{aligned}$$

Mathematica [A] time = 0.352139, size = 111, normalized size = 0.72

$$\frac{3 \sin(c + dx)(b \cos(c + dx))^{10/3} \left((13A + 10C) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right) + 10 \left(B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{13}{6}; \frac{19}{6}; \cos^2(c + dx)\right) \right) \right)}{130b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c
+ d*x]^2), x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(10/3)*Sin[c + d*x]*((13*A + 10*C)*Hypergeometric2F1[1
/2, 5/3, 8/3, Cos[c + d*x]^2] + 10*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, 1
3/6, 19/6, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))) / (130*b^2*d*Sqrt[Sin[
c + d*x]^2])
```

Maple [F] time = 0.322, size = 0, normalized size = 0.

$$\int \cos(dx + c)(b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{4/3} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^4 + Bb \cos(dx + c)^3 + Ab \cos(dx + c)^2\right) (b \cos(dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^4 + B*b*cos(d*x + c)^3 + A*b*cos(d*x + c)^2)*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c), x)

3.346 $\int (b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=154

$$\frac{3(10A + 7C) \sin(c + dx)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{70bd\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)}{10b^2d\sqrt{\sin^2(c + dx)}}$$

[Out] $(3*C*(b*\text{Cos}[c + d*x])^{7/3}*\text{Sin}[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*\text{Cos}[c + d*x])^{7/3}*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(70*b*d*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*(b*\text{Cos}[c + d*x])^{10/3}*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(10*b^2*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rubi [A] time = 0.128679, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3023, 2748, 2643}

$$\frac{3(10A + 7C) \sin(c + dx)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right)}{70bd\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{10/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right)}{10b^2d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^{4/3}*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2), x]$

[Out] $(3*C*(b*\text{Cos}[c + d*x])^{7/3}*\text{Sin}[c + d*x])/(10*b*d) - (3*(10*A + 7*C)*(b*\text{Cos}[c + d*x])^{7/3}*\text{Hypergeometric2F1}[1/2, 7/6, 13/6, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(70*b*d*\text{Sqrt}[\text{Sin}[c + d*x]^2]) - (3*B*(b*\text{Cos}[c + d*x])^{10/3}*\text{Hypergeometric2F1}[1/2, 5/3, 8/3, \text{Cos}[c + d*x]^2]*\text{Sin}[c + d*x])/(10*b^2*d*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 3023

$\text{Int}[(a_.) + (b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(m_.)}*((A_.) + (B_.)*\text{sin}[(e_.) + (f_.)*(x_)] + (C_.)*\text{sin}[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\text{Sin}[e + f*x])^{(m + 1)})/(b*f*(m + 2)), x] + \text{Dist}[1/(b*(m + 2)), \text{Int}[(a + b*\text{Sin}[e + f*x])^m*\text{Simp}[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*\text{Sin}[e + f*x], x], x], x] /;$ $\text{FreeQ}\{a, b, e, f, A, B, C, m\}, x\} \&\amp; !\text{LtQ}[m, -1]$

Rule 2748

$\text{Int}[(b_.)*\text{sin}[(e_.) + (f_.)*(x_)]^{(m_.)}*((c_.) + (d_.)*\text{sin}[(e_.) + (f_.)*(x_)]), x_Symbol] :> \text{Dist}[c, \text{Int}[(b*\text{Sin}[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e + f*x])^{(m + 1)}, x], x] /;$ $\text{FreeQ}\{b, c, d, e, f, m\}, x\}$

Rule 2643

$\text{Int}[(b_.)*\text{sin}[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] :> \text{Simp}[(\text{Cos}[c + d*x]*(b*\text{Sin}[c + d*x])^{(n + 1)}*\text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d*x]^2])/(b*d*(n + 1)*\text{Sqrt}[\text{Cos}[c + d*x]^2]), x] /;$ $\text{FreeQ}\{b, c, d, n\}, x\} \&\amp; !\text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{3C(b \cos(c + dx))^{7/3} \sin(c + dx)}{10bd} + \frac{3 \int (b \cos(c + dx))^{4/3} dx}{b} \\ &= \frac{3C(b \cos(c + dx))^{7/3} \sin(c + dx)}{10bd} + \frac{B \int (b \cos(c + dx))^{4/3} dx}{b} \\ &= \frac{3C(b \cos(c + dx))^{7/3} \sin(c + dx)}{10bd} - \frac{3(10A + 7C)(b \cos(c + dx))^{4/3}}{10bd} \end{aligned}$$

Mathematica [A] time = 0.21952, size = 109, normalized size = 0.71

$$\frac{3 \sin(c + dx)(b \cos(c + dx))^{7/3} \left((10A + 7C) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right) + 7B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{3}; \frac{8}{3}; \cos^2(c + dx)\right) \right)}{70bd \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] (-3*(b*Cos[c + d*x])^(7/3)*Sin[c + d*x]*((10*A + 7*C)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2] + 7*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Cos[c + d*x]^2] - 7*C*Sqrt[Sin[c + d*x]^2]))/(70*b*d*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.278, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3), x)

$$3.347 \quad \int (b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$$

Optimal. Leaf size=148

$$\frac{3(7A + 4C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}; \cos^2(c + dx)\right)}{28d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.148493, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(7A + 4C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{5}{3}; \cos^2(c + dx)\right)}{28d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{7/3} {}_2F_1\left(\frac{1}{2}, \frac{7}{6}, \frac{13}{6}; \cos^2(c + dx)\right)}{7bd\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (3*C*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*d) - (3*(7*A + 4*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(28*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(7/3)*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \\ &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7d} + \frac{3}{7} \int \sqrt[3]{b \cos(c + dx)} \sec(c + dx) dx \\ &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7d} + B \int (b \cos(c + dx))^{4/3} \sec(c + dx) dx \\ &= \frac{3C(b \cos(c + dx))^{4/3} \sin(c + dx)}{7d} - \frac{3(7A + 4C)}{7d} \int \sqrt[3]{b \cos(c + dx)} dx \end{aligned}$$

Mathematica [A] time = 0.224451, size = 109, normalized size = 0.74

$$\frac{3b \sin(2(c + dx)) \sqrt[3]{b \cos(c + dx)} \left((7A + 4C) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) + 4B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{7}{6}; \frac{13}{6}; \cos^2(c + dx)\right) - 4C \cos^2(c + dx) \right)}{56d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]
```

```
[Out] (-3*b*(b*Cos[c + d*x])^(1/3)*((7*A + 4*C)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2] + 4*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2] - 4*C*Sqrt[Sin[c + d*x]^2])*Sin[2*(c + d*x)])/(56*d*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 0.362, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c), x)

3.348 $\int (b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=145

$$\frac{3b(4A + C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx) (b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/ (4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.173694, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3b(4A + C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx) (b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{4d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (3*b*C*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*d) - (3*b*(4*A + C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/ (4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/ (4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c

$+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]$
 $\&\& !IntegerQ[2*n]$

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3bC \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{4d} + \frac{1}{4}(3b) \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3bC \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{4d} + (bB) \int \frac{A + B \cos(c + dx)}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3bC \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{4d} - \frac{3b(4A + B)}{4d} \int \frac{1}{(b \cos(c + dx))^{2/3}} dx \end{aligned}$$

Mathematica [A] time = 0.204434, size = 108, normalized size = 0.74

$$\frac{3b^2 \sin(2(c + dx)) \left((4A + C) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) + B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) + C \left(-\sqrt{\sin^2(c + dx)}\right) \right)}{8d \sqrt{\sin^2(c + dx)} (b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (-3*b^2*((4*A + C)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + B*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])*Sin[2*(c + d*x)])/(8*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.376, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right)(b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2, x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2, x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^2, x)

$$3.349 \quad \int (b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

Optimal. Leaf size=145

$$\frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3(A - 2C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}} - \frac{3bB \sin(c + dx)\sqrt[3]{b \cos(c + dx)}}{d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) - (3*b*B*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.193268, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3(A - 2C) \sin(c + dx)(b \cos(c + dx))^{4/3} {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right)}{8d\sqrt{\sin^2(c + dx)}} - \frac{3bB \sin(c + dx)\sqrt[3]{b \cos(c + dx)}}{d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] (3*A*b^2*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)) - (3*b*B*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*(A - 2*C)*(b*Cos[c + d*x])^(4/3)*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{5/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + \frac{3}{2} \int \frac{\frac{2b^2B}{3} - \frac{1}{3}b^2(A - 2)}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} + (b^2B) \int \frac{1}{(b \cos(c + dx))^{2/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{2d(b \cos(c + dx))^{2/3}} - \frac{3bB\sqrt[3]{b \cos(c + dx)} {}_2F_1}{d} \end{aligned}$$

Mathematica [A] time = 0.229061, size = 117, normalized size = 0.81

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \csc(c + dx) \left(\cos(c + dx) \left(4B {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) + C \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{2}{3}; \frac{5}{3}; \cos^2(c + dx)\right) \right) \right)}{4d(b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Se
c[c + d*x]^3,x]
```

```
[Out] (-3*b^2*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]
) + Cos[c + d*x]*(4*B*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2] + C*
Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]))*Sqrt[Sin[c
+ d*x]^2)]/(4*d*(b*Cos[c + d*x])^(2/3))
```

Maple [F] time = 0.421, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3, x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^3, x)

3.350 $\int (b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=152

$$\frac{3Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} - \frac{3b(2A + 5C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{5d\sqrt{\sin^2(c + dx)}} + \frac{3b^2B \sin(c + dx) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)}(b \cos(c + dx))^{2/3}}$$

[Out] (3*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) + (3*b^2*B*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.19387, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} - \frac{3b(2A + 5C) \sin(c + dx) \sqrt[3]{b \cos(c + dx)} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{5d\sqrt{\sin^2(c + dx)}} + \frac{3b^2B \sin(c + dx) {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right)}{2d\sqrt{\sin^2(c + dx)}(b \cos(c + dx))^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4, x]

[Out] (3*A*b^3*Sin[c + d*x])/(5*d*(b*Cos[c + d*x])^(5/3)) + (3*b^2*B*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*(2*A + 5*C)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := -Simp[(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{8/3}} dx \\ &= \frac{3Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} + \frac{1}{5}(3b) \int \frac{\frac{5b^2B}{3} + \frac{1}{3}b^2}{(b \cos(c + dx))^{5/3}} dx \\ &= \frac{3Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} + (b^3B) \int \frac{1}{(b \cos(c + dx))^{5/3}} dx \\ &= \frac{3Ab^3 \sin(c + dx)}{5d(b \cos(c + dx))^{5/3}} + \frac{3b^2B {}_2F_1\left(-\frac{1}{3}, \frac{1}{2}; \frac{2}{3}; \cos^2(c + dx)\right)}{2d(b \cos(c + dx))^{5/3}} \end{aligned}$$

Mathematica [A] time = 0.208955, size = 124, normalized size = 0.82

$$\frac{3\sqrt{\sin^2(c + dx)} \csc(c + dx) \sec^3(c + dx) (b \cos(c + dx))^{4/3} \left(5 \cos(c + dx) \left(2C \cos(c + dx) {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{7}{6}; \cos^2(c + dx)\right) - \frac{1}{6}\right) + 5 \cos(c + dx) \left(-B \text{Hypergeometric2F1}\left[-\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \cos(c + dx)^2\right] + 2C \cos(c + dx) \text{Hypergeometric2F1}\left[\frac{1}{6}, \frac{1}{2}, \frac{7}{6}, \cos(c + dx)^2\right]\right) \sec(c + dx)^3 \sqrt{\sin(c + dx)^2}\right)}{10d}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]

[Out] (-3*(b*Cos[c + d*x])^(4/3)*Csc[c + d*x]*(-2*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2] + 5*Cos[c + d*x]*(-B*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]) + 2*C*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]))*Sec[c + d*x]^3*Sqrt[Sin[c + d*x]^2])/(10*d)

Maple [F] time = 0.418, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

[Out] int((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{4/3} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \sec(dx + c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4, x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{4}{3}} \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4, x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*sec(d*x + c)^4, x)

$$3.351 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=154

$$\frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^3 d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)}{11b^4 d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^4*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.145611, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^3 d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)}{11b^4 d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^3*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)]) + (C_)*sin[(e_) + (f_)*(x_)]^2, x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} dx &= \frac{\int (b \cos(c + dx))^{5/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^3d} + \frac{3 \int (b \cos(c + dx))^{5/3} \left(\frac{1}{3}b(A + B \cos(c + dx) + C \cos^2(c + dx))\right) dx}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^3d} + \frac{B \int (b \cos(c + dx))^{8/3} dx}{b^3} + \frac{3 \int (b \cos(c + dx))^{5/3} (A + C \cos^2(c + dx)) dx}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^3d} - \frac{3(11A + 8C)(b \cos(c + dx))^{5/3}}{88d} \end{aligned}$$

Mathematica [A] time = 0.269755, size = 114, normalized size = 0.74

$$\frac{3 \sin(c + dx) \cos^3(c + dx) \left((11A + 8C) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) + 8B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right) - 8C \sqrt{\sin^2(c + dx)} \right)}{88d \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(1/3), x]
```

```
[Out] (-3*Cos[c + d*x]^3*Sin[c + d*x]*((11*A + 8*C)*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, C
os[c + d*x]^2] - 8*C*Sqrt[Sin[c + d*x]^2]))/(88*d*(b*Cos[c + d*x])^(1/3)*Sq
rt[Sin[c + d*x]^2])
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c))(b \cos(dx + c))^{\frac{2}{3}}}{b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3)/b, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)

$$3.352 \quad \int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{\sqrt[3]{b\cos(c+dx)}} dx$$

Optimal. Leaf size=154

$$\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.148116, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(8A+5C)\sin(c+dx)(b\cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{40b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c+dx)\right)}{8b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^2*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} dx &= \frac{\int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8b^2d} + \frac{3 \int (b \cos(c + dx))^{2/3} \left(\frac{1}{3}\right) dx}{8b^2d} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8b^2d} + \frac{B \int (b \cos(c + dx))^{5/3} dx}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8b^2d} - \frac{3(8A + 5C)(b \cos(c + dx))^{2/3}}{8b^2d} \end{aligned}$$

Mathematica [A] time = 0.211579, size = 109, normalized size = 0.71

$$\frac{3 \sin(c + dx) (b \cos(c + dx))^{5/3} \left((8A + 5C) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) + 5B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) - 5C \right)}{40b^2d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c +
d*x])^(1/3), x]
```

```
[Out] (-3*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x]*((8*A + 5*C)*Hypergeometric2F1[1/2,
5/6, 11/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3,
7/3, Cos[c + d*x]^2] - 5*C*Sqrt[Sin[c + d*x]^2]))/(40*b^2*d*Sqrt[Sin[c + d*
x]^2])
```

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int \cos(dx + c) (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)
```

```
[Out] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
c))^(1/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}}}{b}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b,
x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
c))^(1/3), x)
```

$$3.353 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=154

$$\frac{3(5A+2C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}} - \frac{3B \sin(c+dx)(b \cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.122842, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3023, 2748, 2643}

$$\frac{3(5A+2C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10bd\sqrt{\sin^2(c+dx)}} - \frac{3B \sin(c+dx)(b \cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{5b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{3 \int \frac{\frac{1}{3}b(5A+2C) + \frac{5}{3}bB \cos(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx}{5b} \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{B \int (b \cos(c + dx))^{2/3} dx}{b} + \frac{1}{5}(5A + 2C) \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{3(5A + 2C)(b \cos(c + dx))^{2/3}}{10bd \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.141897, size = 108, normalized size = 0.7

$$\frac{3 \sin(2(c + dx)) \left((5A + 2C) {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx) \right) + 2B \cos(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx) \right) - 2C \sqrt{\sin^2(c + dx)} \right)}{20d \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(1/3),x]

[Out] (-3*((5*A + 2*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] - 2*C*Sqrt[Sin[c + d*x]^2])*Sin[2*(c + d*x)])/(20*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.246, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)

[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{2}{3}}}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(1/3), x)

$$3.354 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=149

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^2 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3}}{2bd \sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.163131, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^2 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3}}{2bd \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*A*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] :=> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :=> -Simp[(A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :=> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643


```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx = b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx$$

$$= \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + \frac{3 \int \frac{\frac{b^2 B}{3} - \frac{1}{3} b^2 (2A - C) \cos(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx}{b^2}$$

$$= \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} + B \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx - \frac{(2A - C) \int (b \cos(c + dx))^{2/3} dx}{2bd \sqrt{\sin^2(c + dx)}}$$

$$= \frac{3A \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)}} - \frac{3B(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right)}{2bd \sqrt{\sin^2(c + dx)}}$$

Mathematica [A] time = 6.24293, size = 268, normalized size = 1.8

$$(b \cos(c + dx))^{2/3} (A \sec(c + dx) + B + C \cos(c + dx)) \left(\frac{4(2A - C) \sec(c) \sin(\tan^{-1}(\tan(c) + dx)) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{6}; \frac{5}{6}; \cos^2(dx + \tan^{-1}(\tan(c)))\right)}{\sqrt{\sin^2(\tan^{-1}(\tan(c) + dx))}} \right) +$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c +
d*x])^(1/3), x]
```

```
[Out] ((b*Cos[c + d*x])^(2/3)*(B + C*Cos[c + d*x] + A*Sec[c + d*x])*(Csc[c]*(-5*(
2*A - C)*Cos[c - d*x - ArcTan[Tan[c]]]*Sec[c] + (-2*A + C)*Cos[c + d*x + Ar
cTan[Tan[c]]]*Sec[c] + 3*((4*A - C)*Cos[d*x] - C*Cos[2*c + d*x])*Sqrt[Sec[c
]^2]) - (2*B*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2)*
Sqrt[Sec[c]^2]*Sin[2*d*x - 2*ArcTan[Cot[c]]])/(Sin[d*x - ArcTan[Cot[c]]]^2)
^(1/3) + (4*(2*A - C)*HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[d*x + ArcT
an[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/Sqrt[Sin[d*x + ArcTan[Tan[
c]]]^2]))/(2*b*d*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*(c + d*x)])*Sqrt[Sec
[c]^2])
```

Maple [F] time = 0.345, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) \sec(dx + c) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(1/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(1/3), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(1/3), x)

$$3.355 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=145

$$\frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8bd \sqrt{\sin^2(c+dx)}} + \frac{3Ab \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} + \frac{3B \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)} \sqrt[3]{b}}$$

[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.174072, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8bd \sqrt{\sin^2(c+dx)}} + \frac{3Ab \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} + \frac{3B \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx)} \sqrt[3]{b}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*A*b*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_)^(n_)), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx = b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/3}} dx$$

$$= \frac{3Ab \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{3 \int \frac{\frac{4b^2B}{3} + \frac{1}{3}b^2(A+4C) \cos(c+dx)}{(b \cos(c+dx))^{4/3}} dx}{4b}$$

$$= \frac{3Ab \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + (bB) \int \frac{1}{(b \cos(c + dx))^{4/3}} dx + \frac{1}{4}(A + C) \int \frac{1}{(b \cos(c + dx))^{4/3}} dx$$

$$= \frac{3Ab \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{3B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sin(c + dx)}{d \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}}$$

Mathematica [B] time = 6.30457, size = 699, normalized size = 4.82

$$4B \csc(c) \cos^{\frac{7}{3}}(c + dx) (A \sec^2(c + dx) + B \sec(c + dx) + C) \left(\frac{\tan(c) \sin(\tan^{-1}(\tan(c)) + dx) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{6}; \frac{5}{6}; \cos^2(dx + c)\right)}{\sqrt{\tan^2(c) + 1} \sqrt{1 - \cos(\tan^{-1}(\tan(c)) + dx)} \sqrt{\cos(\tan^{-1}(\tan(c)) + dx) + 1} \sqrt[3]{\cos(c + dx)}} \right)$$

$$d \sqrt[3]{b \cos(c + dx)} (2A + 2B \cos(c + dx) + C \cos(2c + 2dx))$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
+ d*x])^(1/3), x]
```

```
[Out] (Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((6*B*Csc[c]*Sec[c]
)/d + (3*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(2*d) + (3*Sec[c]*Sec[c + d*x]*(
A*Sin[c] + 4*B*Sin[d*x]))/(2*d)))/((b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Co
s[c + d*x] + C*Cos[2*c + 2*d*x])) - (A*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[
Cot[c]])*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C +
B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x - ArcTan[Cot[c]]])/(2*d*(b*Cos[
c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])*(Cos[c]*C
os[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)) - (2*
C*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[Cot[c]])*Hypergeometric2F1[1/2, 2/3,
3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*S
in[d*x - ArcTan[Cot[c]]])/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c +
d*x] + C*Cos[2*c + 2*d*x])*(Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d
*x - ArcTan[Cot[c]]]^2)^(1/3)) + (4*B*Cos[c + d*x]^(7/3)*Csc[c]*(C + B*Sec[
c + d*x] + A*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[d
*x + ArcTan[Tan[c]]]^2)*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x
+ ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*(Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (3*Cos[c]^2*Cos[d*x + ArcTan[T
an[c]]]*Sqrt[1 + Tan[c]^2])/(2*(Cos[c]^2 + Sin[c]^2)))/(Cos[c]*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)))/(d*(b*Cos[c + d*x])^(1/3)*(2*A +
C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))
```

Maple [F] time = 0.375, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^2 \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)

[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(1/3), x)
```

$$3.356 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^3(c+dx)}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=149

$$\frac{3Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/3}} + \frac{3(4A+7C) \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} + \frac{3bB \sin(c+dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{4/3}}$$

[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.178657, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3Ab^2 \sin(c+dx)}{7d(b \cos(c+dx))^{7/3}} + \frac{3(4A+7C) \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{7d \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} + \frac{3bB \sin(c+dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*A*b^2*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*b*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx &= b^3 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{10/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3}{7} \int \frac{\frac{7b^2B}{3} + \frac{1}{3}b^2(4A + 7C) \cos(c + dx)}{(b \cos(c + dx))^{7/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + (b^2B) \int \frac{1}{(b \cos(c + dx))^{7/3}} dx + \frac{1}{7}(b(4A + 7C)) \int \frac{\cos(c + dx)}{(b \cos(c + dx))^{7/3}} dx \\ &= \frac{3Ab^2 \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3bB {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) \sin(c + dx)}{4d(b \cos(c + dx))^{4/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.277794, size = 118, normalized size = 0.79

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \csc(c + dx) \left(4A {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right) + 7 \cos(c + dx) \left(B {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) + 4C \cos(c + dx)\right)\right)}{28d(b \cos(c + dx))^{7/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3)/(b*Cos[c + d*x])^(1/3), x]
```

```
[Out] (3*b^2*Csc[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2] + 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(7/3))
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^3 \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3), x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{1/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^3}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^3/(b*cos(d*x + c)), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3/(b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^3}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3/(b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^3/(b*cos(d*x + c))^(1/3), x)

$$3.357 \quad \int \frac{\cos^3(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=154

$$\frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^4 d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)}{11b^5 d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^4*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^5*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.142863, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(11A + 8C) \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{88b^4 d \sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{11/3} {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right)}{11b^5 d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(8/3)*Sin[c + d*x])/(11*b^4*d) - (3*(11*A + 8*C)*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(88*b^4*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(11/3)*Hypergeometric2F1[1/2, 11/6, 17/6, Cos[c + d*x]^2]*Sin[c + d*x])/(11*b^5*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^3(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{4/3}} dx &= \frac{\int (b \cos(c + dx))^{5/3} (A + B \cos(c + dx) + C \cos^2(c + dx))}{b^3} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^4d} + \frac{3 \int (b \cos(c + dx))^{5/3} \left(\frac{1}{2} + \frac{1}{3} \cos^2(c + dx)\right)}{b^3} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^4d} + \frac{B \int (b \cos(c + dx))^{8/3} dx}{b^4} \\ &= \frac{3C(b \cos(c + dx))^{8/3} \sin(c + dx)}{11b^4d} - \frac{3(11A + 8C)(b \cos(c + dx))^{5/3}}{11b^4d} \end{aligned}$$

Mathematica [A] time = 0.335602, size = 114, normalized size = 0.74

$$\frac{3 \sin(c + dx) \cos^4(c + dx) \left((11A + 8C) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) + 8B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{11}{6}; \frac{17}{6}; \cos^2(c + dx)\right) - 8C \right)}{88d \sqrt{\sin^2(c + dx)} (b \cos(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(4/3), x]
```

```
[Out] (-3*Cos[c + d*x]^4*Sin[c + d*x]*((11*A + 8*C)*Hypergeometric2F1[1/2, 4/3, 7
/3, Cos[c + d*x]^2] + 8*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 11/6, 17/6, C
os[c + d*x]^2] - 8*C*Sqrt[Sin[c + d*x]^2]))/(88*d*(b*Cos[c + d*x])^(4/3)*Sq
rt[Sin[c + d*x]^2])
```

Maple [F] time = 0.427, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^3 (A + B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)
```

```
[Out] int(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) (b \cos(dx + c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^(2/3)/b^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^3}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^3/(b*cos(d*x + c))^(4/3), x)

$$3.358 \quad \int \frac{\cos^2(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=154

$$\frac{3(8A + 5C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40b^3d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{8b^4d\sqrt{\sin^2(c + dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.143839, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(8A + 5C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{40b^3d\sqrt{\sin^2(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{8/3} {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right)}{8b^4d\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b^3*d) - (3*(8*A + 5*C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(40*b^3*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(8/3)*Hypergeometric2F1[1/2, 4/3, 7/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^4*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{4/3}} dx &= \frac{\int (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8b^3d} + \frac{3 \int (b \cos(c + dx))^{2/3} \left(\frac{1}{3}b(8A + 5C) + B \sin(c + dx)\right) dx}{b^3} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8b^3d} + \frac{B \int (b \cos(c + dx))^{5/3} dx}{b^3} + \frac{3(8A + 5C)(b \cos(c + dx))^{2/3}}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{5/3} \sin(c + dx)}{8b^3d} - \frac{3(8A + 5C)(b \cos(c + dx))^{2/3}}{40d} \end{aligned}$$

Mathematica [A] time = 0.235796, size = 114, normalized size = 0.74

$$\frac{3 \sin(c + dx) \cos^3(c + dx) \left((8A + 5C) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) + 5B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{4}{3}; \frac{7}{3}; \cos^2(c + dx)\right) - 5C \sqrt{\sin^2(c + dx)} \right)}{40d \sqrt{\sin^2(c + dx)} (b \cos(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c
+ d*x])^(4/3), x]
```

```
[Out] (-3*Cos[c + d*x]^3*Sin[c + d*x]*((8*A + 5*C)*Hypergeometric2F1[1/2, 5/6, 11
/6, Cos[c + d*x]^2] + 5*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Cos
[c + d*x]^2] - 5*C*Sqrt[Sin[c + d*x]^2]))/(40*d*(b*Cos[c + d*x])^(4/3)*Sqrt
[Sin[c + d*x]^2])
```

Maple [F] time = 0.332, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (A + B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)
```

```
[Out] int(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{2}{3}}}{b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/b^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)
```

$$3.359 \quad \int \frac{\cos(c+dx)(A+B\cos(c+dx)+C\cos^2(c+dx))}{(b\cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=154

$$\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.140053, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{3(5A+2C)\sin(c+dx)(b\cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}} - \frac{3B\sin(c+dx)(b\cos(c+dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c+dx)\right)}{5b^3d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d) - (3*(5*A + 2*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(10*b^2*d*Sqrt[Sin[c + d*x]^2]) - (3*B*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{\cos(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{4/3}} dx &= \frac{\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx}{b} \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5b^2 d} + \frac{3 \int \frac{\frac{1}{3}b(5A + 2C) + \frac{5}{3}bB \cos(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx}{5b^2} \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5b^2 d} + \frac{B \int (b \cos(c + dx))^{2/3} dx}{b^2} \\ &= \frac{3C(b \cos(c + dx))^{2/3} \sin(c + dx)}{5b^2 d} - \frac{3(5A + 2C)(b \cos(c + dx))^{2/3}}{5b^2 d} \end{aligned}$$

Mathematica [A] time = 0.205271, size = 111, normalized size = 0.72

$$\frac{3 \sin(2(c + dx)) \left((5A + 2C) {}_2F_1 \left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx) \right) + 2B \cos(c + dx) {}_2F_1 \left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx) \right) - 2C \sqrt{\sin^2(c + dx)} \right)}{20bd \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (-3*((5*A + 2*C)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*B*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2] - 2*C*Sqrt[Sin[c + d*x]^2])*Sin[2*(c + d*x)])/(20*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.297, size = 0, normalized size = 0.

$$\int \cos(dx + c) (A + B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
c))^(4/3), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}}}{b^2 \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^
2*cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3)
,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)/(b*cos(d*x +
c))^(4/3), x)
```

$$3.360 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=152

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^3 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3}}{2b^2 d \sqrt{\sin^2(c + dx)}}$$

[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.139211, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {3021, 2748, 2643}

$$\frac{3(2A - C) \sin(c + dx)(b \cos(c + dx))^{5/3} {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right)}{5b^3 d \sqrt{\sin^2(c + dx)}} + \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} - \frac{3B \sin(c + dx)(b \cos(c + dx))^{2/3}}{2b^2 d \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)) - (3*B*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*b^2*d*Sqrt[Sin[c + d*x]^2]) + (3*(2*A - C)*(b*Cos[c + d*x])^(5/3)*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*b^3*d*Sqrt[Sin[c + d*x]^2])

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m + 1))*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx &= \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} + \frac{3 \int \frac{b^2 B - \frac{1}{3} b^2 (2A - C) \cos(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx}{b^3} \\ &= \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} + \frac{B \int \frac{1}{\sqrt[3]{b \cos(c + dx)}} dx}{b} - \frac{(2A - C) \int (b \cos(c + dx))^{2/3} dx}{b^2} \\ &= \frac{3A \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)}} - \frac{3B(b \cos(c + dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) \sin(c + dx)}{2b^2 d \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.224296, size = 115, normalized size = 0.76

$$\frac{3\sqrt{\sin^2(c + dx)} \cot(c + dx) \left(\cos(c + dx) \left(5B {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c + dx)\right) + 2C \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{11}{6}; \cos^2(c + dx)\right) \right) \right)}{10d(b \cos(c + dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(b*Cos[c + d*x])^(4/3), x]

[Out] (-3*Cot[c + d*x]*(-10*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2] + Cos[c + d*x]*(5*B*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2] + 2*C*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]))*Sqrt[Sin[c + d*x]^2])/(10*d*(b*Cos[c + d*x])^(4/3))

Maple [F] time = 0.249, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{2}{3}}}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c))^(4/3), x)

$$3.361 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=147

$$\frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8b^2 d \sqrt{\sin^2(c+dx)}} + \frac{3A \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} + \frac{3B \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.16783, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3(A+4C) \sin(c+dx)(b \cos(c+dx))^{2/3} {}_2F_1\left(\frac{1}{3}, \frac{1}{2}; \frac{4}{3}; \cos^2(c+dx)\right)}{8b^2 d \sqrt{\sin^2(c+dx)}} + \frac{3A \sin(c+dx)}{4d(b \cos(c+dx))^{4/3}} + \frac{3B \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)) + (3*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*(A + 4*C)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(8*b^2*d*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(b*Sin[c + d*x])^(n+1)*Hypergeometric2F1[1/2, (n+1)/2, (n+3)/2, Sin[c

+ d*x]^2]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx)}{(b \cos(c + dx))^{4/3}} dx = b \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{7/3}} dx$$

$$= \frac{3A \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{3 \int \frac{\frac{4b^2B}{3} + \frac{1}{3}b^2(A+4C) \cos(c+dx)}{(b \cos(c+dx))^{4/3}} dx}{4b^2}$$

$$= \frac{3A \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + B \int \frac{1}{(b \cos(c + dx))^{4/3}} dx + \frac{(A + 4C)}{4b^2}$$

$$= \frac{3A \sin(c + dx)}{4d(b \cos(c + dx))^{4/3}} + \frac{3B {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c + dx)\right) \sin(c + dx)}{bd \sqrt[3]{b \cos(c + dx)} \sqrt{\sin^2(c + dx)}}$$

Mathematica [B] time = 6.27087, size = 703, normalized size = 4.78

$$\frac{4B \csc(c) \cos^{\frac{7}{3}}(c+dx) (A \sec^2(c+dx) + B \sec(c+dx) + C)}{d \sqrt[3]{b \cos(c+dx)} (2A + 2B \cos(c+dx) + C \cos(2c + 2dx) + C)} \left(\frac{\tan(c) \sin(\tan^{-1}(\tan(c)) + dx) {}_2F_1\left(-\frac{1}{2}, -\frac{1}{6}; \frac{5}{6}; \cos^2(dx + \tan^{-1}(\tan(c)))\right)}{\sqrt{\tan^2(c)+1} \sqrt{1-\cos(\tan^{-1}(\tan(c))+dx)} \sqrt{\cos(\tan^{-1}(\tan(c))+dx)+1} \sqrt[3]{\cos(c) \sqrt{\tan^2(c)+1} \cos(\tan^{-1}(\tan(c))+dx)}} \right)$$

Antiderivative was successfully verified.

[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x])/(b*Cos[c + d*x])^(4/3), x]

[Out] ((Cos[c + d*x]^3*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((6*B*Csc[c]*Sec[c])/d + (3*A*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(2*d) + (3*Sec[c]*Sec[c + d*x]*(A*Sin[c] + 4*B*Sin[d*x]))/(2*d)))/(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])) - (A*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[Cot[c]]]*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x - ArcTan[Cot[c]]])/(2*d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))*(Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)) - (2*C*Cos[c + d*x]^(7/3)*Cos[d*x - ArcTan[Cot[c]]]*Hypergeometric2F1[1/2, 2/3, 3/2, Cos[d*x - ArcTan[Cot[c]]]^2]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*Sin[d*x - ArcTan[Cot[c]]])/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x]))*(Cos[c]*Cos[d*x] - Sin[c]*Sin[d*x])^(1/3)*(Sin[d*x - ArcTan[Cot[c]]]^2)^(1/3)) + (4*B*Cos[c + d*x]^(7/3)*Csc[c]*(C + B*Sec[c + d*x] + A*Sec[c + d*x]^2)*((HypergeometricPFQ[{-1/2, -1/6}, {5/6}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*(Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (3*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(2*(Cos[c]^2 + Sin[c]^2)))/(Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])^(1/3)))/(d*(b*Cos[c + d*x])^(1/3)*(2*A + C + 2*B*Cos[c + d*x] + C*Cos[2*c + 2*d*x])))/b

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) \sec(dx + c) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)

[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)/(b*cos(d*x+c))^(4/3), x
, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)/(b*cos(d*x +
c))^(4/3), x)
```

$$3.362 \quad \int \frac{(A+B \cos(c+dx)+C \cos^2(c+dx)) \sec^2(c+dx)}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=149

$$\frac{3(4A+7C) \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{7bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} + \frac{3Ab \sin(c+dx)}{7d(b \cos(c+dx))^{7/3}} + \frac{3B \sin(c+dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{4/3}}$$

[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.188693, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {16, 3021, 2748, 2643}

$$\frac{3(4A+7C) \sin(c+dx) {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{5}{6}; \cos^2(c+dx)\right)}{7bd \sqrt{\sin^2(c+dx)} \sqrt[3]{b \cos(c+dx)}} + \frac{3Ab \sin(c+dx)}{7d(b \cos(c+dx))^{7/3}} + \frac{3B \sin(c+dx) {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c+dx)\right)}{4d \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

[In] Int[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*A*b*Sin[c + d*x])/(7*d*(b*Cos[c + d*x])^(7/3)) + (3*B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) + (3*(4*A + 7*C)*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3021

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[((A*b^2 - a*b*B + a^2*C)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+1)*(a^2 - b^2)), x] + Dist[1/(b*(m+1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m+1)*Simp[b*(a*A - b*B + a*C)*(m+1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C))*(m+1)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx)}{(b \cos(c + dx))^{4/3}} dx &= b^2 \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(b \cos(c + dx))^{10/3}} dx \\ &= \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3 \int \frac{\frac{7b^2B}{3} + \frac{1}{3}b^2(4A+7C) \cos(c+dx)}{(b \cos(c+dx))^{7/3}} dx}{7b} \\ &= \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + (bB) \int \frac{1}{(b \cos(c + dx))^{7/3}} dx + \frac{1}{7}(4C \\ &= \frac{3Ab \sin(c + dx)}{7d(b \cos(c + dx))^{7/3}} + \frac{3B {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) \sin(c + dx)}{4d(b \cos(c + dx))^{4/3} \sqrt{\sin^2(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.345088, size = 118, normalized size = 0.79

$$\frac{3b^2 \sqrt{\sin^2(c + dx)} \cot(c + dx) \left(4A {}_2F_1\left(-\frac{7}{6}, \frac{1}{2}; -\frac{1}{6}; \cos^2(c + dx)\right) + 7 \cos(c + dx) \left(B {}_2F_1\left(-\frac{2}{3}, \frac{1}{2}; \frac{1}{3}; \cos^2(c + dx)\right) + 4C \cos(c + dx)\right)\right)}{28d(b \cos(c + dx))^{10/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2)/(b*Cos[c
+ d*x])^(4/3), x]
```

```
[Out] (3*b^2*Cot[c + d*x]*(4*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]
+ 7*Cos[c + d*x]*(B*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2] + 4*
C*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]))*Sqrt[Sin
[c + d*x]^2])/(28*d*(b*Cos[c + d*x])^(10/3))
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^2 (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{4/3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^{\frac{2}{3}} \sec(dx + c)^2}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*sec(d*x + c)^2/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \sec(dx + c)^2}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*sec(d*x + c)^2/(b*cos(d*x + c))^(4/3), x)

3.363 $\int \cos^m(c+dx)(b \cos(c+dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=232

$$\frac{3b(A(3m+10) + C(3m+7)) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+13); \cos^2(c+dx)\right)}{d(3m+7)(3m+10) \sqrt{\sin^2(c+dx)}}$$

[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3*m)) - (3*b*(C*(7 + 3*m) + A*(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*(10 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.212161, antiderivative size = 222, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{3b \left(\frac{A}{3m+7} + \frac{C}{3m+10} \right) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+2}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+7); \frac{1}{6}(3m+13); \cos^2(c+dx)\right) + 3bB \sin(c+dx) \cos^{m+3}(c+dx)}{d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*b*C*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(10 + 3*m)) - (3*b*(A/(7 + 3*m) + C/(10 + 3*m))*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Cos[c + d*x]^(3 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (10 + 3*m)/6, (16 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(a*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \sin[e + f x]^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x)^{(n)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d x] * (b \sin[c + d x])^{(n + 1)} * \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d x]^2]) / (b d (n + 1) \text{Sqrt}[\text{Cos}[c + d x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\amp; \text{IntegerQ}[2 * n]$

Rubi steps

$$\begin{aligned} \int \cos^m(c + dx) (b \cos(c + dx))^{4/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{(b \sqrt[3]{b \cos(c + dx)}) \int \cos^{4/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= \frac{3bC \cos^{2+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{d(10 + 3m)} \\ &= \frac{3bC \cos^{2+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{d(10 + 3m)} \\ &= \frac{3bC \cos^{2+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{d(10 + 3m)} \end{aligned}$$

Mathematica [A] time = 0.651854, size = 169, normalized size = 0.73

$$\frac{3 \sin(c + dx) (b \cos(c + dx))^{4/3} \cos^{m+1}(c + dx) \left((A(3m + 10) + C(3m + 7)) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 7); \frac{1}{6}(3m + 13); \cos^2(c + dx)\right) + C(7 + 3m) \sqrt{\sin^2(c + dx)} \right)}{d(3m + 7)(3m + 10) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(4/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(4/3)*Sin[c + d*x]*(B*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3 + m/2, 8/3 + m/2, Cos[c + d*x]^2] + (C*(7 + 3*m) + A*(10 + 3*m))*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2] - C*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])/(d*(7 + 3*m)*(10 + 3*m)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (b \cos(dx + c))^{4/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(Cb \cos(dx + c)^3 + Bb \cos(dx + c)^2 + Ab \cos(dx + c)\right) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*b*cos(d*x + c)^3 + B*b*cos(d*x + c)^2 + A*b*cos(d*x + c))*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{4}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(4/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(4/3)*cos(d*x + c)^m, x)

3.364 $\int \cos^m(c+dx)(b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=229

$$\frac{3(A(3m+8) + C(3m+5)) \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+5); \frac{1}{6}(3m+11); \cos^2(c+dx)\right)}{d(3m+5)(3m+8)\sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)
) - (3*(C*(5 + 3*m) + A*(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*(5 + 3*m)*(8 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(
2 + m)*(b*Cos[c + d*x])^(2/3)*Hypergeometric2F1[1/2, (8 + 3*m)/6, (14 + 3*m
)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.212513, antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{3\left(\frac{A}{3m+5} + \frac{C}{3m+8}\right) \sin(c+dx)(b \cos(c+dx))^{2/3} \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+5); \frac{1}{6}(3m+11); \cos^2(c+dx)\right) + 3B \sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2), x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(d*(8 + 3*m)
) - (3*(A/(5 + 3*m) + C/(8 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2
/3)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(2/3)*Hypergeometric2F1[1/2, (8 + 3*m)/6, (14 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```


$b \sin[e + f x]^{(m + 1)}, x, x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + (d \cdot x))^{(n)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d x] * (b \sin[c + d x])^{(n + 1)} * \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d x]^2]) / (b * d * (n + 1) * \text{Sqrt}[\text{Cos}[c + d x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\amp; \text{IntegerQ}[2 * n]$

Rubi steps

$$\begin{aligned} \int \cos^m(c + dx) (b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{(b \cos(c + dx))^{2/3} \int \cos^{2/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\cos^{2/3}(c + dx)} \\ &= \frac{3C \cos^{1+m}(c + dx) (b \cos(c + dx))^{2/3} \sin(c + dx)}{d(8 + 3m)} \\ &= \frac{3C \cos^{1+m}(c + dx) (b \cos(c + dx))^{2/3} \sin(c + dx)}{d(8 + 3m)} \\ &= \frac{3C \cos^{1+m}(c + dx) (b \cos(c + dx))^{2/3} \sin(c + dx)}{d(8 + 3m)} \end{aligned}$$

Mathematica [A] time = 0.425282, size = 166, normalized size = 0.72

$$\frac{3 \sin(c + dx) (b \cos(c + dx))^{2/3} \cos^{m+1}(c + dx) \left((A(3m + 8) + C(3m + 5)) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 5); \frac{1}{6}(3m + 11); \cos^2(c + dx)\right) + (5 + 3m) \text{Sqrt}[\text{Sin}[c + dx]^2] \right)}{d(3m + 5)(3m + 8) \sqrt{\text{Sin}^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(2/3)*Sin[c + d*x]*((C*(5 + 3*m) + A*(8 + 3*m))*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] + (5 + 3*m)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (8 + 3*m)/6, 7/3 + m/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(5 + 3*m)*(8 + 3*m)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (b \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m, x)

3.365 $\int \cos^m(c+dx) \sqrt[3]{b \cos(c+dx)} (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=229

$$\frac{3(A(3m+7) + C(3m+4)) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+4); \frac{1}{6}(3m+10); \cos^2(c+dx)\right)}{d(3m+4)(3m+7) \sqrt{\sin^2(c+dx)}}$$

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m)
) - (3*(C*(4 + 3*m) + A*(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*(4 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(
2 + m)*(b*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m
)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.201424, antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{3\left(\frac{A}{3m+4} + \frac{C}{3m+7}\right) \sin(c+dx) \sqrt[3]{b \cos(c+dx)} \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+4); \frac{1}{6}(3m+10); \cos^2(c+dx)\right) + 3B \sin(c+dx)}{d \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^m*(b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d
*x]^2), x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(d*(7 + 3*m)
) - (3*(A/(4 + 3*m) + C/(7 + 3*m))*Cos[c + d*x]^(1 + m)*(b*Cos[c + d*x])^(1
/3)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2]*Sin[c
+ d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*(b*Cos[c + d*
x])^(1/3)*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2]
*Sin[c + d*x])/(d*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart
[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
!LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

$b \sin[e + f x]^{m+1}, x, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x) \sin[c + d x]^{n-1}, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d x] * (b \sin[c + d x]^{n+1} * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \sin[c + d x]^2]) / (b d (n+1) \sqrt{\cos[c + d x]^2}), x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\amp; \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \cos^m(c + dx) \sqrt[3]{b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\sqrt[3]{b \cos(c + dx)} \int \cos^{\frac{1}{3}+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{\sqrt[3]{\cos(c + dx)}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{d(7 + 3m)} \\ &= \frac{3C \cos^{1+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{d(7 + 3m)} \\ &= \frac{3C \cos^{1+m}(c + dx) \sqrt[3]{b \cos(c + dx)} \sin(c + dx)}{d(7 + 3m)} \end{aligned}$$

Mathematica [A] time = 0.40335, size = 166, normalized size = 0.72

$$\frac{3 \sin(c + dx) \sqrt[3]{b \cos(c + dx)} \cos^{m+1}(c + dx) \left((A(3m + 7) + C(3m + 4)) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 4); \frac{m}{2} + \frac{5}{3}; \cos^2(c + dx)\right) + (3m + 4) \sqrt{\sin^2(c + dx)} \right)}{d(3m + 4)(3m + 7) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^m*(b*cos[c + d*x])^(1/3)*(A + B*cos[c + d*x] + C*cos[c + d*x]^2), x]

[Out] (-3*cos[c + d*x]^(1 + m)*(b*cos[c + d*x])^(1/3)*Sin[c + d*x]*((C*(4 + 3*m) + A*(7 + 3*m))*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] + (4 + 3*m)*(B*cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 3*m)/6, (13 + 3*m)/6, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(4 + 3*m)*(7 + 3*m)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.32, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m \sqrt[3]{b \cos(dx + c)} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**m*(b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^m*(b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m, x)
```

$$3.366 \quad \int \frac{\cos^m(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt[3]{b \cos(c+dx)}} dx$$

Optimal. Leaf size=229

$$\frac{3(A(3m+5)+C(3m+2)) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \cos^2(c+dx)\right)}{d(3m+2)(3m+5) \sqrt{\sin^2(c+dx) \sqrt[3]{b \cos(c+dx)}}} - \frac{3B \sin(c+dx) \cos^{m+2}(c+dx)}{d(3m+5)}$$

[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)) - (3*(C*(2 + 3*m) + A*(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.201154, antiderivative size = 219, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{3\left(\frac{A}{3m+2} + \frac{C}{3m+5}\right) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+2); \frac{1}{6}(3m+8); \cos^2(c+dx)\right)}{d \sqrt{\sin^2(c+dx) \sqrt[3]{b \cos(c+dx)}}} - \frac{3B \sin(c+dx) \cos^{m+2}(c+dx)}{d(3m+5)}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)) - (3*(A/(2 + 3*m) + C/(5 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \cdot \sin[e + f \cdot x]^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \cdot \sin[c + d \cdot x] + (d \cdot x))^{(n)}, x_Symbol] :> \text{Simp}[(\text{Cos}[c + d \cdot x] * (b \cdot \sin[c + d \cdot x])^{(n + 1)} * \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d \cdot x]^2]) / (b \cdot d \cdot (n + 1) * \text{Sqrt}[\text{Cos}[c + d \cdot x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2 \cdot n]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^m(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} dx &= \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{-\frac{1}{3} + m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sin(c + dx)}{d(5 + 3m) \sqrt[3]{b \cos(c + dx)}} + \frac{(3 \sqrt[3]{\cos(c + dx)}) \int \cos^{-\frac{2}{3} + m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sin(c + dx)}{d(5 + 3m) \sqrt[3]{b \cos(c + dx)}} + \frac{(B \sqrt[3]{\cos(c + dx)}) \int \cos^{\frac{2}{3} + m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt[3]{b \cos(c + dx)}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sin(c + dx)}{d(5 + 3m) \sqrt[3]{b \cos(c + dx)}} - \frac{3 \left(\frac{A}{2+3m} + \frac{C}{5+3m} \right) \cos^{1+m}(c + dx)}{\sqrt[3]{b \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 0.427815, size = 166, normalized size = 0.72

$$\frac{3 \sin(c + dx) \cos^{m+1}(c + dx) \left((A(3m + 5) + C(3m + 2)) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 2); \frac{1}{6}(3m + 8); \cos^2(c + dx)\right) + (3m + 2) \left(B \cos(c + dx) + C \cos^2(c + dx) \right) \right)}{d(3m + 2)(3m + 5) \sqrt{\sin^2(c + dx)} \sqrt[3]{b \cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(1/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*((C*(2 + 3*m) + A*(5 + 3*m))*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] + (2 + 3*m)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 3*m)/6, (11 + 3*m)/6, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(2 + 3*m)*(5 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.313, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m}{b \cos(dx + c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \cos^m(c + dx)}{\sqrt[3]{b \cos(c + dx)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(1/3),x)

[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(1/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(1/3),x, algorithm="giac")


```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(1/3), x)
```

$$3.367 \quad \int \frac{\cos^m(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=227

$$\frac{3(A(3m+4)+3Cm+C) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+1); \frac{1}{6}(3m+7); \cos^2(c+dx)\right) - 3B \sin(c+dx) \cos^m(c+dx)}{d(3m+1)(3m+4) \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}}$$

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rubi [A] time = 0.233094, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{3(A(3m+4)+3Cm+C) \sin(c+dx) \cos^{m+1}(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m+1); \frac{1}{6}(3m+7); \cos^2(c+dx)\right) - 3B \sin(c+dx) \cos^m(c+dx)}{d(3m+1)(3m+4) \sqrt{\sin^2(c+dx)} (b \cos(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(2/3), x]
```

```
[Out] (3*C*Cos[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)) - (3*(C + 3*C*m + A*(4 + 3*m))*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(2 + m)*Hypergeometric2F1[1/2, (4 + 3*m)/6, (10 + 3*m)/6, Cos[c + d*x]^2*Sin[c + d*x])/(d*(4 + 3*m)*(b*Cos[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
```

$b \sin[e + f x]^{m+1}, x, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x)^n, x_Symbol] :> \text{Simp}[(\text{Cos}[c + d x] * (b \sin[c + d x])^{n+1} * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d x]^2]) / (b d (n+1) \text{Sqrt}[\text{Cos}[c + d x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \frac{\cos^m(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{2/3}} dx &= \frac{\cos^{2/3}(c + dx) \int \cos^{-2/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{(b \cos(c + dx))^{2/3}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sin(c + dx)}{d(4 + 3m)(b \cos(c + dx))^{2/3}} + \frac{(3 \cos^{2/3}(c + dx)) \int \cos^{-2/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{(b \cos(c + dx))^{2/3}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sin(c + dx)}{d(4 + 3m)(b \cos(c + dx))^{2/3}} + \frac{(B \cos^{2/3}(c + dx)) \int \cos^{-2/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{(b \cos(c + dx))^{2/3}} \\ &= \frac{3C \cos^{1+m}(c + dx) \sin(c + dx)}{d(4 + 3m)(b \cos(c + dx))^{2/3}} - \frac{3(C + 3Cm + A(4 + 3m)) \cos^{2/3}(c + dx) \int \cos^{-2/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{d(4 + 3m)(b \cos(c + dx))^{2/3}} \end{aligned}$$

Mathematica [A] time = 0.401665, size = 164, normalized size = 0.72

$$\frac{3 \sin(c + dx) \cos^{m+1}(c + dx) \left((A(3m + 4) + 3Cm + C) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m + 1); \frac{1}{6}(3m + 7); \cos^2(c + dx)\right) + (3m + 1) \left(B \cos^{2/3}(c + dx) \int \cos^{-2/3+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \right) \right)}{d(3m + 1)(3m + 4) \sqrt{\sin^2(c + dx) (b \cos(c + dx))^{2/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x]^(2/3), x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*((C + 3*C*m + A*(4 + 3*m))*Hypergeometric2F1[1/2, (1 + 3*m)/6, (7 + 3*m)/6, Cos[c + d*x]^2] + (1 + 3*m)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + 3*m)/6, 5/3 + m/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(1 + 3*m)*(4 + 3*m)*(b*Cos[c + d*x]^(2/3)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.328, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (A + B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{1}{3}} \cos(dx + c)^m}{b \cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(1/3)*cos(d*x + c)^m/(b*cos(d*x + c)), x)

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(A + B \cos(c + dx) + C \cos^2(c + dx)) \cos^m(c + dx)}{(b \cos(c + dx))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(2/3),x)

[Out] Integral((A + B*cos(c + d*x) + C*cos(c + d*x)**2)*cos(c + d*x)**m/(b*cos(c + d*x))**(2/3), x)

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(2/3), x)

$$3.368 \quad \int \frac{\cos^m(c+dx)(A+B \cos(c+dx)+C \cos^2(c+dx))}{(b \cos(c+dx))^{4/3}} dx$$

Optimal. Leaf size=235

$$\frac{3(C(1-3m) - A(3m+2)) \sin(c+dx) \cos^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m-1); \frac{1}{6}(3m+5); \cos^2(c+dx)\right) - 3B \sin(c+dx) \cos^{m+1}(c+dx)}{bd(1-3m)(3m+2)\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}}$$

[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) - (3*(C*(1 - 3*m) - A*(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 - 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.232412, antiderivative size = 225, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{3\left(\frac{A}{1-3m} - \frac{C}{3m+2}\right) \sin(c+dx) \cos^m(c+dx) {}_2F_1\left(\frac{1}{2}, \frac{1}{6}(3m-1); \frac{1}{6}(3m+5); \cos^2(c+dx)\right) - 3B \sin(c+dx) \cos^{m+1}(c+dx)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3), x]

[Out] (3*C*Cos[c + d*x]^m*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)) + (3*(A/(1 - 3*m) - C/(2 + 3*m))*Cos[c + d*x]^m*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Cos[c + d*x]^(1 + m)*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(2 + 3*m)*(b*Cos[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \cdot \sin[e + f \cdot x]^{(m + 1)}, x], x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \cdot \sin[c + d \cdot x] + (d \cdot x))^{(n)}, x_Symbol] := \text{Simp}[(\text{Cos}[c + d \cdot x] \cdot (b \cdot \sin[c + d \cdot x])^{(n + 1)} \cdot \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d \cdot x]^2]) / (b \cdot d \cdot (n + 1) \cdot \text{Sqrt}[\text{Cos}[c + d \cdot x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2 \cdot n]$

Rubi steps

$$\int \frac{\cos^m(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{(b \cos(c + dx))^{4/3}} dx = \frac{\sqrt[3]{\cos(c + dx)} \int \cos^{-\frac{4}{3}+m}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx))}{b \sqrt[3]{b \cos(c + dx)}} \\ = \frac{3C \cos^m(c + dx) \sin(c + dx)}{bd(2 + 3m) \sqrt[3]{b \cos(c + dx)}} + \frac{(3 \sqrt[3]{\cos(c + dx)}) \int \cos^{-\frac{4}{3}+m}(c + dx)}{b \sqrt[3]{b \cos(c + dx)}} \\ = \frac{3C \cos^m(c + dx) \sin(c + dx)}{bd(2 + 3m) \sqrt[3]{b \cos(c + dx)}} + \frac{(B \sqrt[3]{\cos(c + dx)}) \int \cos^{-\frac{1}{3}+m}(c + dx)}{b \sqrt[3]{b \cos(c + dx)}} \\ = \frac{3C \cos^m(c + dx) \sin(c + dx)}{bd(2 + 3m) \sqrt[3]{b \cos(c + dx)}} + \frac{3 \left(\frac{A}{1-3m} - \frac{C}{2+3m} \right) \cos^m(c + dx)}{bd(2 + 3m) \sqrt[3]{b \cos(c + dx)}}$$

Mathematica [A] time = 0.536624, size = 166, normalized size = 0.71

$$\frac{3 \sin(c + dx) \cos^{m+1}(c + dx) \left((A(3m + 2) + C(3m - 1)) {}_2F_1 \left(\frac{1}{2}, \frac{1}{6}(3m - 1); \frac{1}{6}(3m + 5); \cos^2(c + dx) \right) + (3m - 1) (B \cos(c + dx) + C \cos^2(c + dx)) \right)}{d(3m - 1)(3m + 2) \sqrt{\sin^2(c + dx) (b \cos(c + dx))^{4/3}}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^m*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/(b*Cos[c + d*x])^(4/3),x]

[Out] (-3*Cos[c + d*x]^(1 + m)*Sin[c + d*x]*((C*(-1 + 3*m) + A*(2 + 3*m))*Hypergeometric2F1[1/2, (-1 + 3*m)/6, (5 + 3*m)/6, Cos[c + d*x]^2] + (-1 + 3*m)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + 3*m)/6, (8 + 3*m)/6, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(-1 + 3*m)*(2 + 3*m)*(b*Cos[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.307, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^m (A + B \cos(dx + c) + C (\cos(dx + c))^2) (b \cos(dx + c))^{-\frac{4}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)

[Out] int(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^{\frac{2}{3}} \cos(dx + c)^m}{b^2 \cos(dx + c)^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^(2/3)*cos(d*x + c)^m/(b^2*cos(d*x + c)^2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**m*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(b*cos(d*x+c))**(4/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) \cos(dx + c)^m}{(b \cos(dx + c))^{\frac{4}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^m*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(b*cos(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*cos(d*x + c)^m/(b*cos(d*x + c))^(4/3), x)

3.369 $\int (a \cos(c+dx))^m (b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos$

Optimal. Leaf size=227

$$\frac{B \sin(c + dx)(a \cos(c + dx))^{m+2}(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m + n + 2); \frac{1}{2}(m + n + 4); \cos^2(c + dx)\right) (A(m + n + 2) + C \cos(c + dx))}{a^2 d(m + n + 2) \sqrt{\sin^2(c + dx)}}$$

[Out] (C*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(a*d*(2 + m + n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2]) - (B*(a*Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.232264, antiderivative size = 227, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{B \sin(c + dx)(a \cos(c + dx))^{m+2}(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m + n + 2); \frac{1}{2}(m + n + 4); \cos^2(c + dx)\right) (A(m + n + 2) + C \cos(c + dx))}{a^2 d(m + n + 2) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(a*Cos[c + d*x])^m*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (C*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(a*d*(2 + m + n)) - ((C*(1 + m + n) + A*(2 + m + n))*(a*Cos[c + d*x])^(1 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a*d*(1 + m + n)*(2 + m + n)*Sqrt[Sin[c + d*x]^2]) - (B*(a*Cos[c + d*x])^(2 + m)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(a^2*d*(2 + m + n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m + n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \cdot \sin[e + f \cdot x]^{(m + 1)}, x], x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \cdot \sin[c + d \cdot x] + d \cdot x)^{(n)}, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d \cdot x] \cdot (b \cdot \sin[c + d \cdot x])^{(n + 1)} \cdot \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \sin[c + d \cdot x]^2]) / (b \cdot d \cdot (n + 1) \cdot \text{Sqrt}[\cos[c + d \cdot x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2 \cdot n]$

Rubi steps

$$\begin{aligned} \int (a \cos(c + dx))^m (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= ((a \cos(c + dx))^{-n} (b \cos(c + dx))^n) \int (a \cos(c + dx))^m (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= \frac{C(a \cos(c + dx))^{1+m} (b \cos(c + dx))^n \sin(c + dx)}{ad(2 + m + n)} \\ &= \frac{C(a \cos(c + dx))^{1+m} (b \cos(c + dx))^n \sin(c + dx)}{ad(2 + m + n)} \\ &= \frac{C(a \cos(c + dx))^{1+m} (b \cos(c + dx))^n \sin(c + dx)}{ad(2 + m + n)} \end{aligned}$$

Mathematica [A] time = 0.255935, size = 161, normalized size = 0.71

$$\frac{\sin(c + dx) \cos(c + dx) (a \cos(c + dx))^m (b \cos(c + dx))^n \left((A(m + n + 2) + C(m + n + 1)) {}_2F_1\left(\frac{1}{2}, \frac{1}{2}(m + n + 1); \frac{1}{2}(m + n + 1), \sin^2(c + dx)\right) \right)}{d(m + n + 1)(m + n + 1)}$$

Antiderivative was successfully verified.

[In] Integrate[(a * Cos[c + d * x])^m * (b * Cos[c + d * x])^n * (A + B * Cos[c + d * x] + C * Cos[c + d * x]^2), x]

[Out] -((Cos[c + d * x] * (a * Cos[c + d * x])^m * (b * Cos[c + d * x])^n * Sin[c + d * x] * ((C * (1 + m + n) + A * (2 + m + n)) * Hypergeometric2F1[1/2, (1 + m + n)/2, (3 + m + n)/2, Cos[c + d * x]^2] + (1 + m + n) * (B * Cos[c + d * x] * Hypergeometric2F1[1/2, (2 + m + n)/2, (4 + m + n)/2, Cos[c + d * x]^2] - C * Sqrt[Sin[c + d * x]^2]))) / (d * (1 + m + n) * (2 + m + n) * Sqrt[Sin[c + d * x]^2]))

Maple [F] time = 2.476, size = 0, normalized size = 0.

$$\int (\cos(dx + c) a)^m (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(d*x+c)*a)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)

[Out] int((cos(d*x+c)*a)^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (a \cos(dx + c))^m (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((C \cos(dx + c)^2 + B \cos(dx + c) + A) (a \cos(dx + c))^m (b \cos(dx + c))^n, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))**m*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (a \cos(dx + c))^m (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*cos(d*x+c))^m*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c))^m*(b*cos(d*x + c))^n, x)
```

3.370 $\int \cos^2(c+dx)(b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=187

$$\frac{(A(n+4) + C(n+3)) \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right) - B \sin(c+dx)(b \cos(c+dx))^{n+3}}{b^3 d(n+3)(n+4) \sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+3}}{b^4 d(n+4) \sqrt{\sin^2(c+dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.218638, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{(A(n+4) + C(n+3)) \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right) - B \sin(c+dx)(b \cos(c+dx))^{n+3}}{b^3 d(n+3)(n+4) \sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+3}}{b^4 d(n+4) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(3 + n)*Sin[c + d*x])/(b^3*d*(4 + n)) - ((C*(3 + n) + A*(4 + n))*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(4 + n)*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^4*d*(4 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int \cos^2(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{\int (b \cos(c + dx))^{2+n} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b^2} \\ &= \frac{C(b \cos(c + dx))^{3+n} \sin(c + dx)}{b^3 d(4 + n)} + \frac{\int (b \cos(c + dx))^{2+n} (A + B \cos(c + dx)) dx}{b^2} \\ &= \frac{C(b \cos(c + dx))^{3+n} \sin(c + dx)}{b^3 d(4 + n)} + \frac{B \int (b \cos(c + dx))^{2+n} dx}{b^2} \\ &= \frac{C(b \cos(c + dx))^{3+n} \sin(c + dx)}{b^3 d(4 + n)} - \frac{(C(3 + n) + A(4 + n)) \int (b \cos(c + dx))^{2+n} dx}{b^2} \end{aligned}$$

Mathematica [A] time = 0.482331, size = 144, normalized size = 0.77

$$\frac{\sin(c + dx) \cos^3(c + dx)(b \cos(c + dx))^n \left((A(n + 4) + C(n + 3)) {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c + dx)\right) + (n + 3) \left(B \cos(c + dx) + C \cos^2(c + dx) \right) \right)}{d(n + 3)(n + 4) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] -((Cos[c + d*x]^3*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(3 + n) + A*(4 + n))*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] + (3 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + n)/2, (6 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(3 + n)*(4 + n)*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 2.026, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^2 (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

```
[Out] int(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^4 + B \cos(dx + c)^3 + A \cos(dx + c)^2\right) (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="fricas")

[Out] integral((C*cos(d*x + c)^4 + B*cos(d*x + c)^3 + A*cos(d*x + c)^2)*(b*cos(d*
x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**2*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),
x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \cos(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x,
algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*
x + c)^2, x)

3.371 $\int \cos(c+dx)(b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c -$

Optimal. Leaf size=187

$$\frac{(A(n+3) + C(n+2)) \sin(c+dx)(b \cos(c+dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right)}{b^2 d(n+2)(n+3) \sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c+dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.213301, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {16, 3023, 2748, 2643}

$$\frac{(A(n+3) + C(n+2)) \sin(c+dx)(b \cos(c+dx))^{n+2} {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c+dx)\right)}{b^2 d(n+2)(n+3) \sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+3} {}_2F_1\left(\frac{1}{2}, \frac{n+3}{2}; \frac{n+5}{2}; \cos^2(c+dx)\right)}{b^3 d(n+3) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(2 + n)*Sin[c + d*x])/(b^2*d*(3 + n)) - ((C*(2 + n) + A*(3 + n))*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(3 + n)*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^3*d*(3 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_)^(n_.), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int \cos(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx = \frac{\int (b \cos(c + dx))^{1+n} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{b}$$

$$= \frac{C(b \cos(c + dx))^{2+n} \sin(c + dx)}{b^2 d(3 + n)} + \frac{\int (b \cos(c + dx))^{1+n} (A + B \cos(c + dx)) dx}{b^2 d(3 + n)}$$

$$= \frac{C(b \cos(c + dx))^{2+n} \sin(c + dx)}{b^2 d(3 + n)} + \frac{B \int (b \cos(c + dx))^{1+n} dx}{b^2 d(3 + n)}$$

$$= \frac{C(b \cos(c + dx))^{2+n} \sin(c + dx)}{b^2 d(3 + n)} - \frac{C(2 + n) \int (b \cos(c + dx))^{1+n} dx}{b^2 d(3 + n)}$$

Mathematica [A] time = 0.330168, size = 144, normalized size = 0.77

$$\frac{\sin(c + dx) \cos^2(c + dx)(b \cos(c + dx))^n \left((A(n + 3) + C(n + 2)) {}_2F_1\left(\frac{1}{2}, \frac{n+2}{2}; \frac{n+4}{2}; \cos^2(c + dx)\right) + (n + 2) \left(B \cos(c + dx) + C \cos^2(c + dx) \right) \right)}{d(n + 2)(n + 3)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] -((Cos[c + d*x]^2*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(2 + n) + A*(3 + n))*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] + (2 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + n)/2, (5 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(2 + n)*(3 + n)*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.522, size = 0, normalized size = 0.

$$\int \cos(dx + c) (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

```
[Out] int(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)\right) (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \cos(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c), x)

3.372 $\int (b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=187

$$\frac{(A(n+2) + C(n+1)) \sin(c+dx)(b \cos(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c+dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+2}}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.167909, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {3023, 2748, 2643}

$$\frac{(A(n+2) + C(n+1)) \sin(c+dx)(b \cos(c+dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c+dx)\right)}{bd(n+1)(n+2)\sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^{n+2}}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (C*(b*Cos[c + d*x])^(1 + n)*Sin[c + d*x])/(b*d*(2 + n)) - ((C*(1 + n) + A*(2 + n))*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(2 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Ssin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Ssin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[c, Int[(b*Ssin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Ssin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Simp[(Cos[c + d*x]*(b*Ssin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x] && !IntegerQ[2*n]

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{C(b \cos(c + dx))^{1+n} \sin(c + dx)}{bd(2 + n)} + \frac{\int (b \cos(c + dx))^n (b(C(1+n) \cos(c + dx) + A)) dx}{bd(2 + n)} \\ &= \frac{C(b \cos(c + dx))^{1+n} \sin(c + dx)}{bd(2 + n)} + \frac{B \int (b \cos(c + dx))^{1+n} dx}{b} \\ &= \frac{C(b \cos(c + dx))^{1+n} \sin(c + dx)}{bd(2 + n)} - \frac{\left(A + \frac{C(1+n)}{2+n}\right) (b \cos(c + dx))^{1+n}}{bd(2 + n)} \end{aligned}$$

Mathematica [A] time = 0.234622, size = 142, normalized size = 0.76

$$\frac{\sin(c + dx) \cos(c + dx) (b \cos(c + dx))^n \left((A(n + 2) + C(n + 1)) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) + (n + 1) \left(B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) - C \sqrt{\sin^2(c + dx)} \right) \right)}{d(n + 1)(n + 2) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] -((Cos[c + d*x]*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(1 + n) + A*(2 + n))*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] + (1 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (2 + n)/2, (4 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2])))/(d*(1 + n)*(2 + n)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 1.466, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n, x)
```

3.373 $\int (b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx$

Optimal. Leaf size=170

$$\frac{(An + A + Cn) \sin(c + dx) (b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right)}{dn(n+1)\sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx) (b \cos(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right)}{bd(n+1)\sqrt{\sin^2(c + dx)}}$$

[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.191191, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.108$, Rules used = {16, 3023, 2748, 2643}

$$\frac{(An + A + Cn) \sin(c + dx) (b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right)}{dn(n+1)\sqrt{\sin^2(c + dx)}} - \frac{B \sin(c + dx) (b \cos(c + dx))^{n+1} {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right)}{bd(n+1)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]

[Out] (C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)) - ((A + A*n + C*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*(1 + n)*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx &= b \int (b \cos(c + dx))^{-1+n} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec(c + dx) dx \\ &= \frac{C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + n)} + \frac{\int (b \cos(c + dx))^{-1+n} (A + B \cos(c + dx)) \sec(c + dx) dx}{d(1 + n)} \\ &= \frac{C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + n)} + B \int (b \cos(c + dx))^{-1+n} \sec(c + dx) dx \\ &= \frac{C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + n)} - \frac{(A + An + Bn^2)}{d(1 + n)} \end{aligned}$$

Mathematica [A] time = 0.203808, size = 127, normalized size = 0.75

$$\frac{\sin(c + dx)(b \cos(c + dx))^n \left((An + A + Cn) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right) + n \left(B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n+1}{2}; \frac{n+3}{2}; \cos^2(c + dx)\right) \right) \right)}{dn(n + 1)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x], x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Sin[c + d*x]*((A + A*n + C*n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2] + n*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))) / (d*n*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.455, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \sec(dx + c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \sec(dx + c) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c), x)

3.374 $\int (b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(dx) dx$

Optimal. Leaf size=173

$$\frac{b(C(1-n) - An) \sin(c+dx)(b \cos(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{d(1-n)n\sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.230114, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{b(C(1-n) - An) \sin(c+dx)(b \cos(c+dx))^{n-1} {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{d(1-n)n\sqrt{\sin^2(c+dx)}} - \frac{B \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^2,x]

[Out] (b*C*(b*Cos[c + d*x])^(-1 + n)*Sin[c + d*x])/(d*n) - (b*(C*(1 - n) - A*n)*(b*Cos[c + d*x])^(-1 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - n)*n*Sqrt[Sin[c + d*x]^2]) - (B*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Cos[c + d*x]^2]*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])

Rule 16

Int[(u_.)*(v_)^(m_.)*((b_.)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^2(c + dx) dx &= b^2 \int (b \cos(c + dx))^{-2+n} (A + B \cos(c + dx) + \\ &= \frac{bC(b \cos(c + dx))^{-1+n} \sin(c + dx)}{dn} + \frac{b \int (b \cos(c + dx))^{-1+n} \sin(c + dx) dx}{dn} \\ &= \frac{bC(b \cos(c + dx))^{-1+n} \sin(c + dx)}{dn} + (bB) \int (b \cos(c + dx))^{-1+n} \sin(c + dx) dx \\ &= \frac{bC(b \cos(c + dx))^{-1+n} \sin(c + dx)}{dn} - \frac{b(C(1 - \cos^2(c + dx)))^{-1+n} \sin(c + dx)}{dn} \end{aligned}$$

Mathematica [A] time = 0.287624, size = 131, normalized size = 0.76

$$\frac{\tan(c + dx)(b \cos(c + dx))^n \left((-An + C(-n) + C) {}_2F_1\left(\frac{1}{2}, \frac{n-1}{2}; \frac{n+1}{2}; \cos^2(c + dx)\right) - (n-1) \left(B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n}{2}; \frac{n+2}{2}; \cos^2(c + dx)\right) - C \sqrt{\sin^2(c + dx)} \right) \right)}{d(n-1)n\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c
+ d*x]^2,x]
```

```
[Out] ((b*Cos[c + d*x])^n*((C - A*n - C*n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1
+ n)/2, Cos[c + d*x]^2] - (-1 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, n
/2, (2 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))*Tan[c + d*x])/(d*
(-1 + n)*n*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.304, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \sec(dx + c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**2,
x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \sec(dx + c)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^2,x,
algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^2, x)

$$3.375 \quad \int (b \cos(c+dx))^n \left(A + B \cos(c + dx) + C \cos^2(c + dx) \right) \sec^3 dx$$

Optimal. Leaf size=194

$$\frac{b^2(A(1-n) + C(2-n)) \sin(c+dx)(b \cos(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}} - \frac{b^2C \sin(c+dx)(b \cos(c+dx))^{n-2}}{d(1-n)} +$$

[Out] $-\left(\frac{b^2 C (b \cos[c + d*x])^{-2 + n} \sin[c + d*x]}{d(1-n)}\right) + (b^2(A(1-n) + C(2-n)) * (b \cos[c + d*x])^{-2 + n} * \text{Hypergeometric2F1}\left[\frac{1}{2}, (-2 + n)/2, n/2, \cos[c + d*x]^2 * \sin[c + d*x]\right] / (d(1-n)(2-n) * \text{Sqrt}[\sin[c + d*x]^2]) + (b * B * (b \cos[c + d*x])^{-1 + n} * \text{Hypergeometric2F1}\left[\frac{1}{2}, (-1 + n)/2, (1 + n)/2, \cos[c + d*x]^2 * \sin[c + d*x]\right] / (d(1-n) * \text{Sqrt}[\sin[c + d*x]^2])$

Rubi [A] time = 0.255924, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{b^2(A(1-n) + C(2-n)) \sin(c+dx)(b \cos(c+dx))^{n-2} {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c+dx)\right)}{d(1-n)(2-n)\sqrt{\sin^2(c+dx)}} - \frac{b^2C \sin(c+dx)(b \cos(c+dx))^{n-2}}{d(1-n)} +$$

Antiderivative was successfully verified.

[In] Int[(b*cos[c + d*x])^n*(A + B*cos[c + d*x] + C*cos[c + d*x]^2)*Sec[c + d*x]^3,x]

[Out] $-\left(\frac{b^2 C (b \cos[c + d*x])^{-2 + n} \sin[c + d*x]}{d(1-n)}\right) + (b^2(A(1-n) + C(2-n)) * (b \cos[c + d*x])^{-2 + n} * \text{Hypergeometric2F1}\left[\frac{1}{2}, (-2 + n)/2, n/2, \cos[c + d*x]^2 * \sin[c + d*x]\right] / (d(1-n)(2-n) * \text{Sqrt}[\sin[c + d*x]^2]) + (b * B * (b \cos[c + d*x])^{-1 + n} * \text{Hypergeometric2F1}\left[\frac{1}{2}, (-1 + n)/2, (1 + n)/2, \cos[c + d*x]^2 * \sin[c + d*x]\right] / (d(1-n) * \text{Sqrt}[\sin[c + d*x]^2])$

Rule 16

Int[(u_)*(v_)^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[1/b^m, Int[u*(b*v)^(m+n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2, x_Symbol] := -Simp[(C*cos[e + f*x]*(a + b*sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*sin[e + f*x])^m, x], x] + Dist[d/b, Int[(b*sin[e + f*x])^(m+1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[(b_.)*sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx = b^3 \int (b \cos(c + dx))^{-3+n} (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^3(c + dx) dx$$

$$= -\frac{b^2 C (b \cos(c + dx))^{-2+n} \sin(c + dx)}{d(1 - n)} - \frac{b^2 (A + B \cos(c + dx))^{-2+n} \sin(c + dx)}{d(1 - n)}$$

$$= -\frac{b^2 C (b \cos(c + dx))^{-2+n} \sin(c + dx)}{d(1 - n)} + \frac{b^2 (A + B \cos(c + dx))^{-2+n} \sin(c + dx)}{d(1 - n)}$$

Mathematica [A] time = 0.457482, size = 137, normalized size = 0.71

$$\frac{b \tan(c + dx) (b \cos(c + dx))^{n-1} \left((A(n-1) + C(n-2)) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c + dx)\right) + (n-2) \left(B \cos(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{n-2}{2}; \frac{n}{2}; \cos^2(c + dx)\right) - C \sqrt{\sin^2(c + dx)} \right) \right)}{d(n-2)(n-1) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^3,x]
```

```
[Out] -((b*(b*Cos[c + d*x])^(-1 + n)*((C*(-2 + n) + A*(-1 + n))*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] + (-2 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))*Tan[c + d*x])/(d*(-2 + n)*(-1 + n)*Sqrt[Sin[c + d*x]^2])
```

Maple [F] time = 1.602, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \sec(dx + c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**3,
x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right) (b \cos(dx + c))^n \sec(dx + c)^3 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^3,x,
algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^3, x)

3.376 $\int (b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(dx) dx$

Optimal. Leaf size=196

$$\frac{b^3(A(2-n) + C(3-n)) \sin(c+dx)(b \cos(c+dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{n-3}{2}; \frac{n-1}{2}; \cos^2(c+dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}} + \frac{b^2B \sin(c+dx)(b \cos(c+dx))^{n-2}}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

[Out] $-\left(\frac{b^3 C (b \cos[c + dx])^{-3+n} \sin[c + dx]}{d(2-n)}\right) + (b^3(A(2-n) + C(3-n)) (b \cos[c + dx])^{-3+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, (-3+n)/2, (-1+n)/2, \cos[c + dx]^2\right] \sin[c + dx]) / (d(2-n)(3-n) \sqrt{\sin[c + dx]^2}) + (b^2 B (b \cos[c + dx])^{-2+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, (-2+n)/2, n/2, \cos[c + dx]^2\right] \sin[c + dx]) / (d(2-n) \sqrt{\sin[c + dx]^2})$

Rubi [A] time = 0.261561, antiderivative size = 196, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 39, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {16, 3023, 2748, 2643}

$$\frac{b^3(A(2-n) + C(3-n)) \sin(c+dx)(b \cos(c+dx))^{n-3} {}_2F_1\left(\frac{1}{2}, \frac{n-3}{2}; \frac{n-1}{2}; \cos^2(c+dx)\right)}{d(2-n)(3-n)\sqrt{\sin^2(c+dx)}} + \frac{b^2B \sin(c+dx)(b \cos(c+dx))^{n-2}}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b \cos[c + dx])^n (A + B \cos[c + dx] + C \cos^2[c + dx]) \sec^4(dx), x]$

[Out] $-\left(\frac{b^3 C (b \cos[c + dx])^{-3+n} \sin[c + dx]}{d(2-n)}\right) + (b^3(A(2-n) + C(3-n)) (b \cos[c + dx])^{-3+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, (-3+n)/2, (-1+n)/2, \cos[c + dx]^2\right] \sin[c + dx]) / (d(2-n)(3-n) \sqrt{\sin[c + dx]^2}) + (b^2 B (b \cos[c + dx])^{-2+n} \text{Hypergeometric2F1}\left[\frac{1}{2}, (-2+n)/2, n/2, \cos[c + dx]^2\right] \sin[c + dx]) / (d(2-n) \sqrt{\sin[c + dx]^2})$

Rule 16

$\text{Int}[(u \cdot)^m (v \cdot)^n ((b \cdot)(v \cdot))^n, x_Symbol] \rightarrow \text{Dist}[1/b^m, \text{Int}[u \cdot (b \cdot v \cdot)^{m+n}, x], x] /;$ FreeQ[{b, n}, x] && IntegerQ[m]

Rule 3023

$\text{Int}[(a \cdot + (b \cdot) \sin[e \cdot] + (f \cdot)(x \cdot)]^m ((A \cdot) + (B \cdot) \sin[e \cdot] + (C \cdot)(x \cdot)^2), x_Symbol] \rightarrow -\text{Simp}[(C \cos[e + f \cdot x] (a + b \sin[e + f \cdot x])^{m+1}) / (b f (m+2)), x] + \text{Dist}[1/(b(m+2)), \text{Int}[(a + b \sin[e + f \cdot x])^m \text{Simp}[A b (m+2) + b C (m+1) + (b B (m+2) - a C) \sin[e + f \cdot x], x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

$\text{Int}[(b \cdot) \sin[e \cdot] + (f \cdot)(x \cdot)]^m ((c \cdot) + (d \cdot) \sin[e \cdot] + (f \cdot)(x \cdot)), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b \sin[e + f \cdot x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b \sin[e + f \cdot x])^{m+1}, x], x] /;$ FreeQ[{b, c, d, e, f, m}, x]

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_.)]^(n_), x_Symbol] := Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2)]/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned} \int (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) \sec^4(c + dx) dx &= b^4 \int (b \cos(c + dx))^{-4+n} (A + B \cos(c + dx) + \\ &= -\frac{b^3 C (b \cos(c + dx))^{-3+n} \sin(c + dx)}{d(2-n)} - \frac{b^3 \int (b \cos(c + dx))^{-3+n} \sin(c + dx) dx}{d(2-n)} \\ &= -\frac{b^3 C (b \cos(c + dx))^{-3+n} \sin(c + dx)}{d(2-n)} + (b^3 B) \int (b \cos(c + dx))^{-3+n} \sin(c + dx) dx \\ &= -\frac{b^3 C (b \cos(c + dx))^{-3+n} \sin(c + dx)}{d(2-n)} + \frac{b^3 (A(2-n) + B)}{d(2-n)} \end{aligned}$$

Mathematica [A] time = 0.331017, size = 142, normalized size = 0.72

$$\frac{\tan(c + dx) \sec^2(c + dx) (b \cos(c + dx))^n \left((A(n-2) + C(n-3)) {}_2F_1\left(\frac{1}{2}, \frac{n-3}{2}; \frac{n-1}{2}; \cos^2(c + dx)\right) + (n-3) \left(B \cos(c + dx) \right) \right)}{d(n-3)(n-2)\sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)*Sec[c + d*x]^4,x]
```

```
[Out] -(((b*Cos[c + d*x])^n*Sec[c + d*x]^2*((C*(-3 + n) + A*(-2 + n))*Hypergeometric2F1[1/2, (-3 + n)/2, (-1 + n)/2, Cos[c + d*x]^2] + (-3 + n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2, n/2, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))*Tan[c + d*x])/(d*(-3 + n)*(-2 + n)*Sqrt[Sin[c + d*x]^2]))
```

Maple [F] time = 1.433, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\sec(dx + c))^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

```
[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^4, x)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*sec(d*x+c)**4,
x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n \sec(dx + c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*sec(d*x+c)^4,x,
algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sec(d*x + c)^4, x)

3.377 $\int \cos^{\frac{3}{2}}(c+dx)(b \cos(c+dx))^n (A + B \cos(c + dx) + C \cos^2(c$

Optimal. Leaf size=223

$$\frac{2(A(2n+7) + C(2n+5)) \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+5); \frac{1}{4}(2n+9); \cos^2(c+dx)\right) + 2B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{d(2n+5)(2n+7)\sqrt{\sin^2(c+dx)}}$$

[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2*(C*(5 + 2*n) + A*(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.237412, antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{2\left(\frac{A}{2n+5} + \frac{C}{2n+7}\right) \sin(c+dx) \cos^{\frac{5}{2}}(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+5); \frac{1}{4}(2n+9); \cos^2(c+dx)\right) + 2B \sin(c+dx) \cos^{\frac{3}{2}}(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (2*C*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(7 + 2*n)) - (2*(A/(5 + 2*n) + C/(7 + 2*n))*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(7/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_.)*((b_.)*(v_))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \sin[e + f x]^{m+1}, x, x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x)^n, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d x] * (b \sin[c + d x])^{n+1} * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \sin[c + d x]^2]) / (b d (n+1) \sqrt{\cos[c + d x]^2}), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2n]$

Rubi steps

$$\begin{aligned} \int \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{\frac{3}{2}+n} \\ &= \frac{2C \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^n \sin(c + dx)}{d(7 + 2n)} \\ &= \frac{2C \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^n \sin(c + dx)}{d(7 + 2n)} \\ &= \frac{2C \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^n \sin(c + dx)}{d(7 + 2n)} \end{aligned}$$

Mathematica [A] time = 0.524583, size = 164, normalized size = 0.74

$$\frac{2 \sin(c + dx) \cos^{\frac{5}{2}}(c + dx) (b \cos(c + dx))^n \left((A(2n + 7) + C(2n + 5)) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 5); \frac{1}{4}(2n + 9); \cos^2(c + dx)\right) + 2 \right)}{d(2n + 5)(2n + 7) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^(3/2)*(b*cos[c + d*x])^n*(A + B*cos[c + d*x] + C*cos[c + d*x]^2), x]

[Out] (-2*cos[c + d*x]^(5/2)*(b*cos[c + d*x])^n*sin[c + d*x]*((C*(5 + 2*n) + A*(7 + 2*n))*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] + (5 + 2*n)*(B*cos[c + d*x]*Hypergeometric2F1[1/2, (7 + 2*n)/4, (11 + 2*n)/4, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(5 + 2*n)*(7 + 2*n)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.674, size = 0, normalized size = 0.

$$\int (\cos(dx + c))^{\frac{3}{2}} (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral} \left((C \cos(dx + c)^3 + B \cos(dx + c)^2 + A \cos(dx + c)) (b \cos(dx + c))^n \sqrt{\cos(dx + c)}, x \right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^3 + B*cos(d*x + c)^2 + A*cos(d*x + c))*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \cos(dx + c)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*cos(d*x + c)^(3/2), x)
```

3.378 $\int \sqrt{\cos(c+dx)}(b \cos(c+dx))^n (A + B \cos(c+dx) + C \cos^2(c+dx)) dx$

Optimal. Leaf size=223

$$\frac{2(A(2n+5) + C(2n+3)) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \cos^2(c+dx)\right)}{d(2n+3)(2n+5)\sqrt{\sin^2(c+dx)}} - 2B \sin(c+dx)$$

[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2*(C*(3 + 2*n) + A*(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(3 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.215999, antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{2\left(\frac{A}{2n+3} + \frac{C}{2n+5}\right) \sin(c+dx) \cos^{\frac{3}{2}}(c+dx) (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+3); \frac{1}{4}(2n+7); \cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}} - 2B \sin(c+dx)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (2*C*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(5 + 2*n)) - (2*(A/(3 + 2*n) + C/(5 + 2*n))*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(5/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_)*((a_)*(v_))^(m_)*((b_)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \sin[e + f x]^{m+1}, x, x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x)^n, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d x] * (b \sin[c + d x])^{n+1} * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \sin[c + d x]^2]) / (b d (n+1) \sqrt{\cos[c + d x]^2}), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \sqrt{\cos(c + dx)} (b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{\frac{1}{2}+n}(c + dx) dx \\ &= \frac{2C \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^n \sin(c + dx)}{d(5 + 2n)} \\ &= \frac{2C \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^n \sin(c + dx)}{d(5 + 2n)} \\ &= \frac{2C \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^n \sin(c + dx)}{d(5 + 2n)} \end{aligned}$$

Mathematica [A] time = 0.428797, size = 164, normalized size = 0.74

$$\frac{2 \sin(c + dx) \cos^{\frac{3}{2}}(c + dx) (b \cos(c + dx))^n \left((A(2n + 5) + C(2n + 3)) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 3); \frac{1}{4}(2n + 7); \cos^2(c + dx)\right) + (2n + 3) \right)}{d(2n + 3)(2n + 5) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] (-2*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Sin[c + d*x]*((C*(3 + 2*n) + A*(5 + 2*n))*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2] + (3 + 2*n)*(B*Cos[c + d*x]*Hypergeometric2F1[1/2, (5 + 2*n)/4, (9 + 2*n)/4, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(3 + 2*n)*(5 + 2*n)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.68, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)

[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}((C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sqrt{\cos(dx + c)}, x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n \sqrt{\cos(dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)*cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n*sqrt(cos(d*x + c)), x)
```

$$3.379 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\sqrt{\cos(c+dx)}} dx$$

Optimal. Leaf size=221

$$\frac{2(A(2n+3)+2Cn+C) \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+1); \frac{1}{4}(2n+5); \cos^2(c+dx)\right) + 2B \sin(c+dx)}{d(2n+1)(2n+3) \sqrt{\sin^2(c+dx)}}$$

[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.196848, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{2(A(2n+3)+2Cn+C) \sin(c+dx) \sqrt{\cos(c+dx)} (b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n+1); \frac{1}{4}(2n+5); \cos^2(c+dx)\right) + 2B \sin(c+dx)}{d(2n+1)(2n+3) \sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Sqrt[Cos[c + d*x]], x]

[Out] (2*C*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)) - (2*(C + 2*C*n + A*(3 + 2*n))*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sin[c + d*x]^2]) - (2*B*Cos[c + d*x]^(3/2)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (3 + 2*n)/4, (7 + 2*n)/4, Cos[c + d*x]^2*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_.))^(m_.)*((b_.)*(v_.))^(n_.), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \sin[e + f x]^{(m + 1)}, x, x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + (d \cdot x))^{(n)}, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d x] * (b \sin[c + d x])^{(n + 1)} * \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d x]^2]) / (b * d * (n + 1) * \text{Sqrt}[\text{Cos}[c + d x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\amp; \text{IntegerQ}[2 * n]$

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx))}{\sqrt{\cos(c + dx)}} dx &= (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{-\frac{1}{2} + n}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= \frac{2C \sqrt{\cos(c + dx)} (b \cos(c + dx))^n \sin(c + dx)}{d(3 + 2n)} + \frac{(2 \cos^{-1}(c + dx) (b \cos(c + dx))^n \sin(c + dx) + (B \cos(c + dx) + C) \sqrt{\cos(c + dx)})}{d(3 + 2n)} \\ &= \frac{2C \sqrt{\cos(c + dx)} (b \cos(c + dx))^n \sin(c + dx)}{d(3 + 2n)} - \frac{2(C + B \cos(c + dx)) \sqrt{\cos(c + dx)}}{d(3 + 2n)} \end{aligned}$$

Mathematica [A] time = 0.374842, size = 162, normalized size = 0.73

$$\frac{2 \sin(c + dx) \sqrt{\cos(c + dx)} (b \cos(c + dx))^n \left((A(2n + 3) + 2Cn + C) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n + 1); \frac{1}{4}(2n + 5); \cos^2(c + dx)\right) + (2n + 1) \sqrt{\sin^2(c + dx)} \right)}{d(2n + 1)(2n + 3) \sqrt{\sin^2(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((b * Cos[c + d * x])^n * (A + B * Cos[c + d * x] + C * Cos[c + d * x]^2)) / Sqrt[Cos[c + d * x]], x]

[Out] (-2 * Sqrt[Cos[c + d * x]] * (b * Cos[c + d * x])^n * Sin[c + d * x] * ((C + 2 * C * n + A * (3 + 2 * n)) * Hypergeometric2F1[1/2, (1 + 2 * n)/4, (5 + 2 * n)/4, Cos[c + d * x]^2] + (1 + 2 * n) * (B * Cos[c + d * x] * Hypergeometric2F1[1/2, (3 + 2 * n)/4, (7 + 2 * n)/4, Cos[c + d * x]^2] - C * Sqrt[Sin[c + d * x]^2])) / (d * (1 + 2 * n) * (3 + 2 * n) * Sqrt[Sin[c + d * x]^2])

Maple [F] time = 0.787, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)

[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n}{\sqrt{\cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/sqrt(cos(d*x + c)), x)
```


$$3.380 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{3}{2}}(c+dx)} dx$$

Optimal. Leaf size=217

$$\frac{2(2An + A - C(1 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right) + 2B \sin(c + dx)\sqrt{\cos(c + dx)}}{d(1 - 4n^2)\sqrt{\sin^2(c + dx)}\sqrt{\cos(c + dx)}}$$

[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rubi [A] time = 0.201545, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{2(2An + A - C(1 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right) + 2B \sin(c + dx)\sqrt{\cos(c + dx)}}{d(1 - 4n^2)\sqrt{\sin^2(c + dx)}\sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Int[((b*Cos[c + d*x])^n*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2))/Cos[c + d*x]^(3/2), x]

[Out] (2*C*(b*Cos[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Cos[c + d*x]]) + (2*(A - C*(1 - 2*n) + 2*A*n)*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 - 4*n^2)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2]) - (2*B*Sqrt[Cos[c + d*x]]*(b*Cos[c + d*x])^n*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d*x]^2])

Rule 20

Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] := Dist[(b^IntPart[n]*(b*v)^FracPart[n])/(a^IntPart[n]*(a*v)^FracPart[n]), Int[u*(a*v)^(m+n), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m+n]

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m+1))/(b*f*(m+2)), x] + Dist[1/(b*(m+2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(

$b \sin[e + f x]^{(m + 1)}, x, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + (d \sin[c + d x])^2)^n, x_Symbol] \rightarrow \text{Simp}[(\cos[c + d x] * (b \sin[c + d x])^{(n + 1)} * \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \sin[c + d x]^2]) / (b d (n + 1) \sqrt{\cos[c + d x]^2}), x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{IntegerQ}[2 * n]$

Rubi steps

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{3}{2}}(c + dx)} dx = (\cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

$$= \frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + 2n)\sqrt{\cos(c + dx)}} + \frac{(2 \cos^{-n}(c + dx)(b \cos(c + dx))^n (A + B \cos(c + dx)))}{d(1 + 2n)\sqrt{\cos(c + dx)}}$$

$$= \frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + 2n)\sqrt{\cos(c + dx)}} + (B \cos^{-n}(c + dx)(b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) dx$$

$$= \frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 + 2n)\sqrt{\cos(c + dx)}} + \frac{2(A - C(1 - 2n) + 2An)}{d(1 + 2n)\sqrt{\cos(c + dx)}}$$

Mathematica [A] time = 0.445906, size = 157, normalized size = 0.72

$$\frac{2 \sin(c + dx)(b \cos(c + dx))^n \left((2An + A + C(2n - 1)) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 1); \frac{1}{4}(2n + 3); \cos^2(c + dx)\right) + (2n - 1) \left(B \cos(c + dx) + C \cos^2(c + dx) \right) \right)}{d(4n^2 - 1) \sqrt{\sin^2(c + dx)} \sqrt{\cos(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b*cos[c + d*x])^n*(A + B*cos[c + d*x] + C*cos[c + d*x]^2)/cos[c + d*x]^(3/2),x]

[Out] (-2*(b*cos[c + d*x])^n*sin[c + d*x]*((A + 2*A*n + C*(-1 + 2*n))*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] + (-1 + 2*n)*(B*cos[c + d*x]*Hypergeometric2F1[1/2, (1 + 2*n)/4, (5 + 2*n)/4, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(-1 + 4*n^2)*Sqrt[Cos[c + d*x]]*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.76, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)

[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{3}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(3/2), x)
```

$$3.381 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{5}{2}}(c+dx)} dx$$

Optimal. Leaf size=221

$$\frac{2(-2An + A + C(3 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right)}{d(1 - 2n)(3 - 2n)\sqrt{\sin^2(c + dx) \cos^{\frac{3}{2}}(c + dx)}} + \frac{2B \sin(c + dx)(b \cos(c + dx))^n}{d(1 - 2n)\sqrt{\sin^2(c + dx) \cos^{\frac{3}{2}}(c + dx)}}$$

[Out] $(-2*C*(b*\text{Cos}[c + d*x])^n*\text{Sin}[c + d*x])/(d*(1 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}) + (2*(A + C*(3 - 2*n) - 2*A*n)*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (2*B*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(d*(1 - 2*n)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rubi [A] time = 0.216755, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$\frac{2(-2An + A + C(3 - 2n)) \sin(c + dx)(b \cos(c + dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right)}{d(1 - 2n)(3 - 2n)\sqrt{\sin^2(c + dx) \cos^{\frac{3}{2}}(c + dx)}} + \frac{2B \sin(c + dx)(b \cos(c + dx))^n}{d(1 - 2n)\sqrt{\sin^2(c + dx) \cos^{\frac{3}{2}}(c + dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c + d*x])^n*(A + B*\text{Cos}[c + d*x] + C*\text{Cos}[c + d*x]^2)/\text{Cos}[c + d*x]^{(5/2)}, x]$

[Out] $(-2*C*(b*\text{Cos}[c + d*x])^n*\text{Sin}[c + d*x])/(d*(1 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}) + (2*(A + C*(3 - 2*n) - 2*A*n)*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*\text{Cos}[c + d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c + d*x]^2]) + (2*B*(b*\text{Cos}[c + d*x])^n*\text{Hypergeometric2F1}[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, \text{Cos}[c + d*x]^2*\text{Sin}[c + d*x])/(d*(1 - 2*n)*\text{Sqrt}[\text{Cos}[c + d*x]]*\text{Sqrt}[\text{Sin}[c + d*x]^2])$

Rule 20

$\text{Int}[(u_*)*((a_*)*(v_*)^m)*((b_*)*(v_*)^n), x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]})], \text{Int}[u*(a*v)^{m+n}, x], x] /;$ FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[m + n]

Rule 3023

$\text{Int}[(a_*) + (b_*)*\sin[(e_*) + (f_*)*(x_*)]^m*((A_*) + (B_*)*\sin[(e_*) + (f_*)*(x_*)] + (C_*)*\sin[(e_*) + (f_*)*(x_*)]^2), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e + f*x]*(a + b*\sin[e + f*x])^{m+1})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a + b*\sin[e + f*x])^m*\text{Simp}[A*b*(m+2) + b*C*(m+1) + (b*B*(m+2) - a*C)*\sin[e + f*x], x], x], x] /;$ FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2748

$\text{Int}[(b_*)*\sin[(e_*) + (f_*)*(x_*)]^m*((c_*) + (d_*)*\sin[(e_*) + (f_*)*(x_*)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\sin[e + f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\sin[e + f*x])^m, x], x]$

$b \sin[e + f x]^{(m + 1)}, x, x] /; \text{FreeQ}[\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x)^n, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d x] * (b \sin[c + d x])^{(n + 1)} * \text{Hypergeometric2F1}[1/2, (n + 1)/2, (n + 3)/2, \text{Sin}[c + d x]^2]) / (b d (n + 1) \text{Sqrt}[\text{Cos}[c + d x]^2]), x] /; \text{FreeQ}[\{b, c, d, n\}, x] \&\& \text{IntegerQ}[2 * n]$

Rubi steps

$$\int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{5}{2}}(c + dx)} dx = (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{-\frac{5}{2}+n}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)} - \frac{(2 \cos^{-n}(c + dx) (b \cos(c + dx))^n) \sin(c + dx)}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)} + (B \cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{-\frac{3}{2}+n}(c + dx) dx$$

$$= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)} + \frac{2 \left(\frac{C}{1 - 2n} + \frac{A}{3 - 2n} \right) (b \cos(c + dx))^n \cos^{-\frac{1}{2}}(c + dx)}{d(1 - 2n) \cos^{\frac{3}{2}}(c + dx)}$$

Mathematica [A] time = 0.408139, size = 163, normalized size = 0.74

$$\frac{2 \sin(c + dx) (b \cos(c + dx))^n \left((-2An + A + C(3 - 2n)) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n - 3); \frac{1}{4}(2n + 1); \cos^2(c + dx)\right) - (2n - 3) \left(B \cos(c + dx) + C \cos^2(c + dx) \right) \right)}{d(2n - 3)(2n - 1) \sqrt{\sin^2(c + dx) \cos^{\frac{3}{2}}(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[((b*cos[c + d*x])^n*(A + B*cos[c + d*x] + C*cos[c + d*x]^2))/Cos[c + d*x]^(5/2), x]

[Out] (2*(b*cos[c + d*x])^n*sin[c + d*x]*((A + C*(3 - 2*n) - 2*A*n)*Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d*x]^2] - (-3 + 2*n)*(B*cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + 2*n)/4, (3 + 2*n)/4, Cos[c + d*x]^2] - C*Sqrt[Sin[c + d*x]^2]))/(d*(-3 + 2*n)*(-1 + 2*n)*Cos[c + d*x]^(3/2)*Sqrt[Sin[c + d*x]^2])

Maple [F] time = 0.648, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)

[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(5/2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c))^n}{\cos(dx + c)^{\frac{5}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(5/2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(5/2), x)

$$3.382 \quad \int \frac{(b \cos(c+dx))^n (A+B \cos(c+dx)+C \cos^2(c+dx))}{\cos^{\frac{7}{2}}(c+dx)} dx$$

Optimal. Leaf size=223

$$\frac{2(A(3-2n)+C(5-2n)) \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c+dx)\right)}{d(3-2n)(5-2n)\sqrt{\sin^2(c+dx) \cos^{\frac{5}{2}}(c+dx)}} + \frac{2B \sin(c+dx)(b \cos(c+dx))^n}{d(3-2n)\sqrt{\sin^2(c+dx) \cos^{\frac{5}{2}}(c+dx)}}$$

[Out] $(-2*C*(b*\text{Cos}[c+d*x])^n*\text{Sin}[c+d*x])/(d*(3-2*n)*\text{Cos}[c+d*x]^{(5/2)}) + (2*(A*(3-2*n)+C*(5-2*n))*(b*\text{Cos}[c+d*x])^n*\text{Hypergeometric2F1}[1/2, (-5+2*n)/4, (-1+2*n)/4, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(3-2*n)*(5-2*n)*\text{Cos}[c+d*x]^{(5/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) + (2*B*(b*\text{Cos}[c+d*x])^n*\text{Hypergeometric2F1}[1/2, (-3+2*n)/4, (1+2*n)/4, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(3-2*n)*\text{Cos}[c+d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rubi [A] time = 0.21204, antiderivative size = 213, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, integrand size = 41, $\frac{\text{number of rules}}{\text{integrand size}} = 0.098$, Rules used = {20, 3023, 2748, 2643}

$$2\left(\frac{A}{5-2n} + \frac{C}{3-2n}\right) \sin(c+dx)(b \cos(c+dx))^n {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c+dx)\right) + \frac{2B \sin(c+dx)(b \cos(c+dx))^n}{d(3-2n)\sqrt{\sin^2(c+dx) \cos^{\frac{5}{2}}(c+dx)}}$$

Antiderivative was successfully verified.

[In] $\text{Int}[(b*\text{Cos}[c+d*x])^n*(A+B*\text{Cos}[c+d*x]+C*\text{Cos}[c+d*x]^2)/\text{Cos}[c+d*x]^{(7/2)}, x]$

[Out] $(-2*C*(b*\text{Cos}[c+d*x])^n*\text{Sin}[c+d*x])/(d*(3-2*n)*\text{Cos}[c+d*x]^{(5/2)}) + (2*(C/(3-2*n)+A/(5-2*n))*(b*\text{Cos}[c+d*x])^n*\text{Hypergeometric2F1}[1/2, (-5+2*n)/4, (-1+2*n)/4, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*\text{Cos}[c+d*x]^{(5/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2]) + (2*B*(b*\text{Cos}[c+d*x])^n*\text{Hypergeometric2F1}[1/2, (-3+2*n)/4, (1+2*n)/4, \text{Cos}[c+d*x]^2]*\text{Sin}[c+d*x])/(d*(3-2*n)*\text{Cos}[c+d*x]^{(3/2)}*\text{Sqrt}[\text{Sin}[c+d*x]^2])$

Rule 20

$\text{Int}[(u_.)*((a_.)*(v_.))^{(m_.)}*((b_.)*(v_.))^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[(b^{\text{IntPart}[n]}*(b*v)^{\text{FracPart}[n]})/(a^{\text{IntPart}[n]}*(a*v)^{\text{FracPart}[n]}), \text{Int}[u*(a*v)^{(m+n)}, x], x] /; \text{FreeQ}\{a, b, m, n\}, x\} \&\& \text{!IntegerQ}[m] \&\& \text{!IntegerQ}[n] \&\& \text{!IntegerQ}[m+n]$

Rule 3023

$\text{Int}[(a_. + (b_.)*\text{sin}[e_. + (f_.)*(x_.)])^{(m_.)}*((A_.) + (B_.)*\text{sin}[e_. + (f_.)*(x_.)] + (C_.)*\text{sin}^2[e_. + (f_.)*(x_.)]), x_Symbol] \rightarrow -\text{Simp}[(C*\text{Cos}[e+f*x]*(a+b*\text{Sin}[e+f*x])^{(m+1)})/(b*f*(m+2)), x] + \text{Dist}[1/(b*(m+2)), \text{Int}[(a+b*\text{Sin}[e+f*x])^m*\text{Simp}[A*b*(m+2)+b*C*(m+1)+(b*B*(m+2)-a*C)*\text{Sin}[e+f*x], x], x], x] /; \text{FreeQ}\{a, b, e, f, A, B, C, m\}, x\} \&\& \text{!LtQ}[m, -1]$

Rule 2748

$\text{Int}[(b_.)*\text{sin}[e_. + (f_.)*(x_.)]^{(m_.)}*((c_.) + (d_.)*\text{sin}[e_. + (f_.)*(x_.)]), x_Symbol] \rightarrow \text{Dist}[c, \text{Int}[(b*\text{Sin}[e+f*x])^m, x], x] + \text{Dist}[d/b, \text{Int}[(b*\text{Sin}[e+f*x])^m, x], x]$

$b \sin[e + f x]^{m+1}, x] /; \text{FreeQ}\{b, c, d, e, f, m\}, x]$

Rule 2643

$\text{Int}[(b \sin[c + d x] + d x)^n, x_Symbol] \rightarrow \text{Simp}[(\text{Cos}[c + d x] * (b \sin[c + d x])^{n+1} * \text{Hypergeometric2F1}[1/2, (n+1)/2, (n+3)/2, \text{Sin}[c + d x]^2]) / (b d (n+1) \text{Sqrt}[\text{Cos}[c + d x]^2]), x] /; \text{FreeQ}\{b, c, d, n\}, x \&\& \text{IntegerQ}[2*n]$

Rubi steps

$$\begin{aligned} \int \frac{(b \cos(c + dx))^n (A + B \cos(c + dx) + C \cos^2(c + dx))}{\cos^{\frac{7}{2}}(c + dx)} dx &= (\cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) (A + B \cos(c + dx) + C \cos^2(c + dx)) dx \\ &= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(3-2n) \cos^{\frac{5}{2}}(c + dx)} - \frac{(2 \cos^{-n}(c + dx) (b \cos(c + dx))^n) \sin(c + dx)}{d(3-2n) \cos^{\frac{5}{2}}(c + dx)} \\ &= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(3-2n) \cos^{\frac{5}{2}}(c + dx)} + (B \cos^{-n}(c + dx) (b \cos(c + dx))^n) \int \cos^{-\frac{7}{2}+n}(c + dx) dx \\ &= -\frac{2C(b \cos(c + dx))^n \sin(c + dx)}{d(3-2n) \cos^{\frac{5}{2}}(c + dx)} + \frac{2 \left(\frac{C}{3-2n} + \frac{A}{5-2n} \right) (b \cos(c + dx))^n}{d(3-2n) \cos^{\frac{5}{2}}(c + dx)} \end{aligned}$$

Mathematica [A] time = 0.410477, size = 164, normalized size = 0.74

$$\frac{2 \sin(c + dx) (b \cos(c + dx))^n \left((A(2n-3) + C(2n-5)) {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2n-5); \frac{1}{4}(2n-1); \cos^2(c + dx)\right) + (2n-5) \left(B \cos(c + dx) + C \cos^2(c + dx) \right) \right)}{d(2n-5)(2n-3) \sqrt{\sin^2(c + dx) \cos^{\frac{5}{2}}(c + dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(b Cos[c + d x])^n * (A + B Cos[c + d x] + C Cos[c + d x]^2) / Cos[c + d x]^(7/2), x]

[Out] (-2*(b Cos[c + d x])^n * Sin[c + d x] * ((C*(-5 + 2*n) + A*(-3 + 2*n)) * Hypergeometric2F1[1/2, (-5 + 2*n)/4, (-1 + 2*n)/4, Cos[c + d x]^2] + (-5 + 2*n) * (B Cos[c + d x] * Hypergeometric2F1[1/2, (-3 + 2*n)/4, (1 + 2*n)/4, Cos[c + d x]^2] - C Sqrt[Sin[c + d x]^2])) / (d*(-5 + 2*n)*(-3 + 2*n)*Cos[c + d x]^(5/2) * Sqrt[Sin[c + d x]^2])

Maple [F] time = 0.653, size = 0, normalized size = 0.

$$\int (b \cos(dx + c))^n (A + B \cos(dx + c) + C (\cos(dx + c))^2) (\cos(dx + c))^{-\frac{7}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x)

[Out] int((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))**n*(A+B*cos(d*x+c)+C*cos(d*x+c)**2)/cos(d*x+c)**(7/2), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(C \cos(dx + c)^2 + B \cos(dx + c) + A) (b \cos(dx + c))^n}{\cos(dx + c)^{\frac{7}{2}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((b*cos(d*x+c))^n*(A+B*cos(d*x+c)+C*cos(d*x+c)^2)/cos(d*x+c)^(7/2), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c))^n/cos(d*x + c)^(7/2), x)

3.383 $\int (a + a \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx))$

Optimal. Leaf size=183

$$\frac{2^{m+\frac{1}{2}} \left(A(m^2 + 3m + 2) + Bm(m + 2) + C(m^2 + m + 1) \right) \sin(e + fx) (\cos(e + fx) + 1)^{-m-\frac{1}{2}} (a \cos(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2}\right)}{f(m+1)(m+2)}$$

[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m)) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(B*m*(2 + m) + C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rubi [A] time = 0.247695, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {3023, 2751, 2652, 2651}

$$\frac{2^{m+\frac{1}{2}} \left(A(m^2 + 3m + 2) + Bm(m + 2) + C(m^2 + m + 1) \right) \sin(e + fx) (\cos(e + fx) + 1)^{-m-\frac{1}{2}} (a \cos(e + fx) + a)^m {}_2F_1\left(\frac{1}{2}, \frac{1}{2}\right)}{f(m+1)(m+2)}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2), x]

[Out] -(((C - B*(2 + m))*(a + a*Cos[e + f*x])^m*Sin[e + f*x])/(f*(1 + m)*(2 + m)) + (C*(a + a*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(a*f*(2 + m)) + (2^(1/2 + m)*(B*m*(2 + m) + C*(1 + m + m^2) + A*(2 + 3*m + m^2))*(1 + Cos[e + f*x])^(-1/2 - m)*(a + a*Cos[e + f*x])^m*Hypergeometric2F1[1/2, 1/2 - m, 3/2, (1 - Cos[e + f*x])/2]*Sin[e + f*x])/(f*(1 + m)*(2 + m))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])], x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2652

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*sin[c + d*x])/a))/2])/(d*Sqrt[a + b*sin[c + d*x]]), x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int (a + a \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx)) dx = \frac{C(a + a \cos(e + fx))^{1+m} \sin(e + fx)}{af(2 + m)} + \frac{\int (a + a \cos(e + fx))^m \sin(e + fx) dx}{f(1 + m)(2 + m)}$$

$$= -\frac{(C - B(2 + m))(a + a \cos(e + fx))^m \sin(e + fx)}{f(1 + m)(2 + m)}$$

$$= -\frac{(C - B(2 + m))(a + a \cos(e + fx))^m \sin(e + fx)}{f(1 + m)(2 + m)}$$

$$= -\frac{(C - B(2 + m))(a + a \cos(e + fx))^m \sin(e + fx)}{f(1 + m)(2 + m)}$$

Mathematica [C] time = 3.66625, size = 557, normalized size = 3.04

$$\cos^{-2m} \left(\frac{1}{2}(e + fx) \right) (a \cos(e + fx) + 1)^m \left(\frac{iA4^{1-m} (1 + e^{i(e+fx)}) \left(e^{-\frac{1}{2}i(e+fx)} (1 + e^{i(e+fx)}) \right)^{2m} {}_2F_1(1, m+1; 1-m; -e^{i(e+fx)})}{m} + \frac{2iBe^{-ifx}(\cos(fx)+i \sin(fx))}{m} \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*cos[e + f*x])^m*(A + B*cos[e + f*x] + C*cos[e + f*x]^2), x]
```

```
[Out] ((a*(1 + Cos[e + f*x]))^m*((I*4^(1 - m)*A*(1 + E^(I*(e + f*x))))*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*Hypergeometric2F1[1, 1 + m, 1 - m, -E^(I*(e + f*x))]/m + (I*2^(1 - 2*m)*C*(1 + E^(I*(e + f*x))))*((1 + E^(I*(e + f*x)))/E^((I/2)*(e + f*x)))^(2*m)*Hypergeometric2F1[1, 1 + m, 1 - m, -E^(I*(e + f*x))]/m + (C*cos[(e + f*x)/2]^(2*m)*(I*E^((4*I)*f*x)*(2 + m)*Hypergeometric2F1[2 - m, -2*m, 3 - m, -(E^(I*f*x)*(Cos[e] + I*Sin[e]))]*(Cos[2*e] + I*Sin[2*e]) + (-2 + m)*Hypergeometric2F1[-2 - m, -2*m, -1 - m, -(E^(I*f*x)*(Cos[e] + I*Sin[e]))]*(I*cos[2*e] + Sin[2*e])))/(E^((2*I)*f*x)*(-4 + m^2)*(1 + E^(I*f*x)*Cos[e] + I*E^(I*f*x)*Sin[e])^(2*m)) + ((2*I)*B*cos[(e + f*x)/2]^(2*m)*(Cos[f*x] + I*Sin[f*x])*((-1 + m)*Hypergeometric2F1[-1 - m, -2*m, -m, -(E^(I*f*x)*(Cos[e] + I*Sin[e]))]*(Cos[e + f*x] - I*Sin[e + f*x]) + (1 + m)*Hypergeometric2F1[1 - m, -2*m, 2 - m, -(E^(I*f*x)*(Cos[e] + I*Sin[e]))]*(Cos[e + f*x] + I*Sin[e + f*x])))/(E^(I*f*x)*(-1 + m^2)*(1 + E^(I*f*x)*Cos[e] + I*E^(I*f*x)*Sin[e])^(2*m)))/(4*f*cos[(e + f*x)/2]^(2*m))
```

Maple [F] time = 1.628, size = 0, normalized size = 0.

$$\int (a + a \cos(fx + e))^m (A + B \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2), x)
```

[Out] `int((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)`

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + B \cos(fx + e) + A \right) \left(a \cos(fx + e) + a \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxima")`

[Out] `integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x)`

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos^2(fx + e) + B \cos(fx + e) + A\right)\left(a \cos(fx + e) + a\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")`

[Out] `integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x)`

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)`

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + B \cos(fx + e) + A \right) \left(a \cos(fx + e) + a \right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] `integrate((a+a*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")`

[Out] `integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(a*cos(f*x + e) + a)^m, x)`

3.384 $\int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=144

$$\frac{(40A + 16B + 19C) \sin(c + dx)(a \cos(c + dx) + a)^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{10 \cdot 2^{5/6} d (\cos(c + dx) + 1)^{7/6}} + \frac{3(8B - 3C) \sin(c + dx)(a \cos(c + dx) + a)^{2/3}}{40d}$$

[Out] (3*(8*B - 3*C)*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(40*d) + (3*C*(a + a*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((40*A + 16*B + 19*C)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(10*2^(5/6)*d*(1 + Cos[c + d*x])^(7/6))

Rubi [A] time = 0.18872, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3023, 2751, 2652, 2651}

$$\frac{(40A + 16B + 19C) \sin(c + dx)(a \cos(c + dx) + a)^{2/3} {}_2F_1\left(-\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{10 \cdot 2^{5/6} d (\cos(c + dx) + 1)^{7/6}} + \frac{3(8B - 3C) \sin(c + dx)(a \cos(c + dx) + a)^{2/3}}{40d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2),x]

[Out] (3*(8*B - 3*C)*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(40*d) + (3*C*(a + a*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*a*d) + ((40*A + 16*B + 19*C)*(a + a*Cos[c + d*x])^(2/3)*Hypergeometric2F1[-1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(10*2^(5/6)*d*(1 + Cos[c + d*x])^(7/6))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2652

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1

$- (b \sin[c + dx])/a)/2]/(d \sqrt{a + b \sin[c + dx]}), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \ \&\& \ \text{EqQ}[a^2 - b^2, 0] \ \&\& \ \text{IntegerQ}[2*n] \ \&\& \ \text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{8ad} + \frac{3 \int (a + a \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{40d} \\ &= \frac{3(8B - 3C)(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{40d} + \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{40d} \\ &= \frac{3(8B - 3C)(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{40d} + \frac{3C(a + a \cos(c + dx))^{5/3} \sin(c + dx)}{40d} \end{aligned}$$

Mathematica [C] time = 0.833666, size = 137, normalized size = 0.95

$$\frac{3 \sec^2\left(\frac{1}{2}(c + dx)\right) (a(\cos(c + dx) + 1))^{2/3} \left(2 \sin(c + dx)(40A + 2(8B + 7C) \cos(c + dx) + 32B + 5C \cos(2(c + dx))) + 28C\right)}{320d}$$

Antiderivative was successfully verified.

[In] Integrate[(a + a*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*(a*(1 + Cos[c + d*x]))^(2/3)*Sec[(c + d*x)/2]^2*((-2*I)*(40*A + 16*B + 19*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x))]*(1 + Cos[c + d*x] + I*Sin[c + d*x])^(2/3) + 2*(40*A + 32*B + 28*C + 2*(8*B + 7*C)*Cos[c + d*x] + 5*C*Cos[2*(c + d*x)])*Sin[c + d*x]))/(320*d)

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int (a + \cos(dx + c))^{2/3} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int((a+cos(d*x+c)*a)^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(a \cos(dx + c) + a)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right)(a \cos(dx + c) + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right)(a \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(2/3), x)

3.385 $\int \sqrt[3]{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=144

$$\frac{(28A + 7B + 13C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{14\sqrt[6]{2d}(\cos(c + dx) + 1)^{5/6}} + \frac{3(7B - 3C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a}}{28d}$$

[Out] (3*(7*B - 3*C)*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(28*d) + (3*C*(a + a*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((28*A + 7*B + 13*C)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(14*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))

Rubi [A] time = 0.178464, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3023, 2751, 2652, 2651}

$$\frac{(28A + 7B + 13C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{14\sqrt[6]{2d}(\cos(c + dx) + 1)^{5/6}} + \frac{3(7B - 3C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a}}{28d}$$

Antiderivative was successfully verified.

[In] Int[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*(7*B - 3*C)*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(28*d) + (3*C*(a + a*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*a*d) + ((28*A + 7*B + 13*C)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(14*2^(1/6)*d*(1 + Cos[c + d*x])^(5/6))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2652

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1

$-(b \sin[c + dx])/a)/2)/(d \sqrt{a + b \sin[c + dx]}), x] /; \text{FreeQ}\{a, b, c, d, n\}, x] \&\& \text{EqQ}[a^2 - b^2, 0] \&\& !\text{IntegerQ}[2*n] \&\& \text{GtQ}[a, 0]$

Rubi steps

$$\begin{aligned} \int \sqrt[3]{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{7ad} + \frac{3 \int \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx) dx}{28d} \\ &= \frac{3(7B - 3C) \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{28d} + \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{28d} \\ &= \frac{3(7B - 3C) \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{28d} + \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{28d} \\ &= \frac{3(7B - 3C) \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{28d} + \frac{3C(a + a \cos(c + dx))^{4/3} \sin(c + dx)}{28d} \end{aligned}$$

Mathematica [F] time = 0.117657, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + a \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] Integrate[(a + a*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

Maple [F] time = 0.338, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + \cos(dx + c)a} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

[Out] int((a+cos(d*x+c)*a)^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(a \cos(dx + c) + a)^{1/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2), x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right)(a \cos(dx + c) + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right)(a \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(a*cos(d*x + c) + a)^(1/3), x)

$$3.386 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx$$

Optimal. Leaf size=144

$$\frac{(10A - 5B + 7C) \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{5 \cdot 2^{5/6} d \sqrt[6]{\cos(c + dx) + 1} \sqrt[3]{a \cos(c + dx) + a}} + \frac{3(5B - 3C) \sin(c + dx)}{10d \sqrt[3]{a \cos(c + dx) + a}} + \frac{3C \sin(c + dx)(a \cos(c + dx) + a)}{5ad}$$

[Out] (3*(5*B - 3*C)*Sin[c + d*x])/(10*d*(a + a*Cos[c + d*x])^(1/3)) + (3*C*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((10*A - 5*B + 7*C)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(5*2^(5/6)*d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))

Rubi [A] time = 0.175508, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3023, 2751, 2652, 2651}

$$\frac{(10A - 5B + 7C) \sin(c + dx) {}_2F_1\left(\frac{1}{2}, \frac{5}{6}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{5 \cdot 2^{5/6} d \sqrt[6]{\cos(c + dx) + 1} \sqrt[3]{a \cos(c + dx) + a}} + \frac{3(5B - 3C) \sin(c + dx)}{10d \sqrt[3]{a \cos(c + dx) + a}} + \frac{3C \sin(c + dx)(a \cos(c + dx) + a)}{5ad}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3), x]

[Out] (3*(5*B - 3*C)*Sin[c + d*x])/(10*d*(a + a*Cos[c + d*x])^(1/3)) + (3*C*(a + a*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*a*d) + ((10*A - 5*B + 7*C)*Hypergeometric2F1[1/2, 5/6, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(5*2^(5/6)*d*(1 + Cos[c + d*x])^(1/6)*(a + a*Cos[c + d*x])^(1/3))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2751

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> -Simp[(d*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(f*(m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(b*(m + 1)), Int[(a + b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && !LtQ[m, -2^(-1)]

Rule 2652

Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n +
1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1
- (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x]]), x] /; FreeQ[{a, b
, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt[3]{a + a \cos(c + dx)}} dx &= \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{3 \int \frac{\frac{1}{3}a(5A+2C) + \frac{1}{3}a(5B-3C) \cos(c+dx)}{\sqrt[3]{a+a \cos(c+dx)}} dx}{5a} \\ &= \frac{3(5B - 3C) \sin(c + dx)}{10d\sqrt[3]{a + a \cos(c + dx)}} + \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{1}{10}(10A - 5B) \\ &= \frac{3(5B - 3C) \sin(c + dx)}{10d\sqrt[3]{a + a \cos(c + dx)}} + \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{((10A - 5B))}{5} \\ &= \frac{3(5B - 3C) \sin(c + dx)}{10d\sqrt[3]{a + a \cos(c + dx)}} + \frac{3C(a + a \cos(c + dx))^{2/3} \sin(c + dx)}{5ad} + \frac{(10A - 5B)}{5} \end{aligned}$$

Mathematica [C] time = 0.586402, size = 105, normalized size = 0.73

$$\frac{3 \sin(c + dx)(5B + 2C \cos(c + dx) - C) - 3i(10A - 5B + 7C) {}_2F_1\left(\frac{1}{3}, \frac{2}{3}; \frac{4}{3}; -e^{i(c+dx)}\right) (i \sin(c + dx) + \cos(c + dx) + 1)^{2/3}}{10d\sqrt[3]{a(\cos(c + dx) + 1)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(1/3), x]
```

```
[Out] ((-3*I)*(10*A - 5*B + 7*C)*Hypergeometric2F1[1/3, 2/3, 4/3, -E^(I*(c + d*x))]*(1 + Cos[c + d*x] + I*Sin[c + d*x])^(2/3) + 3*(5*B - C + 2*C*Cos[c + d*x])*Sin[c + d*x])/(10*d*(a*(1 + Cos[c + d*x]))^(1/3))
```

Maple [F] time = 0.312, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{a + \cos(dx + c)} a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3), x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(1/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="maxima")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))^(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(1/3), x)

$$3.387 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=144

$$\frac{(4A - 8B + 7C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt[6]{2ad}(\cos(c + dx) + 1)^{5/6}} + \frac{3(A - B + C) \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}} + \frac{3C \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}}$$

[Out] (3*(A - B + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A - 8*B + 7*C)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))

Rubi [A] time = 0.196683, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3023, 2750, 2652, 2651}

$$\frac{(4A - 8B + 7C) \sin(c + dx) \sqrt[3]{a \cos(c + dx) + a} {}_2F_1\left(\frac{1}{6}, \frac{1}{2}, \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt[6]{2ad}(\cos(c + dx) + 1)^{5/6}} + \frac{3(A - B + C) \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}} + \frac{3C \sin(c + dx)}{d(a \cos(c + dx) + a)^{2/3}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]

[Out] (3*(A - B + C)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x])^(2/3)) + (3*C*(a + a*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*a*d) - ((4*A - 8*B + 7*C)*(a + a*Cos[c + d*x])^(1/3)*Hypergeometric2F1[1/6, 1/2, 3/2, (1 - Cos[c + d*x])/2]*Sin[c + d*x])/(2*2^(1/6)*a*d*(1 + Cos[c + d*x])^(5/6))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2750

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m)/(a*f*(2*m + 1)), x] + Dist[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]

Rule 2652

Int[((a_.) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_.), x_Symbol] :> Dist[(a^IntPart[n]*(a + b*Sin[c + d*x])^FracPart[n])/(1 + (b*Sin[c + d*x])/a)^FracPart[n]], Int[(1 + (b*Sin[c + d*x])/a)^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]

Rule 2651

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := -Simp[(2^(n + 1/2)*a^(n - 1/2)*b*Cos[c + d*x]*Hypergeometric2F1[1/2, 1/2 - n, 3/2, (1*(1 - (b*Sin[c + d*x])/a))/2])/(d*Sqrt[a + b*Sin[c + d*x])], x] /; FreeQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx = \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} + \frac{3 \int \frac{\frac{1}{3}a(4A+C) + \frac{1}{3}a(4B-3C) \cos(c+dx)}{(a+a \cos(c+dx))^{2/3}} dx}{4a}$$

$$= \frac{3(A - B + C) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} + \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} - \frac{(4A - 8)}{4ad}$$

$$= \frac{3(A - B + C) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} + \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} - \frac{((4A - 8))}{4ad}$$

$$= \frac{3(A - B + C) \sin(c + dx)}{d(a + a \cos(c + dx))^{2/3}} + \frac{3C \sqrt[3]{a + a \cos(c + dx)} \sin(c + dx)}{4ad} - \frac{(4A - 8)}{4ad}$$

Mathematica [F] time = 0.195509, size = 0, normalized size = 0.

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + a \cos(c + dx))^{2/3}} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]
```

```
[Out] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + a*Cos[c + d*x])^(2/3), x]
```

Maple [F] time = 0.315, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) (a + \cos(dx + c) a)^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3), x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+cos(d*x+c)*a)^(2/3), x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3), x, algorithm="maxima")
```

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3), x, algorithm="fricas")

[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+a*cos(d*x+c))**(2/3), x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(a \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+a*cos(d*x+c))^(2/3), x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(a*cos(d*x + c) + a)^(2/3), x)

3.388 $\int (a+b \cos(c+dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=290

$$\frac{\sin(c + dx) (3a^2C - 8abB + 8Ab^2 + 5b^2C) (a + b \cos(c + dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{4\sqrt{2}b^2d\sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}} + (a$$

[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + ((8*A*b^2 - 8*a*b*B + 3*a^2*C + 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))

Rubi [A] time = 0.368263, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sin(c + dx) (3a^2C - 8abB + 8Ab^2 + 5b^2C) (a + b \cos(c + dx))^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{4\sqrt{2}b^2d\sqrt{\cos(c + dx) + 1} \left(\frac{a + b \cos(c + dx)}{a + b}\right)^{2/3}} + (a$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(5/3)*Sin[c + d*x])/(8*b*d) + ((a + b)*(8*b*B - 3*a*C)*AppellF1[1/2, 1/2, -5/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3)) + ((8*A*b^2 - 8*a*b*B + 3*a^2*C + 5*b^2*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(4*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(2/3))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^(n)/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned} \int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{3 \int (a + b \cos(c + dx))^{2/3} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx}{8bd} \\ &= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{(8bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx}{8bd} \\ &= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} - \frac{((8bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx)}{8bd} \\ &= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{((-a - b)(8bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx)}{8bd} \\ &= \frac{3C(a + b \cos(c + dx))^{5/3} \sin(c + dx)}{8bd} + \frac{(a + b)(8bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx}{8bd} \end{aligned}$$

Mathematica [A] time = 3.39797, size = 296, normalized size = 1.02

$$3 \operatorname{csc}(c + dx)(a + b \cos(c + dx))^{2/3} \left(4(-6a^2C + 16abB + 40Ab^2 + 25b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} (a + b \cos(c + dx)) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(2/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (-3*(a + b*cos[c + d*x])^(2/3)*Csc[c + d*x]*(20*(-a^2 + b^2)*(8*b*B - 3*a*C)
)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] *Sqrt[-((b*(1 + Cos
[c + d*x]))/(a - b))] + 4*(40*A*b^2 + 16*a*b*B - 6*a^2*C + 25*b^2*C)*Appell
F1[5/3, 1/2, 1/2, 8/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(
a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] *Sqrt[(b*(1 + Cos[c + d*x]
))/(-a + b)]*(a + b*cos[c + d*x]) - 20*b^2*(8*b*B + 2*a*C + 5*b*C*cos[c + d*
x])*Sin[c + d*x]^2)/(800*b^3*d)
```

Maple [F] time = 0.303, size = 0, normalized size = 0.

$$\int (a + b \cos(dx + c))^{\frac{2}{3}} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right)(b \cos(dx + c) + a)^{\frac{2}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))**(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(d*x+c))^(2/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(2/3), x)

3.389 $\int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx$

Optimal. Leaf size=290

$$\frac{\sqrt{2} \sin(c + dx) (3a^2C - 7abB + 7Ab^2 + 4b^2C) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{7b^2d\sqrt{\cos(c + dx)} + 1\sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(7*A*b^2 - 7*a*b*B + 3*a^2*C + 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))

Rubi [A] time = 0.337235, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2} \sin(c + dx) (3a^2C - 7abB + 7Ab^2 + 4b^2C) \sqrt[3]{a + b \cos(c + dx)} F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \cos(c + dx)), \frac{b(1 - \cos(c + dx))}{a + b}\right)}{7b^2d\sqrt{\cos(c + dx)} + 1\sqrt[3]{\frac{a + b \cos(c + dx)}{a + b}}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(4/3)*Sin[c + d*x])/(7*b*d) + (Sqrt[2]*(a + b)*(7*b*B - 3*a*C)*AppellF1[1/2, 1/2, -4/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + (Sqrt[2]*(7*A*b^2 - 7*a*b*B + 3*a^2*C + 4*b^2*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(7*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_.)*sin[(c_) + (d_.)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^(n)/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned} \int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx) + C \cos^2(c + dx)) dx &= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{3 \int \sqrt[3]{a + b \cos(c + dx)} (A + B \cos(c + dx)) dx}{7bd} \\ &= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{(7bB - 3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx}{7bd} \\ &= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} - \frac{((7bB - 3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx)}{7b^2} \\ &= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{((-a - b)(7bB - 3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx)}{7b^2} \\ &= \frac{3C(a + b \cos(c + dx))^{4/3} \sin(c + dx)}{7bd} + \frac{\sqrt{2}(a + b)(7bB - 3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx}{7bd} \end{aligned}$$

Mathematica [A] time = 3.4235, size = 294, normalized size = 1.01

$$3 \csc(c + dx) \sqrt[3]{a + b \cos(c + dx)} \left((-3a^2C + 7abB + 28Ab^2 + 16b^2C) \sqrt{-\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} (a + b \cos(c + dx)) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[c + d*x])^(1/3)*(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2), x]
```

```
[Out] (-3*(a + b*cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(-a^2 + b^2)*(7*b*B - 3*a*C)
*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c +
d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] *Sqrt[-((b*(1 + Cos[
c + d*x]))/(a - b))] + (28*A*b^2 + 7*a*b*B - 3*a^2*C + 16*b^2*C)*AppellF1[4
/3, 1/2, 1/2, 7/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a +
b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] *Sqrt[(b*(1 + Cos[c + d*x]))/(-
a + b)]*(a + b*cos[c + d*x]) - 4*b^2*(7*b*B + a*C + 4*b*C*cos[c + d*x])*Sin
[c + d*x]^2))/(112*b^3*d)
```

Maple [F] time = 0.292, size = 0, normalized size = 0.

$$\int \sqrt[3]{a + b \cos(dx + c)} (A + B \cos(dx + c) + C (\cos(dx + c))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

```
[Out] int((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(dx + c)^2 + B \cos(dx + c) + A\right)(b \cos(dx + c) + a)^{\frac{1}{3}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))**(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(dx + c)^2 + B \cos(dx + c) + A)(b \cos(dx + c) + a)^{\frac{1}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(d*x+c))^(1/3)*(A+B*cos(d*x+c)+C*cos(d*x+c)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)*(b*cos(d*x + c) + a)^(1/3), x)
```


$$3.390 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx$$

Optimal. Leaf size=287

$$\frac{\sqrt{2} \sin(c+dx) (3a^2C - 5abB + 5Ab^2 + 2b^2C) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) + \sqrt{2}(5bB - 3a^2C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right] + (a+b) \cos(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right]}{5b^2d \sqrt{\cos(c+dx)} + 1 \sqrt[3]{a+b \cos(c+dx)}} + \frac{\sqrt{2}(5bB - 3a^2C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right] + (a+b) \cos(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right]}{5b^2d \sqrt{\cos(c+dx)} + 1 \sqrt[3]{a+b \cos(c+dx)}}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) + (Sqrt[2]*(5*A*b^2 - 5*a*b*B + 3*a^2*C + 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(1/3)

Rubi [A] time = 0.325953, antiderivative size = 287, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2} \sin(c+dx) (3a^2C - 5abB + 5Ab^2 + 2b^2C) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{1}{3}; \frac{3}{2}; \frac{1}{2} (1 - \cos(c+dx)), \frac{b(1-\cos(c+dx))}{a+b}\right) + \sqrt{2}(5bB - 3a^2C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right] + (a+b) \cos(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right]}{5b^2d \sqrt{\cos(c+dx)} + 1 \sqrt[3]{a+b \cos(c+dx)}} + \frac{\sqrt{2}(5bB - 3a^2C) \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, -\frac{2}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right] + (a+b) \cos(c+dx) \sqrt[3]{\frac{a+b \cos(c+dx)}{a+b}} \operatorname{AppellF1}\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{3}, \frac{3}{2}, \frac{(1 - \cos(c+dx))}{2}, \frac{b(1 - \cos(c+dx))}{(a+b)}\right]}{5b^2d \sqrt{\cos(c+dx)} + 1 \sqrt[3]{a+b \cos(c+dx)}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b*d) + (Sqrt[2]*(5*b*B - 3*a*C)*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(2/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*((a + b*Cos[c + d*x])/(a + b))^(2/3) + (Sqrt[2]*(5*A*b^2 - 5*a*b*B + 3*a^2*C + 2*b^2*C)*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(1/3)*Sin[c + d*x])/(5*b^2*d*Sqrt[1 + Cos[c + d*x]])*(a + b*Cos[c + d*x])^(1/3)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_.)] + (C_.)*sin[(e_.) + (f_.)*(x_.)]^2), x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_.)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_.)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
-((d*(a + b*x))/(b*c - a*d)), -(f*(a + b*x)/(b*e - a*f))]/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{\sqrt[3]{a + b \cos(c + dx)}} dx &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{3 \int \frac{\frac{1}{3}b(5A+2C) + \frac{1}{3}(5bB-3aC) \cos(c+dx)}{\sqrt[3]{a+b \cos(c+dx)}} dx}{5b} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{(5bB - 3aC) \int (a + b \cos(c + dx))^{2/3} dx}{5b^2} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{((5bB - 3aC) \sin(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - u}} du, \frac{a + b \cos(c + dx)}{b}\right)}{5b^2 d \sqrt{1 - \cos(c + dx)}} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} - \frac{((5bB - 3aC)(a + b \cos(c + dx))^{2/3} \sin(c + dx)) \operatorname{Subst}\left(\int \frac{1}{\sqrt{1 - u}} du, \frac{a + b \cos(c + dx)}{b}\right)}{5b^2 d \sqrt{1 - \cos(c + dx)}} \\ &= \frac{3C(a + b \cos(c + dx))^{2/3} \sin(c + dx)}{5bd} + \frac{\sqrt{2}(5bB - 3aC) F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{5b^2 d \sqrt{1 - \cos(c + dx)}} \end{aligned}$$

Mathematica [A] time = 2.37751, size = 268, normalized size = 0.93

$$\frac{3 \csc(c + dx)(a + b \cos(c + dx))^{2/3} \left(5(3a^2C - 5abB + 5Ab^2 + 2b^2C) \sqrt{\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} F_1\left(\frac{2}{3}; \frac{1}{2}, \frac{1}{2}, \frac{5}{2}; \frac{a+b \cos(c+dx)}{a-b}\right)\right)}{5b^2 d \sqrt{1 - \cos(c + dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(1/3), x]
```

```
[Out] (-3*(a + b*cos[c + d*x])^(2/3)*Csc[c + d*x]*(5*(5*A*b^2 - 5*a*b*B + 3*a^2*C
+ 2*b^2*C)*AppellF1[2/3, 1/2, 1/2, 5/3, (a + b*cos[c + d*x])/(a - b), (a +
b*cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] *Sqrt[(b*
(1 + Cos[c + d*x]))/(-a + b)] + 2*(5*b*B - 3*a*C)*AppellF1[5/3, 1/2, 1/2, 8
/3, (a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a + b)]*Sqrt[-((b*(
-1 + Cos[c + d*x]))/(a + b))] *Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*
Cos[c + d*x]) - 10*b^2*C*Sin[c + d*x]^2))/(50*b^3*d)
```

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) \frac{1}{\sqrt[3]{a + b \cos(dx + c)}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(1/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{1}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(1/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(1/3), x)

$$3.391 \quad \int \frac{A+B \cos(c+dx)+C \cos^2(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx$$

Optimal. Leaf size=286

$$\frac{\sin(c+dx) \left(3a^2C - 4abB + 4Ab^2 + b^2C\right) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a+b \cos(c+dx))^{2/3}} + \frac{(4bB - 3a^2C)}{(a+b)^2}$$

[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + ((4*A*b^2 - 4*a*b*B + 3*a^2*C + b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))

Rubi [A] time = 0.323789, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sin(c+dx) \left(3a^2C - 4abB + 4Ab^2 + b^2C\right) \left(\frac{a+b \cos(c+dx)}{a+b}\right)^{2/3} F_1\left(\frac{1}{2}; \frac{1}{2}, \frac{2}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c+dx)), \frac{b(1 - \cos(c+dx))}{a+b}\right)}{2\sqrt{2}b^2d\sqrt{\cos(c+dx)+1}(a+b \cos(c+dx))^{2/3}} + \frac{(4bB - 3a^2C)}{(a+b)^2}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3), x]

[Out] (3*C*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(4*b*d) + ((4*b*B - 3*a*C)*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])^(1/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*((a + b*Cos[c + d*x])/(a + b))^(1/3)) + ((4*A*b^2 - 4*a*b*B + 3*a^2*C + b^2*C)*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Cos[c + d*x])/2, (b*(1 - Cos[c + d*x]))/(a + b)]*(a + b*Cos[c + d*x])/(a + b))^(2/3)*Sin[c + d*x])/(2*Sqrt[2]*b^2*d*Sqrt[1 + Cos[c + d*x]]*(a + b*Cos[c + d*x])^(2/3))

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^(n)/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -(f*(a + b*x)/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0]) && SimplifierQ[c + d*x, a + b*x] && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0]) && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \frac{A + B \cos(c + dx) + C \cos^2(c + dx)}{(a + b \cos(c + dx))^{2/3}} dx = \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{3 \int \frac{\frac{1}{3}b(4A+C) + \frac{1}{3}(4bB-3aC) \cos(c+dx)}{(a+b \cos(c+dx))^{2/3}} dx}{4b}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{(4bB - 3aC) \int \sqrt[3]{a + b \cos(c + dx)} dx}{4b^2} + \dots$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} - \frac{((4bB - 3aC) \sin(c + dx)) \text{Subst}\left(\int \frac{\sqrt[3]{a + b \cos(c + dx)}}{\sqrt{1 - \cos(c + dx)}} dx\right)}{4b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} - \frac{((4bB - 3aC) \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx))}{4b^2 d \sqrt{1 - \cos(c + dx)} \sqrt{1 + \cos(c + dx)}}$$

$$= \frac{3C \sqrt[3]{a + b \cos(c + dx)} \sin(c + dx)}{4bd} + \frac{(4bB - 3aC) F_1\left(\frac{1}{2}; \frac{1}{2}, -\frac{1}{3}; \frac{3}{2}; \frac{1}{2}(1 - \cos(c + dx))\right)}{2\sqrt{2}b^2 d \sqrt{1 + \cos(c + dx)}}$$

Mathematica [A] time = 2.38991, size = 266, normalized size = 0.93

$$3 \csc(c + dx) \sqrt[3]{a + b \cos(c + dx)} \left(4(3a^2C - 4abB + 4Ab^2 + b^2C) \sqrt{\frac{b(\cos(c+dx)-1)}{a+b}} \sqrt{\frac{b(\cos(c+dx)+1)}{b-a}} F_1\left(\frac{1}{3}; \frac{1}{2}, \frac{1}{2}; \frac{4}{3}; \frac{a+b \cos(c+dx)}{a-b}\right) \right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Cos[c + d*x] + C*Cos[c + d*x]^2)/(a + b*Cos[c + d*x])^(2/3), x]
```

```
[Out] (-3*(a + b*cos[c + d*x])^(1/3)*Csc[c + d*x]*(4*(4*A*b^2 - 4*a*b*B + 3*a^2*C
+ b^2*C)*AppellF1[1/3, 1/2, 1/2, 4/3, (a + b*cos[c + d*x])/(a - b), (a + b
*cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 + Cos[c + d*x]))/(a + b))] *Sqrt[(b*(1
+ Cos[c + d*x]))/(-a + b)] + (4*b*B - 3*a*C)*AppellF1[4/3, 1/2, 1/2, 7/3,
(a + b*cos[c + d*x])/(a - b), (a + b*cos[c + d*x])/(a + b)]*Sqrt[-((b*(-1 +
Cos[c + d*x]))/(a + b))] *Sqrt[(b*(1 + Cos[c + d*x]))/(-a + b)]*(a + b*cos[
c + d*x]) - 4*b^2*C*Sin[c + d*x]^2))/(16*b^3*d)
```

Maple [F] time = 0.277, size = 0, normalized size = 0.

$$\int (A + B \cos(dx + c) + C (\cos(dx + c))^2) (a + b \cos(dx + c))^{-\frac{2}{3}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="maxima")
```

```
[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorit
hm="fricas")
```

```
[Out] integral((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)**2)/(a+b*cos(d*x+c))**(2/3),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{C \cos(dx + c)^2 + B \cos(dx + c) + A}{(b \cos(dx + c) + a)^{\frac{2}{3}}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*cos(d*x+c)+C*cos(d*x+c)^2)/(a+b*cos(d*x+c))^(2/3),x, algorithm="giac")

[Out] integrate((C*cos(d*x + c)^2 + B*cos(d*x + c) + A)/(b*cos(d*x + c) + a)^(2/3), x)

3.392 $\int (a+b \cos(e+fx))^m \left(A + (A+C) \cos(e+fx) + C \cos^2(e+fx) \right) dx$

Optimal. Leaf size=215

$$\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b} \right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}} + \frac{4\sqrt{2}(A+C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b} \right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}}$$

[Out] (4*Sqrt[2]*C*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]])*((a + b*Cos[e + f*x])/(a + b))^m + (2*Sqrt[2]*(A - C)*AppellF1[1/2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]])*((a + b*Cos[e + f*x])/(a + b))^m

Rubi [A] time = 0.252992, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3017, 2755, 139, 138, 2784}

$$\frac{2\sqrt{2}(A-C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b} \right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}} + \frac{4\sqrt{2}(A+C)\sin(e+fx)(a+b\cos(e+fx))^m \left(\frac{a+b\cos(e+fx)}{a+b} \right)^{-m} F_1\left(\frac{1}{2}; -\frac{1}{2}, -m; \frac{3}{2}; \frac{1}{2}(1-\cos(e+fx)), \frac{b(1-\cos(e+fx))}{a+b}\right)}{f\sqrt{\cos(e+fx)+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*Cos[e + f*x]^2), x]

[Out] (4*Sqrt[2]*C*AppellF1[1/2, -3/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]])*((a + b*Cos[e + f*x])/(a + b))^m + (2*Sqrt[2]*(A - C)*AppellF1[1/2, -1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(f*Sqrt[1 + Cos[e + f*x]])*((a + b*Cos[e + f*x])/(a + b))^m

Rule 3017

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((A_) + (B_)*sin[(e_) + (f_)*(x_)] + (C_)*sin[(e_) + (f_)*(x_)]^2), x_Symbol] := Dist[A - C, Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x]), x], x] + Dist[C, Int[(a + b*Sin[e + f*x])^m*(1 + Sin[e + f*x])^2, x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && EqQ[A - B + C, 0] && !IntegerQ[2*m]

Rule 2755

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)]), x_Symbol] := Dist[(c*Cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]])*Sqrt[1 - Sin[e + f*x]], Subst[Int[(a + b*x)^m*Sqrt[1 + (d*x)/c])/Sqrt[1 - (d*x)/c], x], x, Sin[e + f*x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*m] && EqQ[c^2 - d^2, 0]

Rule 139

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,

m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]

Rule 138

Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])

Rule 2784

Int[((a_) + (b_)*sin[(e_) + (f_)*(x_)])^(m_)*((c_) + (d_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] :> Dist[(a^m*cos[e + f*x])/(f*Sqrt[1 + Sin[e + f*x]]*Sqrt[1 - Sin[e + f*x]]), Subst[Int[((1 + (b*x)/a)^(m - 1/2)*(c + d*x)^n]/Sqrt[1 - (b*x)/a], x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && IntegerQ[m]

Rubi steps

$$\int (a + b \cos(e + fx))^m (A + (A + C) \cos(e + fx) + C \cos^2(e + fx)) dx = (A - C) \int (1 + \cos(e + fx))(a + b \cos(e + fx))^m dx$$

$$= -\frac{(C \sin(e + fx)) \operatorname{Subst}\left(\int \frac{(1+x)^{3/2}(a+bx)^m}{\sqrt{1-x}} dx, x, \cos(e + fx)\right)}{f \sqrt{1 - \cos(e + fx)} \sqrt{1 + \cos(e + fx)}}$$

$$= -\frac{\left(C(a + b \cos(e + fx))^m \left(-\frac{a+b \cos(e+fx)}{-a-b}\right)^{-m} \sin(e + fx)\right)}{f \sqrt{1 - \cos(e + fx)}}$$

$$= \frac{4\sqrt{2}CF_1\left(\frac{1}{2}; -\frac{3}{2}, -m; \frac{3}{2}; \frac{1}{2}(1 - \cos(e + fx))\right), \frac{b(1 - \cos(e + fx))}{f}}{f \sqrt{1 - \cos(e + fx)}}$$

Mathematica [F] time = 3.79979, size = 0, normalized size = 0.

$$\int (a + b \cos(e + fx))^m (A + (A + C) \cos(e + fx) + C \cos^2(e + fx)) dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*cos[e + f*x]^2), x]

[Out] Integrate[(a + b*cos[e + f*x])^m*(A + (A + C)*Cos[e + f*x] + C*cos[e + f*x]^2), x]

Maple [F] time = 1.612, size = 0, normalized size = 0.

$$\int (a + b \cos(fx + e))^m (A + (A + C) \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

```
[Out] int((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + (A + C) \cos(fx + e) + A \right) (b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos^2(fx + e) + (A + C) \cos(fx + e) + A\right)(b \cos(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] integral((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))**m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)**2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(C \cos^2(fx + e) + (A + C) \cos(fx + e) + A \right) (b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+(A+C)*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + (A + C)*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)
```

3.393 $\int (a+b \cos(e+fx))^m (A+B \cos(e+fx) + C \cos^2(e+fx))$

Optimal. Leaf size=303

$$\frac{\sqrt{2} \sin(e+fx) (a^2C - abB(m+2) + Ab^2(m+2) + b^2C(m+1)) (a+b \cos(e+fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}\right)}{b^2 f(m+2) \sqrt{\cos(e+fx)+1}}$$

[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a + b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2*C + b^2*C*(1 + m) + A*b^2*(2 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)

Rubi [A] time = 0.375155, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3023, 2756, 2665, 139, 138}

$$\frac{\sqrt{2} \sin(e+fx) (a^2C - abB(m+2) + Ab^2(m+2) + b^2C(m+1)) (a+b \cos(e+fx))^m \left(\frac{a+b \cos(e+fx)}{a+b}\right)^{-m} F_1\left(\frac{1}{2}; \frac{1}{2}, -m; \frac{3}{2}\right)}{b^2 f(m+2) \sqrt{\cos(e+fx)+1}}$$

Antiderivative was successfully verified.

[In] Int[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2), x]

[Out] (C*(a + b*Cos[e + f*x])^(1 + m)*Sin[e + f*x])/(b*f*(2 + m)) - (Sqrt[2]*(a + b)*(a*C - b*B*(2 + m))*AppellF1[1/2, 1/2, -1 - m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m) + (Sqrt[2]*(a^2*C + b^2*C*(1 + m) + A*b^2*(2 + m) - a*b*B*(2 + m))*AppellF1[1/2, 1/2, -m, 3/2, (1 - Cos[e + f*x])/2, (b*(1 - Cos[e + f*x]))/(a + b)]*(a + b*Cos[e + f*x])^m*Sin[e + f*x])/(b^2*f*(2 + m)*Sqrt[1 + Cos[e + f*x]]*((a + b*Cos[e + f*x])/(a + b))^m)

Rule 3023

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]) + (C_.)*sin[(e_.) + (f_.)*(x_)^2], x_Symbol] := -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m + 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] && !LtQ[m, -1]

Rule 2756

Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] := Dist[(b*c - a*d)/b, Int[(a + b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]

Rule 2665

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c + d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)^(n)/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 139

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))]/(b*(m + 1)*(b/(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d), 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c*f), 0] && SimplifierQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f/(f*c - e*d), 0] && SimplifierQ[e + f*x, a + b*x])
```

Rubi steps

$$\int (a + b \cos(e + fx))^m (A + B \cos(e + fx) + C \cos^2(e + fx)) dx = \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{\int (a + b \cos(e + fx))^m (A + B \cos(e + fx)) dx}{b^2}$$

$$= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{(-aC + bB(2 + m)) \int (a + b \cos(e + fx))^m dx}{b^2}$$

$$= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{((-aC + bB(2 + m)) \int (a + b \cos(e + fx))^m dx)}{b^2}$$

$$= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} + \frac{((-a - b)(-aC + bB(2 + m)) \int (a + b \cos(e + fx))^m dx)}{b^2}$$

$$= \frac{C(a + b \cos(e + fx))^{1+m} \sin(e + fx)}{bf(2 + m)} - \frac{\sqrt{2}(a + b)(aC - bB(2 + m)) \int (a + b \cos(e + fx))^m dx}{b^2}$$

Mathematica [B] time = 26.8395, size = 16189, normalized size = 53.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Cos[e + f*x])^m*(A + B*Cos[e + f*x] + C*Cos[e + f*x]^2),x]
```

```
[Out] Result too large to show
```

Maple [F] time = 1.438, size = 0, normalized size = 0.

$$\int (a + b \cos(fx + e))^m (A + B \cos(fx + e) + C (\cos(fx + e))^2) dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)

[Out] int((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x)

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e) + A)(b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="maxima")

[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(C \cos(fx + e)^2 + B \cos(fx + e) + A\right)(b \cos(fx + e) + a)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="fricas")

[Out] integral((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*cos(f*x+e))**m*(A+B*cos(f*x+e)+C*cos(f*x+e)**2),x)

[Out] Timed out

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (C \cos(fx + e)^2 + B \cos(fx + e) + A)(b \cos(fx + e) + a)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cos(f*x+e))^m*(A+B*cos(f*x+e)+C*cos(f*x+e)^2),x, algorithm="giac")
```

```
[Out] integrate((C*cos(f*x + e)^2 + B*cos(f*x + e) + A)*(b*cos(f*x + e) + a)^m, x)
```


Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
1 (* Original version thanks to Albert Rich emailed on 03/21/2017 *)
2 (* ::Package:: *)
3
4 (* ::Subsection:: *)
5 (*GradeAntiderivative[result,optimal]*)
6
7
8 (* ::Text:: *)
9 (*If result and optimal are mathematical expressions, *)
10 (*      GradeAntiderivative[result,optimal] returns*)
11 (* "F" if the result fails to integrate an expression that*)
12 (*   is integrable*)
13 (* "C" if result involves higher level functions than necessary*)
14 (* "B" if result is more than twice the size of the optimal*)
15 (*   antiderivative*)
16 (* "A" if result can be considered optimal*)
17
18
19 GradeAntiderivative[result_,optimal_] :=
20   If[ExpnType[result]<=ExpnType[optimal],
21     If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
22       If[LeafCount[result]<=2*LeafCount[optimal],
23         "A",
24         "B"],
25       "C"],
26   If[FreeQ[result,Integrate] && FreeQ[result,Int],
27     "C",
28     "F"]]
29
30
31 (* ::Text:: *)
32 (*The following summarizes the type number assigned an *)
33 (*expression based on the functions it involves*)
34 (*1 = rational function*)
35 (*2 = algebraic function*)
36 (*3 = elementary function*)
37 (*4 = special function*)
```

```

38 (*5 = hyperpergeometric function*)
39 (*6 = appell function*)
40 (*7 = rootsum function*)
41 (*8 = integrate function*)
42 (*9 = unknown function*)
43
44
45 ExpnType[expn_] :=
46   If[AtomQ[expn],
47     1,
48     If[ListQ[expn],
49       Max[Map[ExpnType,expn]],
50       If[Head[expn]===Power,
51         If[IntegerQ[expn[[2]]],
52           ExpnType[expn[[1]]],
53           If[Head[expn[[2]]]===Rational,
54             If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
55               1,
56               Max[ExpnType[expn[[1]],2]],
57             Max[ExpnType[expn[[1]],ExpnType[expn[[2]],3]],
58             If[Head[expn]===Plus || Head[expn]===Times,
59               Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
60             If[ElementaryFunctionQ[Head[expn]],
61               Max[3,ExpnType[expn[[1]]],
62             If[SpecialFunctionQ[Head[expn]],
63               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
64             If[HypergeometricFunctionQ[Head[expn]],
65               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
66             If[AppellFunctionQ[Head[expn]],
67               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
68             If[Head[expn]===RootSum,
69               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
70             If[Head[expn]===Integrate || Head[expn]===Int,
71               Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
72             9]]]]]]]]]]
73
74
75 ElementaryFunctionQ[func_] :=
76   MemberQ[{
77     Exp,Log,
78     Sin,Cos,Tan,Cot,Sec,Csc,
79     ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
80     Sinh,Cosh,Tanh,Coth,Sech,Csch,
81     ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
82   },func]
83
84
85 SpecialFunctionQ[func_] :=
86   MemberQ[{
87     Erf, Erfc, Erfi,
88     FresnelS, FresnelC,
89     ExpIntegralE, ExpIntegralEi, LogIntegral,
90     SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
91     Gamma, LogGamma, PolyGamma,
92     Zeta, PolyLog, ProductLog,
93     EllipticF, EllipticE, EllipticPi
94   },func]
95
96
97 HypergeometricFunctionQ[func_] :=
98   MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
99
100

```

```

101 AppellFunctionQ[func_] :=
102   MemberQ[{AppellF1},func]

```

4.0.2 Maple grading function

```

1 # File: GradeAntiderivative.mpl
2 # Original version thanks to Albert Rich emailed on 03/21/2017
3
4 #Nasser 03/22/2017 Use Maple leaf count instead since buildin
5 #Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
6 #Nasser 03/24/2017 corrected the check for complex result
7 #Nasser 10/27/2017 check for leafsize and do not call ExpnType()
8 # if leaf size is "too large". Set at 500,000
9 #Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
10 # see problem 156, file Apostol_Problems
11
12 GradeAntiderivative := proc(result,optimal)
13 local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
14     debug:=false;
15
16     leaf_count_result:=leafcount(result);
17     #do NOT call ExpnType() if leaf size is too large. Recursion problem
18     if leaf_count_result > 500000 then
19         return "B";
20     fi;
21
22     leaf_count_optimal:=leafcount(optimal);
23
24     ExpnType_result:=ExpnType(result);
25     ExpnType_optimal:=ExpnType(optimal);
26
27     if debug then
28         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
29             ExpnType_optimal);
30     fi;
31
32 # If result and optimal are mathematical expressions,
33 # GradeAntiderivative[result,optimal] returns
34 # "F" if the result fails to integrate an expression that
35 # is integrable
36 # "C" if result involves higher level functions than necessary
37 # "B" if result is more than twice the size of the optimal
38 # antiderivative
39 # "A" if result can be considered optimal
40
41 #This check below actually is not needed, since I only
42 #call this grading only for passed integrals. i.e. I check
43 #for "F" before calling this. But no harm of keeping it here.
44 #just in case.
45
46 if not type(result,freeof('int')) then
47     return "F";
48 end if;
49
50 if ExpnType_result<=ExpnType_optimal then
51     if debug then
52         print("ExpnType_result<=ExpnType_optimal");
53     fi;
54     if is_contains_complex(result) then
55         if is_contains_complex(optimal) then
56             if debug then

```

```

57         print("both result and optimal complex");
58         fi;
59         #both result and optimal complex
60         if leaf_count_result<=2*leaf_count_optimal then
61             return "A";
62         else
63             return "B";
64         end if
65     else #result contains complex but optimal is not
66         if debug then
67             print("result contains complex but optimal is not");
68         fi;
69         return "C";
70     end if
71 else # result do not contain complex
72     # this assumes optimal do not as well
73     if debug then
74         print("result do not contain complex, this assumes optimal do
not as well");
75     fi;
76     if leaf_count_result<=2*leaf_count_optimal then
77         if debug then
78             print("leaf_count_result<=2*leaf_count_optimal");
79         fi;
80         return "A";
81     else
82         if debug then
83             print("leaf_count_result>2*leaf_count_optimal");
84         fi;
85         return "B";
86     end if
87 end if
88 else #ExpnType(result) > ExpnType(optimal)
89     if debug then
90         print("ExpnType(result) > ExpnType(optimal)");
91     fi;
92     return "C";
93 end if
94
95 end proc:
96
97 #
98 # is_contains_complex(result)
99 # takes expressions and returns true if it contains "I" else false
100 #
101 #Nasser 032417
102 is_contains_complex:= proc(expression)
103     return (has(expression,I));
104 end proc:
105
106 # The following summarizes the type number assigned an expression
107 # based on the functions it involves
108 # 1 = rational function
109 # 2 = algebraic function
110 # 3 = elementary function
111 # 4 = special function
112 # 5 = hyperpergeometric function
113 # 6 = appell function
114 # 7 = rootsum function
115 # 8 = integrate function
116 # 9 = unknown function
117
118 ExpnType := proc(expn)

```

```

119   if type(expn,'atomic') then
120     1
121   elif type(expn,'list') then
122     apply(max,map(ExpnType,expn))
123   elif type(expn,'sqrt') then
124     if type(op(1,expn),'rational') then
125       1
126     else
127       max(2,ExpnType(op(1,expn)))
128     end if
129   elif type(expn,'`^`') then
130     if type(op(2,expn),'integer') then
131       ExpnType(op(1,expn))
132     elif type(op(2,expn),'rational') then
133       if type(op(1,expn),'rational') then
134         1
135       else
136         max(2,ExpnType(op(1,expn)))
137       end if
138     else
139       max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
140     end if
141   elif type(expn,'`+`') or type(expn,'`*`') then
142     max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
143   elif ElementaryFunctionQ(op(0,expn)) then
144     max(3,ExpnType(op(1,expn)))
145   elif SpecialFunctionQ(op(0,expn)) then
146     max(4,apply(max,map(ExpnType,[op(expn)])))
147   elif HypergeometricFunctionQ(op(0,expn)) then
148     max(5,apply(max,map(ExpnType,[op(expn)])))
149   elif AppellFunctionQ(op(0,expn)) then
150     max(6,apply(max,map(ExpnType,[op(expn)])))
151   elif op(0,expn)='int' then
152     max(8,apply(max,map(ExpnType,[op(expn)]))) else
153     9
154   end if
155 end proc:
156
157
158 ElementaryFunctionQ := proc(func)
159   member(func,[
160     exp,log,ln,
161     sin,cos,tan,cot,sec,csc,
162     arcsin,arccos,arctan,arccot,arcsec,arccsc,
163     sinh,cosh,tanh,coth,sech,csch,
164     arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
165 end proc:
166
167 SpecialFunctionQ := proc(func)
168   member(func,[
169     erf,erfc,erfi,
170     FresnelS,FresnelC,
171     Ei,Ei,Li,Si,Ci,Shi,Chi,
172     GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
173     EllipticF,EllipticE,EllipticPi])
174 end proc:
175
176 HypergeometricFunctionQ := proc(func)
177   member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
178 end proc:
179
180 AppellFunctionQ := proc(func)
181   member(func,[AppellF1])

```

```

182 end proc:
183
184 # u is a sum or product. rest(u) returns all but the
185 # first term or factor of u.
186 rest := proc(u) local v;
187     if nops(u)=2 then
188         op(2,u)
189     else
190         apply(op(0,u),op(2..nops(u),u))
191     end if
192 end proc:
193
194 #leafcount(u) returns the number of nodes in u.
195 #Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
196 leafcount := proc(u)
197     MmaTranslator[Mma][LeafCount](u);
198 end proc:

```

4.0.3 Sympy grading function

```

1 #Dec 24, 2019. Nasser M. Abbasi:
2 #     Port of original Maple grading function by
3 #     Albert Rich to use with Sympy/Python
4 #Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
5 #     added 'exp_polar'
6 from sympy import *
7
8 def leaf_count(expr):
9     #sympy do not have leaf count function. This is approximation
10    return round(1.7*count_ops(expr))
11
12 def is_sqrt(expr):
13     if isinstance(expr,Pow):
14         if expr.args[1] == Rational(1,2):
15             return True
16         else:
17             return False
18     else:
19         return False
20
21 def is_elementary_function(func):
22     return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
23                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
24                    asinh,acosh,atanh,acoth,asech,acsch
25                    ]
26
27 def is_special_function(func):
28     return func in [ erf,erfc,erfi,
29                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
30                    gamma,loggamma,digamma,zeta,polylog,LambertW,
31                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
32                    ]
33
34 def is_hypergeometric_function(func):
35     return func in [hyper]
36
37 def is_appell_function(func):
38     return func in [appellf1]
39
40 def is_atom(expn):
41     try:
42         if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
43             return True

```

```

44     else:
45         return False
46
47     except AttributeError as error:
48         return False
49
50 def expnType(expn):
51     debug=False
52     if debug:
53         print("expn=",expn,"type(expn)=",type(expn))
54
55     if is_atom(expn):
56         return 1
57     elif isinstance(expn,list):
58         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
59     elif is_sqrt(expn):
60         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
61             return 1
62         else:
63             return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
64     elif isinstance(expn,Pow): #type(expn,``^`)
65         if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
66             return expnType(expn.args[0]) #ExpnType(op(1,expn))
67         elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
68             if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
69                 return 1
70             else:
71                 return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn
72 )))
73     else:
74         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
75 ExpnType(op(1,expn)),ExpnType(op(2,expn)))
76     elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,``+`) or
77 type(expn,``*`)
78     m1 = expnType(expn.args[0])
79     m2 = expnType(list(expn.args[1:]))
80     return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
81     elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
82     return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
83     elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
84     m1 = max(map(expnType, list(expn.args)))
85     return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
86     elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
87 expn))
88     m1 = max(map(expnType, list(expn.args)))
89     return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
90     elif is_appell_function(expn.func):
91     m1 = max(map(expnType, list(expn.args)))
92     return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
93     elif isinstance(expn,RootSum):
94     m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
95 ,Apply[List,expn]],7]],
96     return max(7,m1)
97     elif str(expn).find("Integral") != -1:
98     m1 = max(map(expnType, list(expn.args)))
99     return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
100     else:
101     return 9
102
103 #main function
104 def grade_antiderivative(result,optimal):
105
106     leaf_count_result = leaf_count(result)

```

```

102 leaf_count_optimal = leaf_count(optimal)
103
104 expnType_result = expnType(result)
105 expnType_optimal = expnType(optimal)
106
107 if str(result).find("Integral") != -1:
108     return "F"
109
110 if expnType_result <= expnType_optimal:
111     if result.has(I):
112         if optimal.has(I): #both result and optimal complex
113             if leaf_count_result <= 2*leaf_count_optimal:
114                 return "A"
115             else:
116                 return "B"
117         else: #result contains complex but optimal is not
118             return "C"
119     else: # result do not contain complex, this assumes optimal do not as
well
120         if leaf_count_result <= 2*leaf_count_optimal:
121             return "A"
122         else:
123             return "B"
124 else:
125     return "C"

```

4.0.4 SageMath grading function

```

1 #Dec 24, 2019. Nasser: Ported original Maple grading function by
2 #     Albert Rich to use with Sagemath. This is used to
3 #     grade Fracas, Giac and Maxima results.
4 #Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
5 #     'arctan2','floor','abs','log_integral'
6
7 from sage.all import *
8 from sage.symbolic.operators import add_vararg, mul_vararg
9
10 def tree(expr):
11     debug=False;
12     if debug:
13         print ("Enter tree(expr), expr=",expr)
14         print ("expr.operator()=",expr.operator())
15         print ("expr.operands()=",expr.operands())
16         print ("map(tree, expr.operands()=",map(tree, expr.operands()))
17
18     if expr.operator() is None:
19         return expr
20     else:
21         return [expr.operator()+list(map(tree, expr.operands()))
22
23 def leaf_count(anti):
24     debug=False;
25
26     if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))
27
28     if len(anti) == 0: #special check for optimal being 0 for some test cases.
29         if debug: print ("len(anti) == 0")
30         return 1
31     else:
32         if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
(flatten(tree(anti))))
33         return round(1.35*len(flatten(tree(anti)))) #fudge factor
34             #since this estimate of leaf count is bit lower than

```



```

35         #what it should be compared to Mathematica's
36
37 def is_sqrt(expr):
38     debug=False;
39     if expr.operator() == operator.pow: #isinstance(expr,Pow):
40         if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
41             if debug: print ("expr is sqrt")
42             return True
43         else:
44             return False
45     else:
46         return False
47
48 def is_elementary_function(func):
49     debug = False
50
51     m = func.name() in ['exp','log','ln',
52         'sin','cos','tan','cot','sec','csc',
53         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
54         'sinh','cosh','tanh','coth','sech','csch',
55         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
56         'arctan2','floor','abs'
57     ]
58     if debug:
59         if m:
60             print ("func ", func , " is elementary_function")
61         else:
62             print ("func ", func , " is NOT elementary_function")
63
64
65     return m
66
67 def is_special_function(func):
68     debug = False
69
70     if debug: print ("type(func)=", type(func))
71
72     m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
73         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
74     sinh_integral'
75         'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
76         'polylog','lambert_w','elliptic_f','elliptic_e',
77         'elliptic_pi','exp_integral_e','log_integral']
78
79     if debug:
80         print ("m=",m)
81         if m:
82             print ("func ", func , " is special_function")
83         else:
84             print ("func ", func , " is NOT special_function")
85
86     return m
87
88
89 def is_hypergeometric_function(func):
90     return func.name() in ['hypergeometric','hypergeometric_M','
91     hypergeometric_U']
92
93 def is_appell_function(func):
94     return func.name() in ['hypergeometric'] #[appellf1] can't find this in
95     sagemath

```

```

95 def is_atom(expn):
96
97     #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
sagemath-equivalent-to-atomic-type-in-maple/
98     try:
99         if expn.parent() is SR:
100             return expn.operator() is None
101         if expn.parent() in (ZZ, QQ, AA, QQbar):
102             return expn in expn.parent() # Should always return True
103         if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
:
104             return expn in expn.parent().base_ring() or expn in expn.parent().
gens()
105         return False
106
107     except AttributeError as error:
108         return False
109
110
111 def expnType(expn):
112     debug=False
113
114     if debug:
115         print(">>>>Enter expnType, expn=", expn)
116         print(">>>>is_atom(expn)=", is_atom(expn))
117
118     if is_atom(expn):
119         return 1
120     elif type(expn)==list: #isinstance(expn,list):
121         return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
122     elif is_sqrt(expn):
123         if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
Rational):
124             return 1
125         else:
126             return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.
args[0]))
127     elif expn.operator() == operator.pow: #isinstance(expn,Pow)
128         if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer
)
129             return expnType(expn.operands()[0]) #expnType(expn.args[0])
130         elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],
Rational)
131             if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
Rational)
132                 return 1
133             else:
134                 return max(2,expnType(expn.operands()[0])) #max(2,expnType(
expn.args[0]))
135         else:
136             return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
137     elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
isinstance(expn,Add) or isinstance(expn,Mul)
138         m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
139         m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
140         return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn)))
141     elif is_elementary_function(expn.operator()): #is_elementary_function(expn
.func)
142         return max(3,expnType(expn.operands()[0]))
143     elif is_special_function(expn.operator()): #is_special_function(expn.func)
144         m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(
expn.args)))

```

```

145     return max(4,m1)    #max(4,m1)
146     elif is_hypergeometric_function(expn.operator()): #
is_hypergeometric_function(expn.func)
147         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
148         return max(5,m1)    #max(5,m1)
149     elif is_appell_function(expn.operator()):
150         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
151         return max(6,m1)    #max(6,m1)
152     elif str(expn).find("Integral") != -1: #this will never happen, since it
153         #is checked before calling the grading function that is passed.
154         #but kept it here.
155         m1 = max(map(expnType, expn.operands()))    #max(map(expnType, list(
expn.args)))
156         return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
157     else:
158         return 9
159
160 #main function
161 def grade_antiderivative(result,optimal):
162     debug = False;
163
164     if debug: print ("Enter grade_antiderivative for sagemath")
165
166     leaf_count_result = leaf_count(result)
167     leaf_count_optimal = leaf_count(optimal)
168
169     if debug: print ("leaf_count_result=", leaf_count_result, "
leaf_count_optimal=",leaf_count_optimal)
170
171
172     expnType_result = expnType(result)
173     expnType_optimal = expnType(optimal)
174
175     if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
expnType_optimal)
176
177     if expnType_result <= expnType_optimal:
178         if result.has(I):
179             if optimal.has(I): #both result and optimal complex
180                 if leaf_count_result <= 2*leaf_count_optimal:
181                     return "A"
182                 else:
183                     return "B"
184             else: #result contains complex but optimal is not
185                 return "C"
186         else: # result do not contain complex, this assumes optimal do not as
well
187             if leaf_count_result <= 2*leaf_count_optimal:
188                 return "A"
189             else:
190                 return "B"
191     else:
192         return "C"

```